

RIVER FLOW PROCESSES

Trial Exam

October 24, 2020

Name: _____

Student Number: _____

- Use a black or blue pen. Do not use a pencil or a red pen.
- You can write your answers below the question on this form. If you need more space use the blank sheet on the left side.
- The final mark for the exam is computed as: $1 + 9 \frac{\text{your score}}{\text{total points}}$
- The numbers in the parentheses next to the questions indicate the number of points to be earned with each question
- Given data for the exam:

Gravitational acceleration:	$g = 9.81 \text{ m/s}^2$
Density of water:	$\rho = 1000 \text{ kg/m}^3$
Density of sediment:	$\rho_s = 2650 \text{ kg/m}^3$
Kinematic viscosity of water:	$\nu = 10^{-6} \text{ m}^2/\text{s}$
von Karman constant:	$\kappa = 0.41$

1. (10 points) Name the following set of equations and provide an explanation of the physical meaning of the groups of terms indicated by the numbers below the equations:

$$\begin{aligned}
 \underbrace{\frac{\partial u}{\partial t}}_1 + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}_2 &= \underbrace{g_x}_3 - \underbrace{\frac{1}{\rho} \frac{\partial P}{\partial x}}_4 + \underbrace{\nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)}_5 \\
 \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= g_y - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
 \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= g_z - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
 \end{aligned}$$

Answer:

2. (15 points) When solving river engineering problems the Reynolds' averaging is applied to the hydrodynamic variables. Explain what the Reynolds' averaging is and why it is needed to solve river engineering problems.

Answer:

3. (30 points) A dam is constructed for electricity production. The dam blocks a river and creates a lake with a surface area of 50 km^2 and an average depth of 30 m. The river is in equilibrium and has an average discharge of $1500 \text{ m}^3/\text{s}$ and a width of 150 m. The bed sediment of the river is relatively uniform and the median grain size is 0.2 mm. The roughness is described with a Chezy coefficient of $50 \text{ m}^{1/2}/\text{s}$. The bed slope is 10^{-4} . Estimate how long it will take for the river to fill the reservoir with sediment reducing its water storage by 50%. You can assume that all sediment coming from the river is captured by the dam reservoir, the river is very wide, and the flow is steady and uniform. The porosity (ratio of volume of voids to total volume of sediment and voids) of the deposited sediment in the reservoir is $\epsilon = 0.4$.

Answer:

4. (20 points) Consider a river with a bottom slope $i_b = 1 \cdot 10^{-4}$ and a depth of 10 m. The sediment has a median diameter of 0.25 mm. Use the following equation that describes the concentration of suspended sediment across the depth with constant sediment diffusivity

$$c(z) = c(z_a) e^{-6Z \frac{z - z_a}{h}}, \quad \text{where} \quad Z = \frac{w_s}{\kappa u_*}$$

to determine at what relative depth, z/h , the suspended sediment concentration, $c(z)$, has dropped by 90% with respect to the reference concentration $c(z_a)$ at a reference level z_a above the bed. You can assume the height of the reference level to be negligible compared to the depth. Assume the flow is steady and uniform under equilibrium conditions in a very wide river.

Answer:

5. (15 points) The figure below shows a modification of the Shields diagram together with data from bankfull flows in rivers. In such flows there is always sediment transport; however, a large number of data is below the Shields curve for $D_*^{3/2} > 100000$, implying that there is no sediment transport. This area of the diagram corresponds to gravel-bed rivers.

Why does the Shield diagram sometimes fail to predict the initiation of sediment motion in gravel-bed rivers?

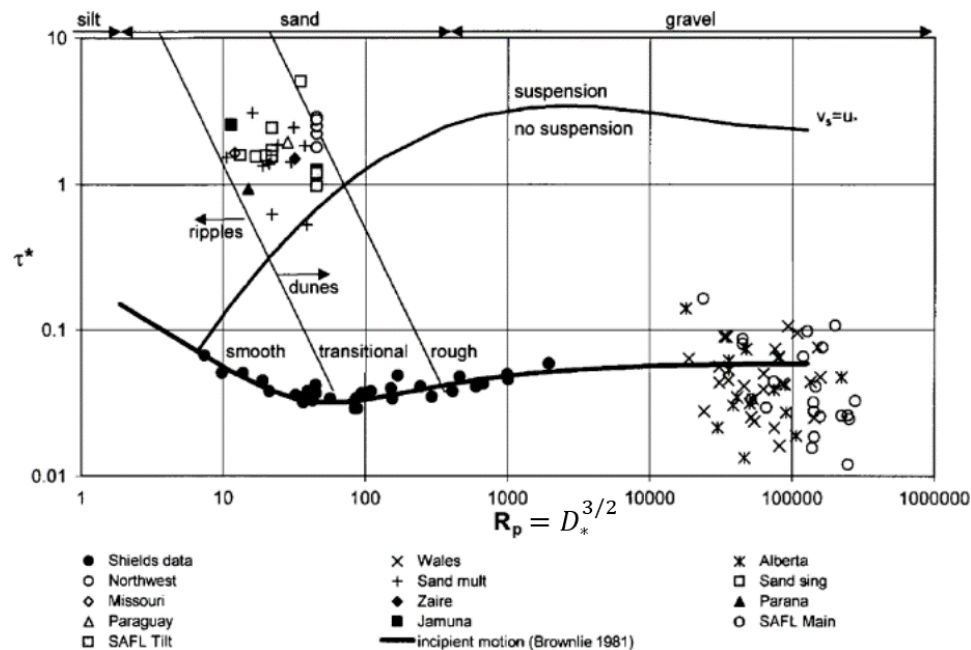


Figure 1: Shields diagram for the initiation of sediment motion with data from bankfull flows (figure adapted from Garcia, 2008)

Answer:

Formula sheet

Submerged specific gravity of the sediment: $R = \frac{\rho_s - \rho}{\rho}$

$$\text{Particle settling velocity: } w_s = \begin{cases} \frac{Rg}{18\nu} D^2, & \text{for } 0.001 < D \leq 0.1 \text{ mm} \\ \frac{10\nu}{D} \left(\sqrt{1 + \frac{0.01RgD^3}{\nu^2}} - 1 \right), & \text{for } 0.1 < D \leq 1 \text{ mm} \\ 1.1\sqrt{RgD}, & \text{for } D > 1 \text{ mm} \end{cases}$$

Hydraulic radius: $R_h = \frac{hW}{2h + W}$

Chezy equation for steady and uniform flow: $U = C\sqrt{R_h i_b}$

Manning's equation for steady and uniform flow: $Q = \frac{1}{n} A R_h^{2/3} i_b^{1/2}$

Manning's coefficient: $n = 0.04 k_s^{1/6}$

Roughness height: $k_s \approx D_{50}$

Logarithmic velocity profile: $\bar{u}(z) = \frac{u_*}{\kappa} \ln \frac{z}{z_0}$

$$\text{Level above the bed where the flow velocity is zero: } z_0 = \begin{cases} 0.11 \frac{\nu}{u_*} + \frac{k_s}{33}, & \text{for } 5 \leq \frac{u_* k_s}{\nu} \leq 70 \\ \frac{k_s}{33}, & \text{for } \frac{u_* k_s}{\nu} > 70 \end{cases}$$

Bed shear stress for steady and uniform flow conditions: $\tau_b = \rho g R_h i$

Shear velocity: $u_* = \sqrt{\frac{\tau_b}{\rho}}$

Shields parameter: $\tau^* = \frac{\tau_b}{g(\rho_s - \rho) D_{50}}$

Particle parameter: $D_* = D_{50} \left[\frac{(\rho_s - \rho) g}{\rho \nu^2} \right]^{1/3}$

Critical Shields parameter from particle parameter: $\tau_{cr}^* = \begin{cases} \frac{0.24}{D_*} & \text{if } 1 < D_* \leq 4 \\ \frac{0.14}{D_*^{0.64}} & \text{if } 4 < D_* \leq 10 \\ \frac{0.04}{D_*^{0.1}} & \text{if } 10 < D_* \leq 20 \\ 0.013 D_*^{0.29} & \text{if } 20 < D_* \leq 150 \\ 0.055 & \text{if } D_* > 150 \end{cases}$

Sediment transport parameter: $\phi = \frac{q_s}{\sqrt{gRD^3}}$

Flow parameter: $\psi = \mu_r \tau^*$

Sediment transport formula of Meyer-Peter and Muller:

$$\phi = 8 (\psi - 0.047)^{3/2}, \quad \text{with} \quad \mu_r = \left(\frac{C}{C_{90}} \right)^{3/2} \quad \text{and} \quad C_{90} = 18 \log \left(\frac{12h}{D_{90}} \right)$$

valid for

- $\mu_r \tau^* < 0.2$
- $D > 0.4 \text{ mm}$
- $w_s/u_* > 1$

Sediment transport formula of Engelund and Hansen:

$$\phi = 0.05 \psi^{5/2}, \quad \text{with} \quad \mu_r = \left(\frac{C^2}{g} \right)^{2/5}$$

valid for

- $w_s/u_* < 1$
- $0.07 < \tau^* < 6$
- $0.19 < D_{50} < 0.93 \text{ mm}$

Suspended sediment concentration across depth with constant sediment diffusivity:

$$c(z) = c(z_a) e^{-6Z \frac{z - z_a}{h}}$$

Rouse parameter: $Z = \frac{w_s}{\kappa u_*}$

List of symbols in the formula sheet

A	area [L^2]
C	Chézy coefficient [$L^{1/2} T^{-1}$]
C_{90}	Chézy coefficient related to D_{90} [$L^{1/2} T^{-1}$]
D	sediment diameter [L]
D_{50}	median sediment diameter [L]
D_x	sediment diameter from which $x\%$ of the sediment mixture by weight is finer [L]
R	submerged specific gravity of the sediment [-]
R_h	hydraulic radius [L]
U	time- and depth-averaged flow velocity for the longitudinal direction [$L T^{-1}$]
Z	Rouse parameter [-]
c	sediment concentration [-]
g	gravitational acceleration [$L T^{-2}$]
h	water depth [L]
i_b	slope [-]
k_s	roughness height [L]
n	Manning coefficient [$T L^{-1/3}$]
q_s	volumetric sediment transport rate per unit width [$L^3 T^{-1} L^{-1}$]
$\bar{u}$	time-averaged flow velocity for the longitudinal direction [$L T^{-1}$]
u_*	shear velocity [$L T^{-1}$]
w_s	settling or fall velocity of a sediment particle [$L T^{-1}$]
z	vertical coordinate [L]
z_0	level above the bed where the flow velocity is zero [L]
z_a	reference level [L]
κ	von Karman constant [-]
μ_r	ripple factor [-]
ν	kinematic viscosity [$L^2 T^{-1}$]
ρ	density of water [$M L^{-3}$]
ρ_s	density of sediment [$M L^{-3}$]
τ_b	bed shear stress [$M L^{-1} T^{-2}$]
τ_{bcr}	critical bed shear stress according to Shields criterion [$M L^{-1} T^{-2}$]
τ^*	dimensionless shear stress or Shields parameter [-]
τ_c^*	critical dimensionless shear stress or critical Shields parameter [-]
ϕ	sediment transport parameter [-]
ψ	flow parameter [-]