

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

A

Only for sorting

Answer form

Calculus 2 - Exam - Friday 12 June, 2026

Full name in BLOCK LETTERS

CORRECTION MODEL A

Student number

Programme

☐

BMT

☐

CE

☐

CSE

☐

IEM

☐

Other:

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the components of the parametrization of C .

$$y(t) = 2\sin(t) \quad [\text{or: } -2\sin(t)]$$

$$z(t) = 4\cos(t) + 5$$

Question 2(a). Coefficient of x^2 in $P(x, y)$:

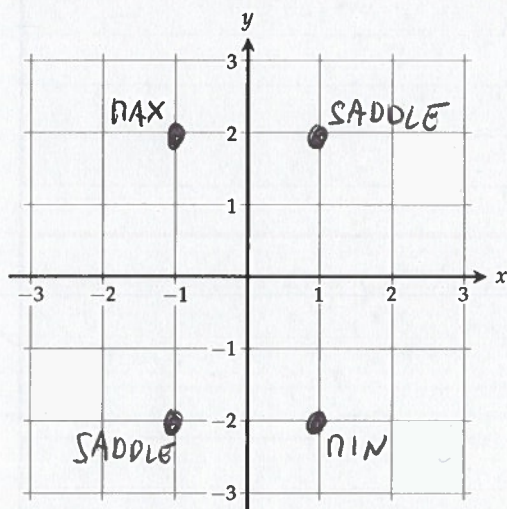
$$9$$

Question 2(b). Coefficient of x^2 in $B(x, y)$:

$$3$$

Question 3 (a) and (b). Mark the critical points and label them.

↓ Use this space for corrections. ↓



Question 4(a). Rewrite the integral using the order of integration $dydx$:

$$\int_0^5 \int_{2x-4}^{x+1} (x+1) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_{-4}^1 \int_0^{y/2+2} (x+1) dx dy + \int_1^6 \int_{y-1}^{y/2+2} (x+1) dx dy$$

$$\int_0^{2\pi/3} \int_{1/(\sin\theta + \frac{1}{2}\cos\theta)}^3 r \, dr \, d\theta$$
 $\sqrt{29}$
$$(0, 2, -5)$$

see version D

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

B

Only for sorting

Answer form

Calculus 2 - Exam - Friday 12 June, 2026

Full name in BLOCK LETTERS CORRECTION MODEL B

Student number _____

Programme

☐

BMT

☐

CE

☐

CSE

☐

IEM

☐

Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the components of the parametrization of C .

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Question 2(a). Coefficient of x^2 in $P(x, y)$:

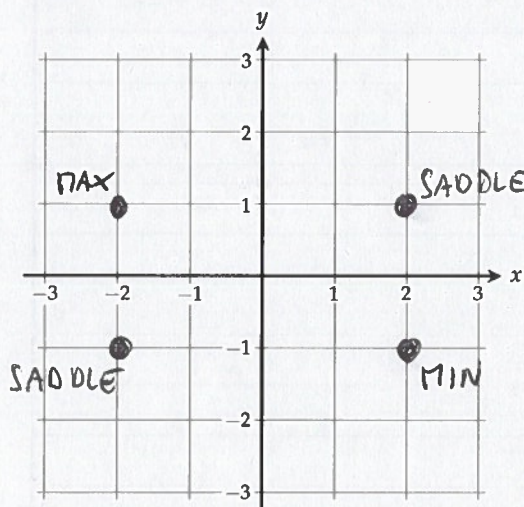
8

Question 2(b). Coefficient of x^2 in $B(x, y)$:

4

Question 3 (a) and (b). Mark the critical points and label them.

↓ Use this space for corrections. ↓



Question 4(a). Rewrite the integral using the order of integration $dydx$:

$$\int_0^7 \int_{2x-6}^{x+1} (x+1) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_{-6}^1 \int_0^{y/2+3} (x+1) dx dy + \int_1^8 \int_{y-1}^{y/2+3} (x+1) dx dy$$

Question 5. The area of D as an iterated double integral in polar coordinates is:

$$\int_0^{2\pi/3} \int_{1/(\sin\theta + \frac{1}{2}\cos\theta)}^3 r \, dr \, d\theta$$

Question 6(a). Radius of S :

$$\sqrt{34}$$

Question 6(b). (x, y, z) -coordinates of the center of S :

$$(5, 0, -3)$$

Question 7. Write a full calculation/argumentation clearly in the box.

See version D

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

C

Only for sorting

Answer form

Calculus 2 – Exam – Friday 12 June, 2026

Full name in BLOCK LETTERS COMPLETION MODEL C

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the components of the parametrization of C .

$$y(t) = 3 \sin(t) \text{ [or: } -3 \sin(t)] \quad z(t) = 12 \cos(t) + 13$$

Question 2(a). Coefficient of x^2 in $P(x, y)$:

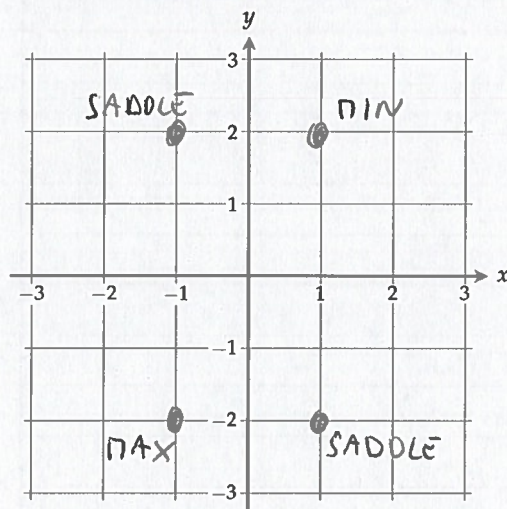
6

Question 2(b). Coefficient of x^2 in $B(x, y)$:

5

Question 3 (a) and (b). Mark the critical points and label them.

↓ Use this space for corrections. ↓



Question 4(a). Rewrite the integral using the order of integration $dy dx$:

$$\int_0^8 \int_{2x-2}^{x+6} (x+6) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_{-2}^6 \int_0^{y/2+1} (x+6) dx dy + \int_6^{14} \int_{y-6}^{y/2+1} (x+6) dx dy$$

Question 5. The area of D as an iterated double integral in polar coordinates is:

$$\int_0^{2\pi/3} \int_{1/(\sin\theta + \frac{1}{2}\cos\theta)}^3 r \, dr \, d\theta$$

Question 6(a). Radius of S :

$$\sqrt{34}$$

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$$(0, 3, -5)$$

Question 7. Write a full calculation/argumentation clearly in the box.

See version D

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

D

Only for sorting

Answer form

Calculus 2 – Exam – Friday 12 June, 2026

Full name in BLOCK LETTERS

CORRECTION MODEL D + OPEN QUESTIONS

Student number

Programme

☐

BMT

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CE

☐

CSE

☐

IEM

☐

Other:

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the components of the parametrization of C .

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Question 2(a). Coefficient of x^2 in $P(x, y)$:

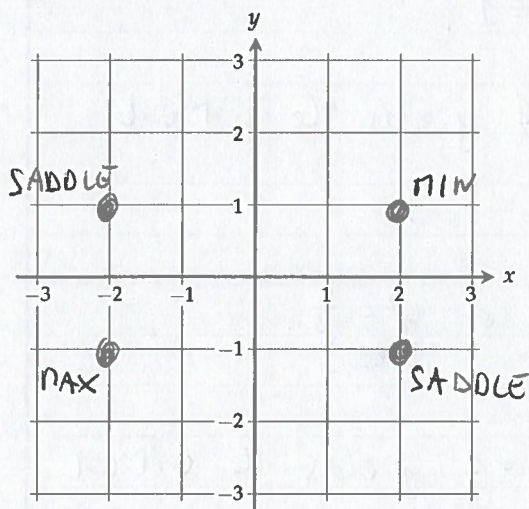
2

Question 2(b). Coefficient of x^2 in $B(x, y)$:

6

Question 3 (a) and (b). Mark the critical points and label them.

↓ Use this space for corrections. ↓



Question 4(a). Rewrite the integral using the order of integration $dydx$:

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$$\sqrt{29}$$

Question 6(b). (x, y, z) -coordinates of the center of S :

$$(5, 0, -2)$$

Question 7. Write a full calculation/argumentation clearly in the box.

Use the square of the distance: $f(x, y) = x^2 + y^2$ on the constraint $g(x, y) = x^2 + y^4 - 1$.

Lagrange system:

$$\begin{cases} f_x(x, y) = \lambda g_x(x, y) \\ f_y(x, y) = \lambda g_y(x, y) \\ g(x, y) = 0 \end{cases} \quad \text{i.e.} \quad \begin{cases} 2x = 2\lambda x \\ 2y = 4\lambda y^3 \\ x^2 + y^4 = 1 \end{cases} \quad \text{i.e.} \quad \begin{cases} x(1 - \lambda) = 0 \\ y(1 - 2\lambda y^2) = 0 \\ x^2 + y^4 = 1 \end{cases}$$

From the 1st equation: $x = 0$ or $\lambda = 1$.

- if $x = 0$, from the 3rd: $0 + y^4 = 1$ yields the critical points $(0, 1)$ and $(0, -1)$
- if $\lambda = 1$: $\begin{cases} \lambda = 1 \\ y(1 - 2y^2) = 0 \\ x^2 + y^4 = 1 \end{cases} \leftarrow y = 0 \text{ or } y^2 = \frac{1}{2}.$
 - if $y = 0$, from the 3rd: $x^2 + 0 = 1$ yields the critical points $(1, 0)$ and $(-1, 0)$.
 - if $y^2 = \frac{1}{2}$, from the 3rd: $x^2 + \frac{1}{4} = 1$ yields the critical points $(\sqrt{3}/2, \sqrt{2}/2), (-\sqrt{3}/2, \sqrt{2}/2), (\sqrt{3}/2, -\sqrt{2}/2), (-\sqrt{3}/2, -\sqrt{2}/2)$.

$f(0, \pm 1) = f(\pm 1, 0) = 1$, $f(\pm \sqrt{3}/2, \pm \sqrt{2}/2) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}$.

So the maximum distance is $\sqrt{5}/2$ attained at $(\pm \sqrt{3}/2, \pm \sqrt{2}/2)$, and the minimum distance is 1 attained at $(\pm 1, 0)$ and $(0, \pm 1)$.

Question 8. Write a full calculation/argumentation clearly in the box.

$$f(x, y) = x^2 + 9y^2 + x + 3y \quad \text{in} \quad x^2 + 9y^2 \leq 1.$$

• INTERIOR: $\nabla f = \langle 0, 0 \rangle$ iff
$$\begin{cases} 2x + 1 = 0 \\ 18y + 3 = 0 \end{cases} \Rightarrow \text{crit. pt. } \left(-\frac{1}{2}, -\frac{1}{6}\right)$$

• BOUNDARY: the ellipse is parametrized by
$$\underline{r}(t) = \left(\cos(t), \frac{1}{3}\sin(t)\right), \quad t \in [0, 2\pi].$$

$$g(t) = f(\underline{r}(t)) = \cos^2(t) + \sin^2(t) + \cos(t) + \sin(t) = \cos(t) + \sin(t) + 1.$$

$$g'(t) = -\sin(t) + \cos(t). \quad g'(t) = 0 \text{ iff } \sin(t) = \cos(t) \\ \text{iff } t = \pi/4 \text{ or } t = 5\pi/4.$$

critical points: $\underline{r}(\pi/4) = (\sqrt{2}/2, \sqrt{2}/6)$
 $\underline{r}(5\pi/4) = (-\sqrt{2}/2, -\sqrt{2}/6).$

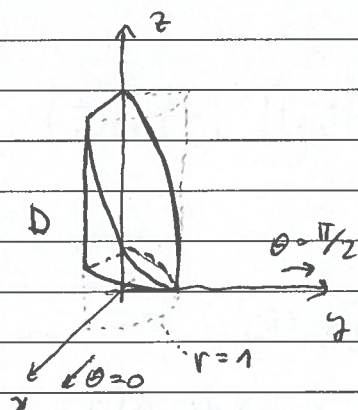
• EVALUATION: $f\left(-\frac{1}{2}, -\frac{1}{6}\right) = \frac{1}{4} + \frac{1}{4} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$

$$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{6}\right) = \frac{1}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 1 + \sqrt{2}$$

$$f\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{6}\right) = \frac{1}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 1 - \sqrt{2}$$

So the maximum is $1 + \sqrt{2}$ attained at $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{6}\right)$, and
the minimum is $-1/2$ attained at $\left(-\frac{1}{2}, -\frac{1}{6}\right)$.

Question 9. Write a full calculation/argumentation clearly in the box.



Cylindrical coordinates:

$$z = 4 - 4y^2 \rightarrow z = 4 - 4r^2 \sin^2 \theta$$

$$z = 1 - y^2 \rightarrow z = 1 - r^2 \sin^2 \theta$$

$$x^2 + y^2 = 1 \rightarrow r = 1$$

$$\bar{x} = \pi y z / M \text{ where } M = \frac{45}{16} \pi.$$

$$\pi y z = \int_0^1 \int_0^{\pi/2} \int_{1-r^2 \sin^2 \theta}^{4-4r^2 \sin^2 \theta} 5 \cdot r \cos \theta \cdot r \, dz \, d\theta \, dr =$$

$$= 5 \int_0^1 \int_0^{\pi/2} 3r^2 \cos \theta (1 - r^2 \sin^2 \theta) \, d\theta \, dr =$$

$$= 15 \int_0^1 \int_0^{\pi/2} (r^2 \cos \theta - r^4 \sin^2 \theta \cos \theta) \, d\theta \, dr =$$

$$= 15 \int_0^1 \left(r^2 (\sin \frac{\pi}{2} - \sin 0) - \frac{1}{3} r^4 (\sin^3 \frac{\pi}{2} - \sin^3 0) \right) dr =$$

$$= 15 \int_0^1 \left(r^2 - \frac{1}{3} r^4 \right) dr = 15 \left(\frac{1}{3} - \frac{1}{15} \right) = 5 - 1 = 4.$$

$$\bar{x} = 4 \cdot \frac{16}{45\pi} = \frac{64}{45\pi}.$$

Question 10. Write a full calculation/argumentation clearly in the box.

• REGION: $xy = \frac{1}{4} \Leftrightarrow 4xy = 1 \Leftrightarrow u = 1$ } $u \in [1, 2]$
 $xy = \frac{1}{2} \Leftrightarrow 4xy = 2 \Leftrightarrow u = 2$ }
 $y = x \Leftrightarrow y - x = 0 \Leftrightarrow v = 0$ } $v \in [0, 3]$
 $y = x + 3 \Leftrightarrow y - x = 3 \Leftrightarrow v = 3$ }

• JACOBIAN: $\left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} \right| =$

$$= \left| \frac{1}{4\sqrt{u+v^2}} \cdot \left(\frac{1}{2} + \frac{v}{2\sqrt{u+v^2}} \right) - \left(-\frac{1}{2} + \frac{v}{2\sqrt{u+v^2}} \right) \cdot \frac{1}{4\sqrt{u+v^2}} \right| =$$

$$= \left| \frac{1}{8\sqrt{u+v^2}} + \frac{v}{8\sqrt{u+v^2}} + \frac{1}{8\sqrt{u+v^2}} - \frac{v}{8\sqrt{u+v^2}} \right| = \frac{1}{4\sqrt{u+v^2}}.$$

• INTEGRAND: $x+y = \sqrt{u+v^2}$

So $\iint_R (x+y) dA = \int_1^2 \int_0^3 \sqrt{u+v^2} \cdot \frac{1}{4\sqrt{u+v^2}} dv du =$

$$= \frac{1}{4} \int_1^2 \int_0^3 dv du = \frac{1}{4} \cdot 3 \cdot 1 = \frac{3}{4}.$$

