

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 12 June, 2026

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2526-Q4-EXAM

Calculus 2

Exam

Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠

Final answer questions

Report all your answers on the answer form.

1. The paraboloid $z = x^2 + y^2$ intersects the plane $z = 2x + 3$ in a curve $\mathcal{C}$. This curve can be parametrized by the vector function [2 pt]

$$\mathbf{r}(t) = \langle 2\cos(t) + 1, y(t), z(t) \rangle$$

for some functions $y(t)$ and $z(t)$, with $t \in [0, 2\pi]$. Determine valid $y(t)$ and $z(t)$.

2. (a) Let $P(x, y)$ be the second-order Taylor polynomial generated by the function [2 pt]

$$f(x, y) = 2e^{xy}$$

at the point $(0, 3)$. Determine the coefficient of x^2 in the expansion of $P(x, y)$.

- (b) Let $B(x, y)$ be the second-order Taylor polynomial generated by the function [1 pt]

$$g(x, y) = 3x^2 + 19xy + y^2 + 16x + 5y + 3$$

at the point $(-4, 7)$. Determine the coefficient of x^2 in the expansion of $B(x, y)$.

3. Consider the function $f(x, y) = x^3 - y^3 - 3x + 12y$.

- (a) Mark with a dot the critical points of $f(x, y)$ on the figure provided in the answer form. [1 pt]
- (b) Label each of the critical points on the figure with "MAX" if $f(x, y)$ attains a local maximum at that point, "MIN" if $f(x, y)$ attains a local minimum at that point, or "SADDLE" if it is a saddle point. [2 pt]

Continues on the following page.

4. Let R be the region in the xy -plane bounded by the lines $x = 0$, $y = 2x - 4$ and $y = x + 1$. [3 pt]
Rewrite

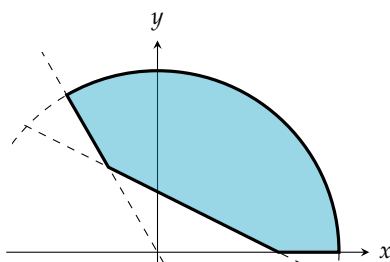
$$\iint_R (x + 1) \, dA$$

as iterated double integrals:

- (a) using the order of integration $dydx$.
(b) using the order of integration $dx dy$.

5. The region D of the plane, shaded in the figure, is bounded by the lines $y = 0$, $y = -\sqrt{3}x$, [3 pt]
and $y = -\frac{1}{2}x + 1$, and by the circle of radius 3 centered at the origin.

Express the area of D as an iterated double integral in polar coordinates, using the order of integration $dr d\theta$. (Do not evaluate the integral.)



6. The following equation in spherical coordinates describes a sphere S :

$$\rho = 4 \sin \theta \sin \phi - 10 \cos \phi.$$

- (a) Determine the radius of S . [1 pt]
(b) Determine the (x, y, z) -coordinates of the center of S . [1 pt]

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

7. Use the method of Lagrange multipliers to find the points closest to and farthest from the origin $(0, 0)$ on the curve described by equation [5 pt]

$$x^2 + y^4 = 1.$$

Clearly indicate which points have the minimum distance and which points have the maximum distance from the origin.

8. Find the absolute maximum and minimum values attained by the function [5 pt]

$$f(x, y) = x^2 + 9y^2 + x + 3y$$

on the closed region bounded by the ellipse of equation $x^2 + 9y^2 = 1$.

Clearly specify at which points these values are attained.

Continues on the following page.

9. The solid object D lies in the first octant and is bounded by the planes $x = 0$ and $y = 0$, by the circular cylinder $x^2 + y^2 = 1$, and by the parabolic cylinders $z = 4 - 4y^2$ and $z = 1 - y^2$. The solid D has constant density $\delta = 5$, its mass is $M = \frac{45}{16}\pi$ and its center of mass is denoted by $(\bar{x}, \bar{y}, \bar{z})$. [5 pt]

Compute $\bar{x}$. Use cylindrical coordinates for your integral.

10. The region R in the first quadrant of the xy -plane is bounded by the curves: [5 pt]

$$xy = \frac{1}{4}, \quad xy = \frac{1}{2}, \quad y = x, \quad y = x + 3.$$

You are given the following transformation of the plane:

$$\begin{cases} u = 4xy \\ v = y - x \end{cases} \quad \text{or, equivalently,} \quad \begin{cases} x = -\frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \\ y = \frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \end{cases} \quad (\text{in the first quadrant}).$$

Use the given transformation to compute

$$\iint_R (x + y) \, dA.$$

Total: 36 pt

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 12 June, 2026

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2526-Q4-EXAM

Calculus 2

Exam

Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠

Final answer questions

Report all your answers on the answer form.

1. The paraboloid $z = x^2 + y^2$ intersects the plane $z = 2y + 3$ in a curve $\mathcal{C}$. This curve can be parametrized by the vector function [2 pt]

$$\mathbf{r}(t) = \langle x(t), 2\sin(t) + 1, z(t) \rangle$$

for some functions $x(t)$ and $z(t)$, with $t \in [0, 2\pi]$. Determine valid $x(t)$ and $z(t)$.

2. (a) Let $P(x, y)$ be the second-order Taylor polynomial generated by the function [2 pt]

$$f(x, y) = 4e^{xy}$$

at the point $(0, 2)$. Determine the coefficient of x^2 in the expansion of $P(x, y)$.

- (b) Let $B(x, y)$ be the second-order Taylor polynomial generated by the function [1 pt]

$$g(x, y) = 4x^2 + 17xy + 2y^2 + 8x + 18y + 6$$

at the point $(-4, 7)$. Determine the coefficient of x^2 in the expansion of $B(x, y)$.

3. Consider the function $f(x, y) = x^3 - y^3 - 12x + 3y$.

- (a) Mark with a dot the critical points of $f(x, y)$ on the figure provided in the answer form. [1 pt]
- (b) Label each of the critical points on the figure with "MAX" if $f(x, y)$ attains a local maximum at that point, "MIN" if $f(x, y)$ attains a local minimum at that point, or "SADDLE" if it is a saddle point. [2 pt]

Continues on the following page.

4. Let R be the region in the xy -plane bounded by the lines $x = 0$, $y = 2x - 6$ and $y = x + 1$. [3 pt]
Rewrite

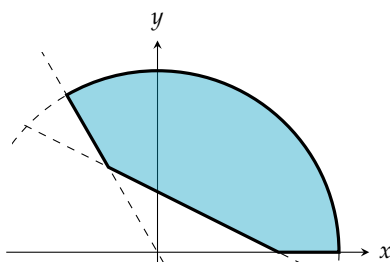
$$\iint_R (x + 1) \, dA$$

as iterated double integrals:

- (a) using the order of integration $dydx$.
(b) using the order of integration $dx dy$.

5. The region D of the plane, shaded in the figure, is bounded by the lines $y = 0$, $y = -\sqrt{3}x$, [3 pt]
and $y = -\frac{1}{2}x + 1$, and by the circle of radius 3 centered at the origin.

Express the area of D as an iterated double integral in polar coordinates, using the order of integration $dr d\theta$. (Do not evaluate the integral.)



6. The following equation in spherical coordinates describes a sphere S :

$$\rho = 10 \cos \theta \sin \phi - 6 \cos \phi.$$

- (a) Determine the radius of S . [1 pt]
(b) Determine the (x, y, z) -coordinates of the center of S . [1 pt]

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

7. Use the method of Lagrange multipliers to find the points closest to and farthest from the origin $(0, 0)$ on the curve described by equation [5 pt]

$$x^2 + y^4 = 1.$$

Clearly indicate which points have the minimum distance and which points have the maximum distance from the origin.

8. Find the absolute maximum and minimum values attained by the function [5 pt]

$$f(x, y) = x^2 + 9y^2 + x + 3y$$

on the closed region bounded by the ellipse of equation $x^2 + 9y^2 = 1$.

Clearly specify at which points these values are attained.

Continues on the following page.

9. The solid object D lies in the first octant and is bounded by the planes $x = 0$ and $y = 0$, by the circular cylinder $x^2 + y^2 = 1$, and by the parabolic cylinders $z = 4 - 4y^2$ and $z = 1 - y^2$. The solid D has constant density $\delta = 5$, its mass is $M = \frac{45}{16}\pi$ and its center of mass is denoted by $(\bar{x}, \bar{y}, \bar{z})$. [5 pt]

Compute $\bar{x}$. Use cylindrical coordinates for your integral.

10. The region R in the first quadrant of the xy -plane is bounded by the curves: [5 pt]

$$xy = \frac{1}{4}, \quad xy = \frac{1}{2}, \quad y = x, \quad y = x + 3.$$

You are given the following transformation of the plane:

$$\begin{cases} u = 4xy \\ v = y - x \end{cases} \quad \text{or, equivalently,} \quad \begin{cases} x = -\frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \\ y = \frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \end{cases} \quad (\text{in the first quadrant}).$$

Use the given transformation to compute

$$\iint_R (x + y) \, dA.$$

Total: 36 pt

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 12 June, 2026

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2526-Q4-EXAM

Calculus 2

Exam

Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠

Final answer questions

Report all your answers on the answer form.

1. The paraboloid $z = x^2 + y^2$ intersects the plane $z = 4x + 5$ in a curve $\mathcal{C}$. This curve can be parametrized by the vector function [2 pt]

$$\mathbf{r}(t) = \langle 3 \cos(t) + 2, y(t), z(t) \rangle$$

for some functions $y(t)$ and $z(t)$, with $t \in [0, 2\pi]$. Determine valid $y(t)$ and $z(t)$.

2. (a) Let $P(x, y)$ be the second-order Taylor polynomial generated by the function [2 pt]

$$f(x, y) = 3e^{xy}$$

at the point $(0, 2)$. Determine the coefficient of x^2 in the expansion of $P(x, y)$.

- (b) Let $B(x, y)$ be the second-order Taylor polynomial generated by the function [1 pt]

$$g(x, y) = 5x^2 + 13xy + y^2 + 15x + 4y + 1$$

at the point $(-4, 7)$. Determine the coefficient of x^2 in the expansion of $B(x, y)$.

3. Consider the function $f(x, y) = x^3 + y^3 - 3x - 12y$.

- (a) Mark with a dot the critical points of $f(x, y)$ on the figure provided in the answer form. [1 pt]
- (b) Label each of the critical points on the figure with "MAX" if $f(x, y)$ attains a local maximum at that point, "MIN" if $f(x, y)$ attains a local minimum at that point, or "SADDLE" if it is a saddle point. [2 pt]

Continues on the following page.

4. Let R be the region in the xy -plane bounded by the lines $x = 0$, $y = 2x - 2$ and $y = x + 6$. [3 pt]
Rewrite

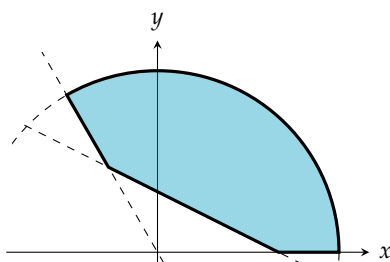
$$\iint_R (x + 6) \, dA$$

as iterated double integrals:

- (a) using the order of integration $dydx$.
(b) using the order of integration $dx dy$.

5. The region D of the plane, shaded in the figure, is bounded by the lines $y = 0$, $y = -\sqrt{3}x$, [3 pt]
and $y = -\frac{1}{2}x + 1$, and by the circle of radius 3 centered at the origin.

Express the area of D as an iterated double integral in polar coordinates, using the order of integration $dr d\theta$. (Do not evaluate the integral.)



6. The following equation in spherical coordinates describes a sphere S :

$$\rho = 6 \sin \theta \sin \phi - 10 \cos \phi.$$

- (a) Determine the radius of S . [1 pt]
(b) Determine the (x, y, z) -coordinates of the center of S . [1 pt]

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

7. Use the method of Lagrange multipliers to find the points closest to and farthest from the origin $(0, 0)$ on the curve described by equation [5 pt]

$$x^2 + y^4 = 1.$$

Clearly indicate which points have the minimum distance and which points have the maximum distance from the origin.

8. Find the absolute maximum and minimum values attained by the function [5 pt]

$$f(x, y) = x^2 + 9y^2 + x + 3y$$

on the closed region bounded by the ellipse of equation $x^2 + 9y^2 = 1$.

Clearly specify at which points these values are attained.

Continues on the following page.

9. The solid object D lies in the first octant and is bounded by the planes $x = 0$ and $y = 0$, by the circular cylinder $x^2 + y^2 = 1$, and by the parabolic cylinders $z = 4 - 4y^2$ and $z = 1 - y^2$. The solid D has constant density $\delta = 5$, its mass is $M = \frac{45}{16}\pi$ and its center of mass is denoted by $(\bar{x}, \bar{y}, \bar{z})$. [5 pt]

Compute $\bar{x}$. Use cylindrical coordinates for your integral.

10. The region R in the first quadrant of the xy -plane is bounded by the curves: [5 pt]

$$xy = \frac{1}{4}, \quad xy = \frac{1}{2}, \quad y = x, \quad y = x + 3.$$

You are given the following transformation of the plane:

$$\begin{cases} u = 4xy \\ v = y - x \end{cases} \quad \text{or, equivalently,} \quad \begin{cases} x = -\frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \\ y = \frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \end{cases} \quad (\text{in the first quadrant}).$$

Use the given transformation to compute

$$\iint_R (x + y) \, dA.$$

Total: 36 pt

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 12 June, 2026

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2526-Q4-EXAM

Calculus 2

Exam

Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠

Final answer questions

Report all your answers on the answer form.

1. The paraboloid $z = x^2 + y^2$ intersects the plane $z = 4y + 5$ in a curve $\mathcal{C}$. This curve can be parametrized by the vector function [2 pt]

$$\mathbf{r}(t) = \langle x(t), 3\sin(t) + 2, z(t) \rangle$$

for some functions $x(t)$ and $z(t)$, with $t \in [0, 2\pi]$. Determine valid $x(t)$ and $z(t)$.

2. (a) Let $P(x, y)$ be the second-order Taylor polynomial generated by the function [2 pt]

$$f(x, y) = 4e^{xy}$$

at the point $(0, 1)$. Determine the coefficient of x^2 in the expansion of $P(x, y)$.

- (b) Let $B(x, y)$ be the second-order Taylor polynomial generated by the function [1 pt]

$$g(x, y) = 6x^2 + 9xy + 19y^2 + 5x + 14y + 2$$

at the point $(-4, 7)$. Determine the coefficient of x^2 in the expansion of $B(x, y)$.

3. Consider the function $f(x, y) = x^3 + y^3 - 12x - 3y$.

- (a) Mark with a dot the critical points of $f(x, y)$ on the figure provided in the answer form. [1 pt]
- (b) Label each of the critical points on the figure with "MAX" if $f(x, y)$ attains a local maximum at that point, "MIN" if $f(x, y)$ attains a local minimum at that point, or "SADDLE" if it is a saddle point. [2 pt]

Continues on the following page.

4. Let R be the region in the xy -plane bounded by the lines $x = 0$, $y = 2x - 2$ and $y = x + 3$. [3 pt]
Rewrite

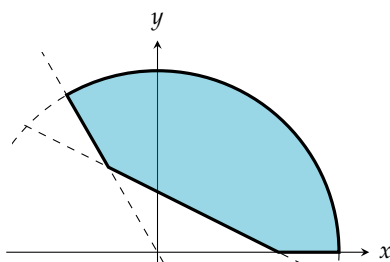
$$\iint_R (x + 3) \, dA$$

as iterated double integrals:

- (a) using the order of integration $dydx$.
(b) using the order of integration $dx dy$.

5. The region D of the plane, shaded in the figure, is bounded by the lines $y = 0$, $y = -\sqrt{3}x$, [3 pt]
and $y = -\frac{1}{2}x + 1$, and by the circle of radius 3 centered at the origin.

Express the area of D as an iterated double integral in polar coordinates, using the order of integration $dr d\theta$. (Do not evaluate the integral.)



6. The following equation in spherical coordinates describes a sphere S :

$$\rho = 10 \cos \theta \sin \phi - 4 \cos \phi.$$

- (a) Determine the radius of S . [1 pt]
(b) Determine the (x, y, z) -coordinates of the center of S . [1 pt]

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

7. Use the method of Lagrange multipliers to find the points closest to and farthest from the origin $(0, 0)$ on the curve described by equation [5 pt]

$$x^2 + y^4 = 1.$$

Clearly indicate which points have the minimum distance and which points have the maximum distance from the origin.

8. Find the absolute maximum and minimum values attained by the function [5 pt]

$$f(x, y) = x^2 + 9y^2 + x + 3y$$

on the closed region bounded by the ellipse of equation $x^2 + 9y^2 = 1$.

Clearly specify at which points these values are attained.

Continues on the following page.

9. The solid object D lies in the first octant and is bounded by the planes $x = 0$ and $y = 0$, by the circular cylinder $x^2 + y^2 = 1$, and by the parabolic cylinders $z = 4 - 4y^2$ and $z = 1 - y^2$. The solid D has constant density $\delta = 5$, its mass is $M = \frac{45}{16}\pi$ and its center of mass is denoted by $(\bar{x}, \bar{y}, \bar{z})$. [5 pt]

Compute $\bar{x}$. Use cylindrical coordinates for your integral.

10. The region R in the first quadrant of the xy -plane is bounded by the curves: [5 pt]

$$xy = \frac{1}{4}, \quad xy = \frac{1}{2}, \quad y = x, \quad y = x + 3.$$

You are given the following transformation of the plane:

$$\begin{cases} u = 4xy \\ v = y - x \end{cases} \quad \text{or, equivalently,} \quad \begin{cases} x = -\frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \\ y = \frac{1}{2}v + \frac{1}{2}\sqrt{u + v^2} \end{cases} \quad (\text{in the first quadrant}).$$

Use the given transformation to compute

$$\iint_R (x + y) \, dA.$$

Total: 36 pt