

Tag : Toetsen/21-22/I2MC1A.22-23 [v2]
 Course : **Calculus 1A**
 Date : Friday October 28th, 2022
 Time : 13:45 – 15:45

Motivate all your open answers.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Final answer

On the answer form, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

1. [3 pt] Using the definition of the projection of u onto v , show by direct calculation that

$$(u - \text{proj}_v u) \cdot \text{proj}_v u = 0$$

2. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

The three points given $P = (0, 0, 4)$, $Q = (3, 0, 0)$, and $R = (1, 1, 1)$.

- (a) [2 pt] Find an equation of the plane M that contains the points P , Q and R .
 (b) [1 pt] Find an equation of the plane parallel to M , that passes through the origin.

Use the plane $N : z = x + y + 11$ if you have no answer to part (a)

3. [2 pt] We are given a function $f(x)$ which is continuous at the point $x = 2$ and that

$$\lim_{x \rightarrow 2} \frac{2 \cdot f(x) - 8}{x - 2} = 5$$

Find $f(2)$.

4. [2 pt] *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Let the function $g(x)$ defined as below:

$$g(x) = \frac{x^4 + 3x^2 + 2x + 4}{x^3 - 3x^2}$$

For each of the following statements, state whether it is true or false.

- (i) $g(x)$ has an oblique asymptote at $y = x - 3$.
- (ii) $g(x)$ has an oblique asymptote at $y = x + 3$.
- (iii) $g(x)$ has a vertical asymptote at $x = 0$.
- (iv) $g(x)$ has a horizontal asymptote at $y = 0$.

5. Define the function

$$h(x) = x\sqrt{4 - x^2}.$$

- (a) [2 pt] Find all critical points of the above function h .
- (b) [2 pt] Find the absolute extreme values of h on the interval $[-1, 2]$, and specify where the extreme values are attained (that is: specify all x_0 for which $h(x_0)$ is a maximum or minimum value).

6. [2 pt] Evaluate the following limit:

$$\lim_{x \rightarrow \infty} x^2 e^{-x}$$

7. [3 pt] Calculate

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x-y)^2}{x^2 + y^2}.$$

or show that this limit does not exist. Explain all your reasoning.

8. [3 pt] Find an equation for the tangent plane to the graph of the function

$$f(x, y) = \ln(x + y)$$

at the point $(\frac{e}{2}, \frac{e}{2}, 1)$. Simplify the equation as much as possible.

Total: 22 points