

First read these instructions carefully:

This test contains 9 exercises. The complete solutions of Exercises 5, 6, 7, 8c, 8d, and 9 including calculations and argumentation, must be accurately written down on the provided separate sheet at the end of this test. For the other exercises you are only required to fill in the final answers on the answer sheet at the end of this test. You must hand in all of your answers and solutions.

The use of electronic devices is not allowed.

1. *This is a final answer question. Provide your final answer (and only your final answer) on the supplied answer sheet.*

Let $A \in \mathbb{R}^{3 \times 3}$ be the invertible matrix given as follows:

$$A = \begin{pmatrix} x & y & 0 \\ 3 & 5 & 0 \\ 1 & z & -1 \end{pmatrix}$$

Determine the values $x, y, z \in \mathbb{R}$ for which $A = A^{-1}$

2. *This is a final answer question. Provide your final answer (and only your final answer) on the supplied answer sheet.*

Decide, for each transformation given below, whether it is one-to-one and whether it is onto.

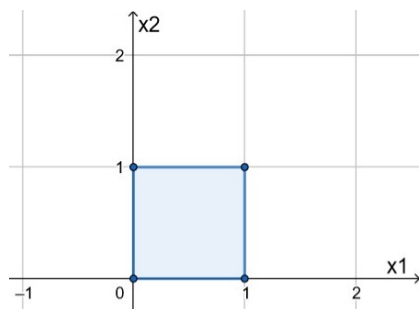
$$S_1 : \mathbb{R}^3 \rightarrow \mathbb{R}^2, S_1 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + 2x_3 \\ -3x_1 + 3x_2 - 6x_3 \end{pmatrix},$$

$$S_2 : \mathbb{R}^3 \rightarrow \mathbb{R}^2, S_2 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \end{pmatrix},$$

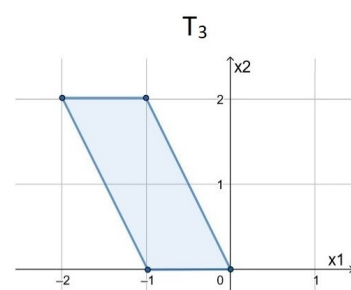
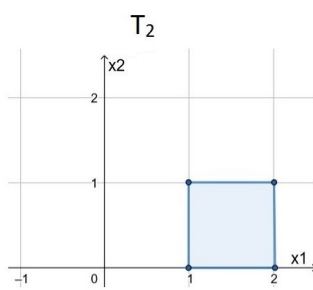
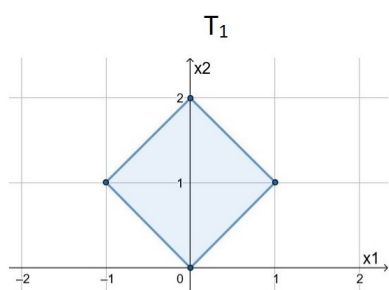
$$S_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ is the linear transformation with representation matrix: } A_3 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 4 & 0 \end{pmatrix},$$

$S_4 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the linear transformation that turns the square with base 1 *unit* and height 1 *unit*, into a parallelogram with base 1 *unit* and height 1 *unit*.

3. This is a final answer question. Provide your final answer (and only your final answer) on the supplied answer sheet.



The square shown above is transformed by three different transformations T_1 , T_2 , and T_3 into the shapes below.



For each transformation T_1 , T_2 , and T_3 :

- Decide whether the transformation is linear or non-linear,
- If the transformation is linear, find the representation matrix.

4. This is a final answer question. Provide your final answer (and only your final answer) on the supplied answer sheet.

The eigenvalues of the matrix $B \in \mathbb{R}^{3 \times 3}$ are given as 1, 3, and 5.

The respective eigenspaces are also given as:

$$E_1 = \text{Span} \left\{ \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right\}, \quad E_3 = \text{Span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}, \quad E_5 = \text{Span} \left\{ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\}.$$

Determine all eigenvalues for B^5 and the respective eigenspace for each eigenvalue.

5. Use the supplied answer sheet and include clear argumentation and calculation.

Consider the planes p_1 and p_2 in $\mathbb{R}^3$:

The plane p_1 is given by :

$$p_1 : x_1 + 3x_2 - 2x_3 + 2 = 0$$

The plane p_2 is given by :

$$p_2 : \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \text{Span} \left\{ \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$$

Find the intersection of the planes p_1 and p_2 . State the intersection in **parametric vector form**.

6. Use the supplied answer sheet and include clear argumentation and calculation.

We are given the following matrices in $\mathbb{R}^{3 \times 3}$

$$A = \begin{pmatrix} 2 & 4 & -1 \\ 1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 3 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & -1 \end{pmatrix}, \quad C = (2B)A^{-1}B^T A$$

Determine $\det C$, without calculating the matrix C .

7. Use the supplied answer sheet and include clear argumentation and calculation.

The following is given as a linearly independent set of vectors in $\mathbb{R}^n$:

$$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}.$$

Show that the set $\{2\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 - \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_3\}$ is also a linearly independent set.

8. Use the supplied answer sheet and include clear argumentation and calculation.

Let $T_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that rotates each point in $\mathbb{R}^2$ about the origin through an angle of $\frac{5\pi}{4}$ radians (counterclockwise), and A_1 be the representation matrix of T_1 .

Let $T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that projects each point in $\mathbb{R}^2$ orthogonally onto the line: $x_2 = x_1$, and A_2 be the representation matrix of T_2 .

Let $F = T_1 \circ T_2$, and B be the representation matrix of F .

- Determine the representation matrix A_1 of T_1 .
- Determine the representation matrix A_2 of T_2 .

c) Determine the representation matrix B of F .

d) Determine all the vectors whose F image is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

If you have no answer for part c, you can take $B = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}$ in part d) even though it is not the correct representation matrix.

9. Use the supplied answer sheet and include clear argumentation and calculation.

The matrix $M \in \mathbb{R}^{3 \times 3}$ is given as follows:

$$M = \begin{pmatrix} -3 & 0 & -1 \\ 1 & 1 & 2 \\ 5 & 0 & 1 \end{pmatrix}$$

Determine the eigenvalues of M together with the corresponding eigenspaces.

For the exercises the following number of points can be obtained:

Exercise 1.	3 pts	Exercise 2.	4 pts	Exercise 3.	3 pts	Exercise 4.	4 pts
Exercise 5.	4 pt	Exercise 6.	3 pt	Exercise 7.	4 pt	Exercise 8a.	1 pt
Exercise 8b.	1 pt	Exercise 8c.	2 pts	Exercise 8d.	2 pts	Exercise 9.	5 pts

The grade is determined by dividing the total number of points by 4 and adding 1.

Answer sheet

Legibly fill in your answers in the corresponding boxes

Linear Algebra_B April 22, 2022

Name:

Student Number:

Programme:

1. The values x, y, z : (Provide only your final answer)

2. Check for each transformation whether it is one-to one and/or onto:
(Provide only your final answer)

S_1	:	one-to-one	()	onto	()
S_2	:	one-to-one	()	onto	()
S_3	:	one-to-one	()	onto	()
S_4	:	one-to-one	()	onto	()

3. Decide whether the transformation is linear or non-linear, and write the representation matrix for the linear transformation(s): (Provide only your final answer)

T_1	:	linear	()	non-linear	()
T_2	:	linear	()	non-linear	()
T_3	:	linear	()	non-linear	()
Representation matrix of T_1 (if linear):					
Representation matrix of T_2 (if linear):					
Representation matrix of T_3 (if linear):					

4. All eigenvalues for B^5 and respective eigenspaces for each eigenvalue:
(Provide only your final answer)

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5. The intersection of the planes p_1 and p_2 in **parametric vector form**:
(Include clear argumentation and calculation)

6. Find $\det C$: (Include clear argumentation and calculation)

7. Show that the given set is linearly independent:
(Include clear argumentation and calculation)

8. a) The representation matrix A_1 of T_1 : (Provide only your final answer)

- b) The representation matrix A_2 of T_2 : (Provide only your final answer)

- c) The representation matrix B of F : (Include clear argumentation and calculation)

- d) The vectors whose F image is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$: (Include clear argumentation and calculation)

9. The eigenvalues of M and the corresponding eigenspaces:
(Include clear argumentation and calculation)

Extra writing space: