

Test: Structural Mechanics in Module 1
Unit of Study code: 2020-000-43

Date: 26/10/2020

Start of the exam: 09:00

End of the exam: 10:45

End of the exam for students with additional time: 11:10

Examiner: ing. G.H.Snellink

GRADING

- ✓ Weight of this test is 50% of the grade for Structural Mechanics.
- ✓ Within this test:
 - Assignment 1: 30%
 - Assignment 2: 70%

INSTRUCTIONS & PRACTICALITIES

- ✓ All Rules of Order for Testing as established by the examination board apply to this test. In case of inconsistencies with other rules set by the examiner, the Rules of Order for Testing prevail.
- ✓ Please read each question carefully before answering the question.
- ✓ All paper work has to be handed in at the end of the test, including scrap paper.
- ✓ For answering questions, use the attached working sheets. If one sheet is not enough, use the sheet with the text "Continuation of ..." in the top right corner. Enter the number of the question where this sheet is the follow-up. Do not make different tasks on the same working sheet.
- ✓ The elaboration of the question must be clear to all (i.e. the teacher). Show what you do, so use a lot of drawings, sketches, or schemas.

RESOURCES ALLOWED TO BE USED DURING THE EXAM:

You are allowed to use the following resources during the exam (i.e. any other resource not list here is not allowed to be used during the exam):

- ✓ (graphical) calculator (not a communication device),
- ✓ pen or/and pencil,
- ✓ eraser,
- ✓ ruler.

Assignment 1. (30 %) Cross-section properties

Given:

You are given the cross-section of a beam(see Figure 1). The cross section is loaded by a moment $M_z=+300\text{kNm}$ and a normal force $N=-2000\text{kN}$

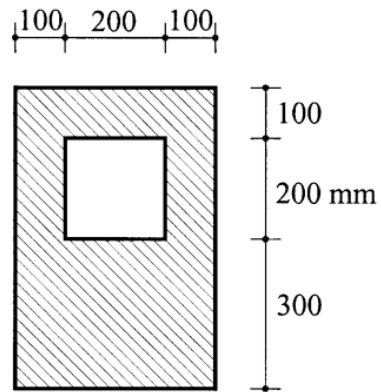


Figure 1

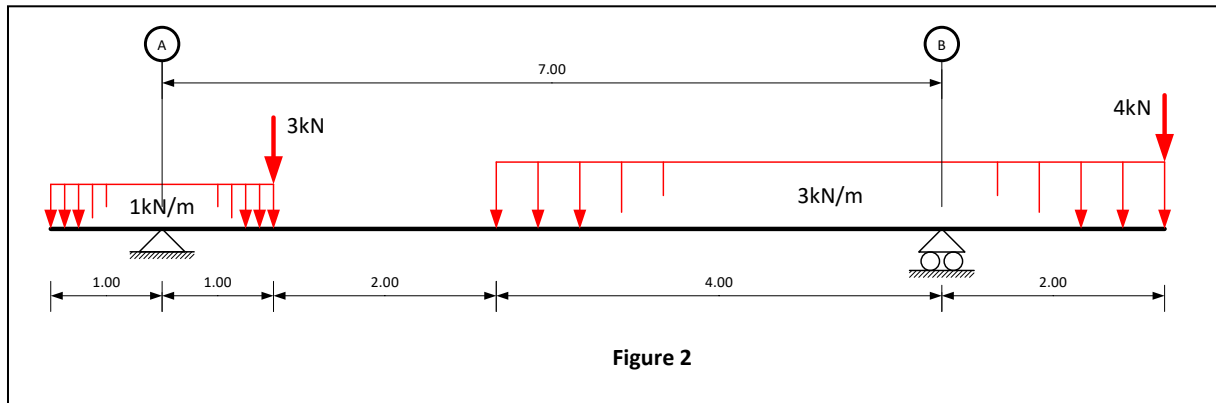
Question:

Determine the extreme stresses in the cross section.

Assignment 2. (70 %) Share force- & Moment- diagram, Displacement.

Given:

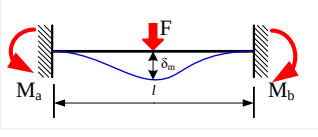
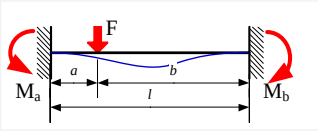
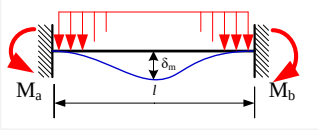
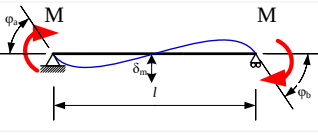
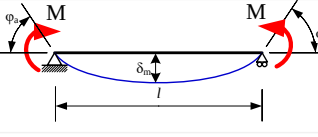
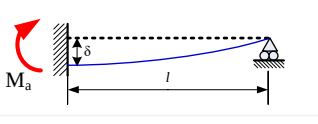
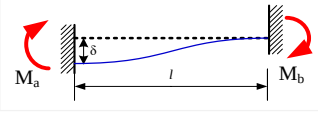
A rigid beam, simply supported at A and B has two overhangs (see figure 2). The bending stiffness of the beam is $EI=20\text{MNm}^2$



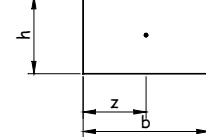
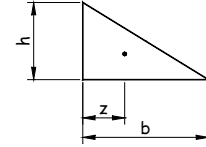
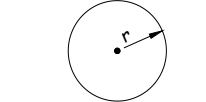
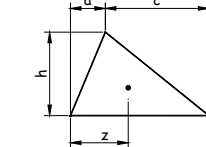
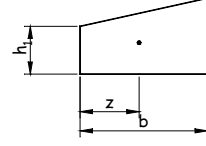
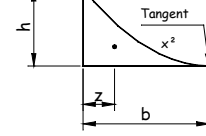
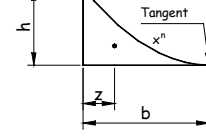
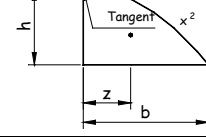
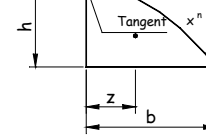
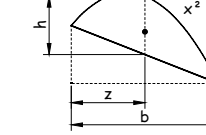
Questions:

- Determine and sketch the V- and M-diagram.
- Determine the deflection in the middle of AB (hint: use results from a).

nr	Mechanical system:	Moment M_a at clamped support M_M at $1/2l$	Angle: φ_a = left side φ_b = right side	Displacement: δ on free end, δ_m at $1/2l$
1		$M_a = M$	$\varphi_b = \frac{Ml}{EI}$	$\delta = \frac{1}{2} \frac{Ml^2}{EI}$
2		$M_a = Fl$	$\varphi_b = \frac{1}{2} \frac{Fl^2}{EI}$	$\delta = \frac{1}{3} \frac{Fl^3}{EI}$
3		$M_a = \frac{1}{2} ql^2$	$\varphi_b = \frac{1}{6} \frac{ql^3}{EI}$	$\delta = \frac{1}{8} \frac{ql^4}{EI}$
4			$\varphi_a = \frac{1}{3} \frac{Ml}{EI}$ $\varphi_b = \frac{1}{6} \frac{Ml}{EI}$	$\delta_m = \frac{1}{16} \frac{Ml^2}{EI}$
5		$M_M = \frac{1}{4} Fl$	$\varphi_a = \varphi_b = \frac{1}{16} \frac{Fl^2}{EI}$	$\delta_m = \frac{1}{48} \frac{Fl^3}{EI}$
6			$\varphi_a = \frac{1}{6} \frac{Fab(l+b)}{EI}$	
7		$M_M = \frac{1}{8} ql^2$	$\varphi_a = \varphi_b = \frac{1}{24} \frac{ql^3}{EI}$	$\delta_m = \frac{5}{384} \frac{ql^4}{EI}$
8		$M_a = \frac{1}{2} M$	$\varphi_b = \frac{1}{4} \frac{Ml}{EI}$	$\delta_m = \frac{1}{32} \frac{Ml^2}{EI}$
9		$M_a = \frac{3}{16} Fl$ $M_M = \frac{5}{32} Fl$	$\varphi_b = \frac{1}{32} \frac{Fl^2}{EI}$	$\delta_m = \frac{7}{768} \frac{Fl^3}{EI}$
10		$M_a = \frac{Fb(l^2 - b^2)}{2l^2}$	$\varphi_b = \frac{1}{4} \frac{Fa^2b}{EI}$	
11		$M_a = \frac{1}{8} ql^2$	$\varphi_b = \frac{1}{48} \frac{ql^3}{EI}$	$\delta_m = \frac{1}{192} \frac{ql^4}{EI}$

nr	Mechanical system:	Moment M_a at clamped support M_M at $\frac{1}{2}l$	Angle: φ_a = left side φ_b = right side	Displacement: δ on free end, δ_m at $\frac{1}{2}l$
12		$M_a = M_b = \frac{1}{8} Fl$ $M_M = \frac{1}{8} Fl$		$\delta_m = \frac{1}{192} \frac{Fl^3}{EI}$
13		$M_a = \frac{Fab^2}{l^2}$ $M_b = \frac{Fa^2b}{l^2}$		
14		$M_a = M_b = \frac{1}{12} ql^2$ $M_M = \frac{1}{24} ql^2$		$\delta_m = \frac{1}{384} \frac{ql^4}{EI}$
15			$\varphi_a = \varphi_b = \frac{1}{6} \frac{Ml}{EI}$	$\delta_m = 0$
16			$\varphi_a = \varphi_b = \frac{1}{2} \frac{Ml}{EI}$	$\delta_m = \frac{1}{8} \frac{Ml^2}{EI}$
17		$M_b = \frac{3EI\delta}{l^2}$		
18		$M_a = M_b = \frac{6EI\delta}{l^2}$		

Area+point of gravity

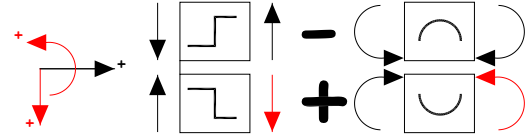
	$A = bh$ $z = \frac{1}{2}b$ $I = \frac{1}{12}bh^3$
	$A = \frac{1}{2}bh$ $z = \frac{1}{3}b$ $I = \frac{1}{36}bh^3$
	$A = \pi r^2$ $I = \frac{1}{4}\pi r^4$
	$A = \frac{1}{2}(a+c)h$ $z = \frac{1}{3}(2a+c)$
	$A = \frac{1}{2}b(h_1+h_2)$ $z = \frac{1}{3}b \frac{h_1+2h_2}{h_1+h_2}$
	$A = \frac{1}{3}bh$ $z = \frac{1}{4}b$
	$A = \frac{1}{n+1}bh$ $z = \frac{1}{n+2}b$
	$A = \frac{2}{3}bh$ $z = \frac{3}{8}b$
	$A = \frac{n}{n+1}bh$ $z = \frac{1(n+1)}{2(n+2)}b$
	$A = \frac{2}{3}bh$ $z = \frac{1}{2}b$

Tangent=parallel to x-axis

Section properties

$A =$	$S_z =$	$I_{zz} =$
$\int_A dA$	$\int_A z dA$	$\int_A z^2 dA$

Sign conventions



Displacement rule of Steiner

$$I_{\bar{z}\bar{z}} = I_{zz(\text{eigen})} + \bar{z}_c^2 A$$

Basic equation for bending

$$Q(x) = -EIw''''$$

$$V(x) = -EIw'''$$

$$M(x) = -EIw''$$

$$EI\varphi(x) = -EIw'$$

$$\Delta\varphi = \varphi_B - \varphi_A = \int_A^B \frac{M}{EI} dx$$

$$\Delta w = w_B - w_A = -\varphi_{A(x_B-x_A)} - \int_A^B \frac{M}{EI} (x_B - x) dx$$

Normal tension

$$\sigma(z) = \sigma_{N(z)} + \sigma_{M(z)} = \frac{N}{A} + \frac{M \cdot z}{I}$$

Extension by axial force

$$EA_x u'_x = N_x = \int -q_x dx$$

$$EA_x u_x = \int N_x dx = \iint -q_x d^2x$$

Prismatic bar & constant force

$$\Delta l = \frac{Nl}{EA}$$

Unit load method

$$\Delta = \sum \frac{F_u \cdot F_l \cdot l}{E \cdot A}$$

Shear stress

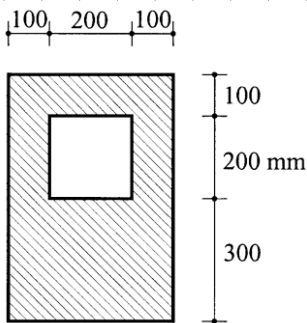
$$s_x^a = -\frac{V_z S_z^a}{I_{zz}} \quad \tau = -\frac{V_z S_z^a}{I_{zz} b^a}$$

Combinend stresses

$$\sigma_{x1} = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi - \tau_{zx} \sin 2\varphi$$

$$\sigma_{z1} = \frac{\sigma_x + \sigma_z}{2} - \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi + \tau_{zx} \sin 2\varphi$$

$$\tau_{x1z1} = \tau_{xz} \cos 2\varphi + \frac{\sigma_x - \sigma_z}{2} \sin 2\varphi$$



Question:

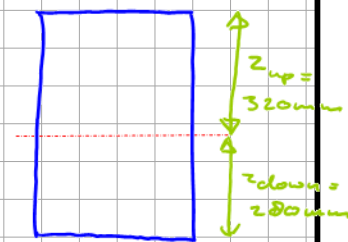
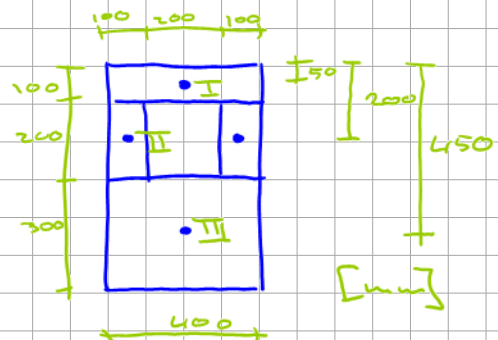
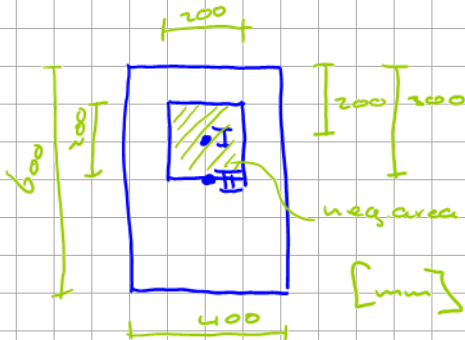
Determine the extreme stresses in the cross section due to a moment of 300 kNm and a Normal force of -2000 kN

Key Activities:

$$\sigma = \frac{N}{A} + \frac{M \cdot z}{I_y}$$

$N = -2000 \text{ kN}$
 $M = +300 \text{ kNm}$ } given
 $\left. \begin{matrix} z \\ A \\ I_y \end{matrix} \right\}$ section properties.

Solving:



Dimensions in dm:

i	b _i	h _i	A _i =b _i ·h _i	z _i	A _i z _i	d _i	A _i d _i ²	1/12 b _i ·h _i ³
1	-2	2	-4	2	-8	-1.2	-5.76	-1.333333
2	4	6	24	3	72	-0.2	0.96	72

Total $\Sigma A = 20$ $\Sigma A z_i = 64$ $\Sigma A d_i^2 = -4.8$ $\Sigma A d_i^2 + \Sigma 1/12 b_i h_i^3 = 65.86667 \text{ dm}^4$
 $d = \frac{\Sigma A z_i}{\Sigma A} = 3.2$
 $d_i = z_i - d$

i	b _i	h _i	A _i =b _i ·h _i	z _i	A _i z _i	d _i	A _i d _i ²	1/12 b _i ·h _i ³
1	4	1	4	0.5	2	-2.7	29.16	0.333333
2	2	2	4	2	8	-1.2	5.76	1.333333
3	4	3	12	4.5	54	1.3	20.28	9

Total $\Sigma A = 20$ $\Sigma A z_i = 64$ $\Sigma A d_i^2 = 55.2$ $\Sigma A d_i^2 + \Sigma 1/12 b_i h_i^3 = 65.86667 \text{ dm}^4$
 $d = \frac{\Sigma A z_i}{\Sigma A} = 3.2$

$$\sigma = \frac{N}{A} + \frac{M \cdot z_{up}}{I} = -\frac{2000 \times 10^3}{20 \times 10^4} + \frac{300 \times 10^6 \times [-320]}{65.87 \times 10^8}$$

$$= -10 - 14.6 = -24.6 \text{ N/mm}^2$$

$$\sigma = \frac{N}{A} + \frac{M \cdot z_{down}}{I} = -\frac{2000 \times 10^3}{20 \times 10^4} + \frac{300 \times 10^6 \times 280}{65.87 \times 10^8}$$

$$= -10 + 12.8 = +2.8 \text{ N/mm}^2$$

Answer

extreme stresses are: -24.6 N/mm^2 and $+2.8 \text{ N/mm}^2$
 (=compression) (=tension)



Question a) Determine and sketch V- & M-diagram

Key Activities

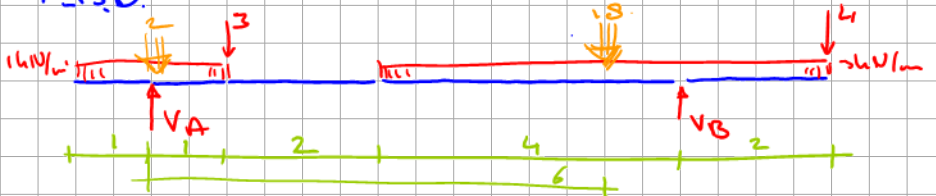
$$\Delta M = A_V$$

$$\Delta V = A_Q$$

$$Q = \text{reaction forces} \Rightarrow \sum V = 0, \sum H = 0, \sum M = 0$$

Solving:

F.B.D.



$$\sum V = 0 \quad 1 \times 2 + 3 + 3 \times 6 + 4 - V_A - V_B = 0$$

$$2 + 3 + 18 + 4 - V_A - V_B = 0$$

$$27 - V_A - V_B = 0$$

$$\sum M_A = 0 \quad 2 \times 0 - 3 \times 1 - 18 \times 6 - 4 \times 9 + 7V_B = 0$$

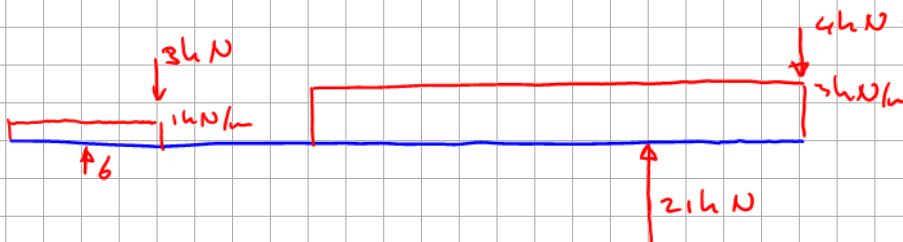
$$0 - 3 - 108 - 36 + 7V_B = 0$$

$$-147 + 7V_B = 0$$

$$V_B = \frac{147}{7} = 21 \text{ kN}$$

$$V_A = 27 - V_B$$

$$= 27 - 21 = 6 \text{ kN}$$

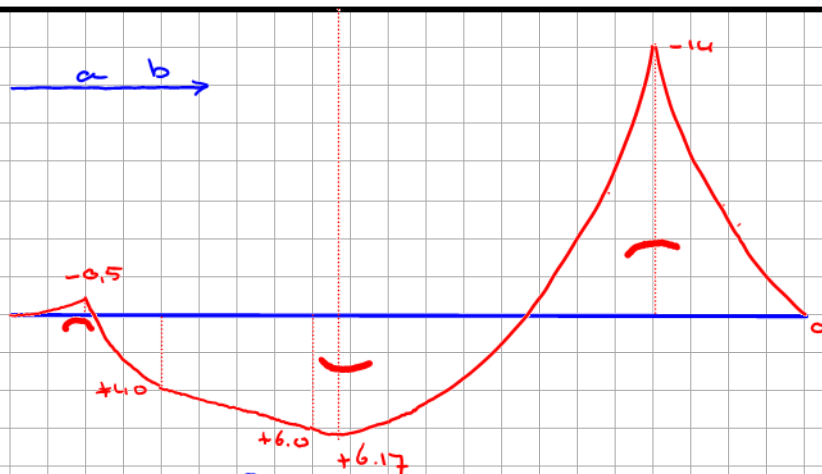


V-diagram [in kN]

$$x = \frac{V}{q} = \frac{1}{2} = 0,533 \text{ m}$$

← c b a

$$\begin{array}{l} 0 + 4 = +4 \text{ kN} \\ 4 + 2 \times 3 = +10 \text{ kN} \\ +10 - 21 = -11 \text{ kN} \\ -11 + 4 \times 3 = +1 \text{ kN} \\ +1 + 3 = +4 \text{ kN} \\ +4 + 1 \times 1 = +5 \text{ kN} \\ +5 - 6 = -1 \text{ kN} \\ -1 + 1 \times 1 = 0 \text{ kN} \end{array}$$



$$\begin{aligned}
 a) & 0 + \frac{1}{2} \times 1 \times -1 = -0.5 \\
 b) & -0.5 + \frac{1}{2} \times (4+5) \times 1 = +4.0 \\
 c) & +4.0 + \frac{1}{2} \times 1 \times 2 = +6.0 \\
 d) & +6.0 + \frac{1}{2} \times 1 \times \frac{1}{2} = +6.17 \\
 e) & +6.17 + \frac{1}{2} \times (-11) \times \frac{3}{2} = -14 \\
 f) & -14 + \frac{1}{2} \times (10+11) \times 2 = 0
 \end{aligned}$$

M-diagram [in kNm]

Answer: see diagrams above.

Question: b): determine deflection in middle of AB

Key Activity: Use Macaulay's method.
- define $M(x) \Rightarrow$

Use Macaulay's method.

- define $M(x)$

$\Rightarrow$ integrate to $EI\phi(x)$ and $-EI\delta(x)$

- Solve unknown parameters with boundary conditions



$$\sum M(x) = 0 \quad \frac{1 \times (x-0)^2}{2} - 6(x-1) + 3(x-2) - \frac{1 \times (x-2)^2}{2} + \frac{3 \times (x-4)^2}{2} + 21(x-8) + 4 \times (x-10) + M(x) = 0$$

Macaulay notation:

$$M(x) = -\frac{1 \times [x-0]^2}{2} + 6[x-1] - 3[x-2] + \frac{1 \times [x-2]^2}{2} - \frac{3 \times [x-4]^2}{2} + 21[x-8] - 4[x-10]$$

$$EI\phi(x) = -\frac{[x-0]^3}{6} + \frac{6[x-1]^2}{2} - \frac{3[x-2]^2}{2} + \frac{[x-2]^3}{6} - \frac{3[x-4]^3}{6} + \frac{21[x-8]^2}{2} - \frac{4[x-10]^2}{2} + C_1$$

$$-EI\delta(x) = -\frac{[x-0]^4}{24} + \frac{6[x-1]^3}{6} - \frac{3[x-2]^3}{6} + \frac{[x-2]^4}{24} - \frac{3[x-4]^4}{24} + \frac{21[x-8]^3}{6} - \frac{4[x-10]^3}{6} + C_1x + C_2$$

boundary conditions:

$$-EI\delta(1) = 0$$

$$-EI\delta(0) = 0$$

Use this sheet for ONE assignment only!

Solving:

$$-EI \delta(1) = \frac{-[x^1]_0^4}{24} + \frac{6[x^0]_1^3}{6} - \frac{3[x^1]_1^3}{6} + \frac{[x^1]_2^4}{24} - \frac{3[x^1]_2^4}{24} + \frac{2[x^0]_3^3}{6} - \frac{4[x^1]_3^3}{6} + C_1 x + C_2$$

$$= -\frac{1}{24} + C_1 + C_2 = 0 \quad (1)$$

$$-EI \delta(8) = \frac{-[x^8]_0^4}{24} + \frac{6[x^7]_1^3}{6} - \frac{3[x^6]_1^3}{6} + \frac{[x^6]_2^4}{24} - \frac{3[x^4]_2^4}{24} + \frac{2[x^0]_3^3}{6} - \frac{4[x^2]_3^3}{6} + C_1 x + C_2$$

$$= -\frac{512}{3} + 343 - 108 + 54 - 32 + 0 + 8C_1 + C_2 = 0$$

$$\frac{259}{3} + 8C_1 + C_2 = 0 \quad (2)$$

$$(1) + (2) \Rightarrow -\frac{1}{24} + C_1 = \frac{259}{3} + 8C_1 \Rightarrow C_1 = -\frac{691}{56} \quad (-12,34)$$

$$(1) \Rightarrow -\frac{1}{24} - \frac{691}{56} + C_2 = 0 \Rightarrow C_2 = \frac{260}{21} \quad (+12,38)$$

$$-EI \delta(4,5) = \frac{-[x^{4,5}]_0^4}{24} + \frac{6[x^{3,5}]_1^3}{6} - \frac{3[x^{2,5}]_1^3}{6} + \frac{[x^{2,5}]_2^4}{24} - \frac{3[x^{0,5}]_2^4}{24} + \frac{2[x^0]_3^3}{6} - \frac{4[x^{1,5}]_3^3}{6} - \frac{691}{56} x + \frac{260}{21}$$

$$= -17,09 + 42,88 - 7,81 + 1,63 - 0,01 - 55,53 + 12,38$$

$$= -23,55$$

$$\delta(4,5) = \frac{-23,55}{-EI} = \frac{23,55}{20\,000} = 0,00118 \text{ m}$$

$$= 1,18 \text{ mm}$$

Answer:

$$\delta(4,5) = +1,18 \text{ mm (is in downward direction)}$$

Name:..... Course: Structural Mechanics in Module 1

Studentnumber:..... Date: 26/10/2020

Test 2:

Continuation of

111

Use this sheet for ONE assignment only!

Name:..... Course: Structural Mechanics in Module 1

Studentnumber:..... Date: 26/10/2020

Test 2:

Continuation of

111

Use this sheet for ONE assignment only!

