

Exam: Vector Calculus for BMT, CE, CSE, ME

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Provide suitable explanations of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

Question 1. Consider the ellipse

$$C: \frac{(x-1)^2}{4} + y^2 = 1.$$

- (a) Find a parametrization of C .
- (b) Calculate the mass of a wire shaped along C with mass density

$$\rho(x, y) = \sqrt{\frac{1}{4}(x-1)^2 + 4y^2}.$$

Question 2. Let the region R be the lower half disk, $x^2 + y^2 < 1$ and $y < 0$, with counterclock-wise oriented boundary curve C . Let

$$\mathbf{F}(x, y) = x^2\mathbf{i} + xy\mathbf{j}.$$

- (a) Calculate the flux of $\mathbf{F}$ across C using a parametrization of C .
- (b) Calculate the flux of $\mathbf{F}$ across C using a double integral.

Question 3. Let $a, b \in \mathbb{R}$ be two parameters. Consider the vector field

$$\mathbf{F}(x, y, z) = (ay + 2xe^{x^2-z^2})\mathbf{i} + (bx)\mathbf{j} + (z - 2ze^{x^2-z^2})\mathbf{k}.$$

- (a) Determine all values of a and b such that $\mathbf{F}$ is conservative.
- (b) Let $a = b = 1$. Determine a potential for $\mathbf{F}$.
- (c) Let $a = b = 1$. Use the potential found in (b) to calculate the work done by $\mathbf{F}$ when moving an object from $A = (2, 0, 2)$ to $B = (4, 1, 4)$.

Question 4. The surface S is the triangular portion of the plane $x + y + z = 1$ lying in the first octant ($x \geq 0, y \geq 0, z \geq 0$). The curve C is the boundary curve of S , oriented clockwise when viewed from above.

- (a) Make a clear sketch of S and C , including the orientation of C .
- (b) Find a parametrization of S .
- (c) Calculate the circulation of $\mathbf{F}(x, y, z) = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$ around C by evaluating a surface integral over S .

Points:

Q1. (a) 1	Q2. (a) 3	Q3. (a) 1	Q4. (a) 1
(b) 2	(b) 2	(b) 1.5	(b) 1
		(c) 1.5	(c) 4

Total: $18 + 2 = 20$ points