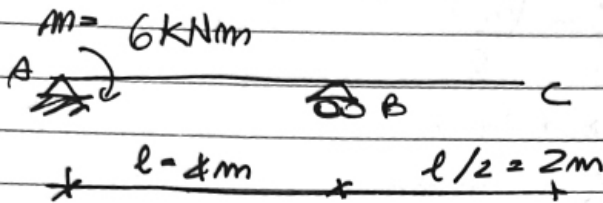


## STRUCTURAL MECHANICS 2 EXAM

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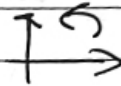
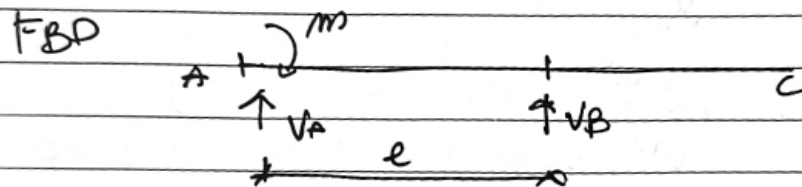
### EXERCISE 1 [25 pt]



$$m = 6 \text{ kNm}$$

$$l = 4 \text{ m}$$

### ② Exercise equilibrium

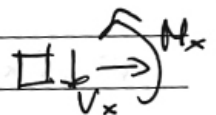
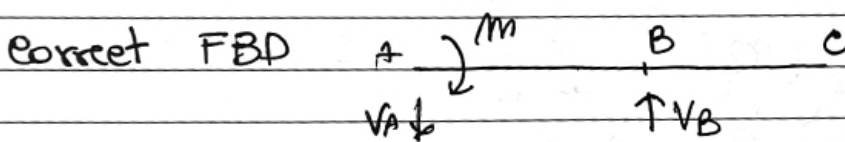


②

$$\sum F_v = V_A + V_B = 0$$

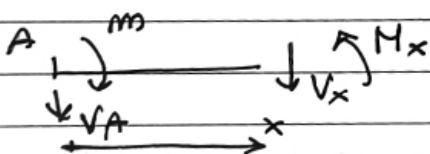
$$\sum M(A) = -m + V_B l = 0 \rightarrow V_B = + m/l = \frac{6 \text{ kNm}}{4 \text{ m}} = 1.5 \text{ kN}$$

$$V_A = -V_B = -1.5 \text{ kN}$$



Equations  $M(x)$ ,  $V(x)$

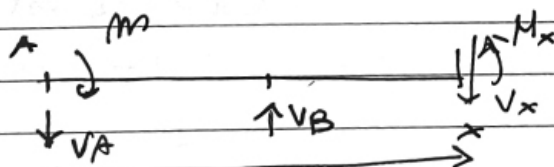
①  $0 \leq x < l$



$$\sum F_v = V_A + V_x = 0 \rightarrow V_x^I = -V_A = -1.5 \text{ kN}$$

$$\sum M_x = -m + M_x + V_A x = 0$$

$$\hookrightarrow M_x^I = m - V_A x$$



$$\cancel{M_x^I} \quad V_x^II = 0$$

$$M_x^II = 0$$

②  $l < x \leq 4l$

④

Curvature:

$$\sigma^{(I)} = \frac{1}{EI} \int M_x^{(I)} dx = \frac{1}{EI} \int (m - V_A x) dx = \frac{1}{EI} \left( mx - V_A \frac{x^2}{2} + C_1 \right) \quad 0 \leq x < l$$

$$\sigma^{(II)} = \frac{1}{EI} \int M_x^{(II)} dx = \frac{1}{EI} \int 0 dx = \frac{1}{EI} C_2 \quad l < x \leq 1.5l \quad (4)$$

Displacement:

$$v^{(I)} = \frac{1}{EI} \int \sigma^{(I)} dx = \frac{1}{EI} \left( \frac{mx^2}{2} - \frac{V_A x^3}{6} + C_1 x + C_3 \right) \quad 0 \leq x < l$$

$$v^{(II)} = \int \sigma^{(II)} dx = \frac{1}{EI} (C_2 x + C_4) \quad l < x \leq 1.5l$$

Boundary conditions :  $v^{(I)}(x=0) = 0$

$v^{(II)}(x=l) = 0$  (4)

$\sigma^{(I)}(x=l) = \sigma^{(II)}(x=l)$

$v^{(I)}(x=l) = v^{(II)}(x=l) = 0$

$$v^{(II)}(x=l) = \frac{1}{EI} (C_2 l + C_4) = 0 \rightarrow C_4 = -C_2 l$$

$$v^{(I)}(x=0) = \frac{1}{EI} \left( \frac{m \cdot 0}{2} + \frac{V_A \cdot 0}{6} + C_1 \cdot 0 + C_3 \right) = 0 \rightarrow \underline{C_3 = 0}$$

$$v^{(I)}(x=l) = 0 \quad \frac{1}{EI} \left( \frac{m l^2}{2} + \frac{m l^2}{6} + C_1 l + 0 \right) = 0$$

$$\underline{C_1 = -\frac{m l}{3}}$$

$\sigma^{(I)}(x=l) = \sigma^{(II)}(x=l)$

$$\frac{1}{EI} \left( m l - \frac{m l^2}{2} + \frac{1}{3} m l \right) = \frac{1}{EI} C_2 \rightarrow \underline{C_2 = +\frac{1}{6} m l} \quad 4$$

$$\rightarrow \underline{C_4 = -\frac{1}{6} m l^2} \quad -46$$

Equations are:

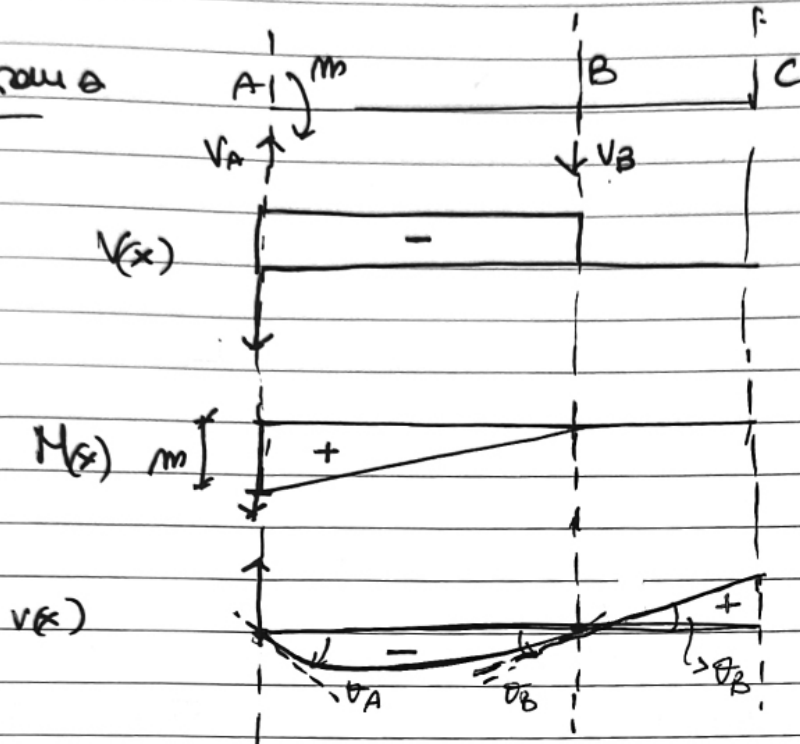
$$\theta^{(I)} = \frac{1}{EI} \left( 6x - \frac{3}{4} \frac{x^2}{2} - \frac{8}{3} ml \right)$$

$$\theta^{(II)} = \frac{4}{EI} \frac{1}{6} ml \quad (2)$$

$$v^{(I)} = \frac{1}{EI} \left( \frac{3}{2} \frac{x^2}{2} - \frac{1}{4} \frac{x^3}{6} - \frac{8}{3} mlx \right)$$

$$v^{(II)} = \frac{1}{EI} \left( \frac{4}{6} \frac{1}{6} mlx - \frac{1}{6} \frac{1}{6} ml^2 \right)$$

② Diagrams



The branch BC is only "carried" by AB because there is no load

③ These can be derived from the equations at point 1, using.

$$\theta^{(I)}(x=0) = -\frac{1}{EI} \frac{ml}{3} = -\frac{1}{EI} 8 \text{ kNm}^2$$

$$v^{(II)}(x=1.5l) = \frac{1}{EI} \left( +\frac{1}{6} ml \times 1.5l - \frac{1}{6} \frac{1}{6} ml^2 \right) = +\frac{1}{12} \frac{ml^2}{EI} = +8 \text{ kNm}^3$$

The same result can also be derived from forget-me-not as AB behaves the same as

$$\theta(x=0) = \frac{1}{3} ml$$

$$v(x=1.5l) \approx \theta_B \frac{l}{2} = \frac{1}{6} \frac{ml}{EI} \times \frac{l}{2} = \frac{1}{12} \frac{ml^2}{EI}$$

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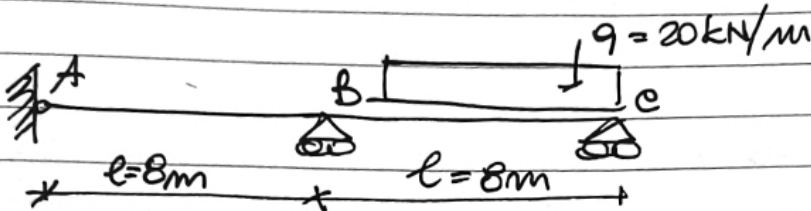
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## EXERCISE 2 [35pt]

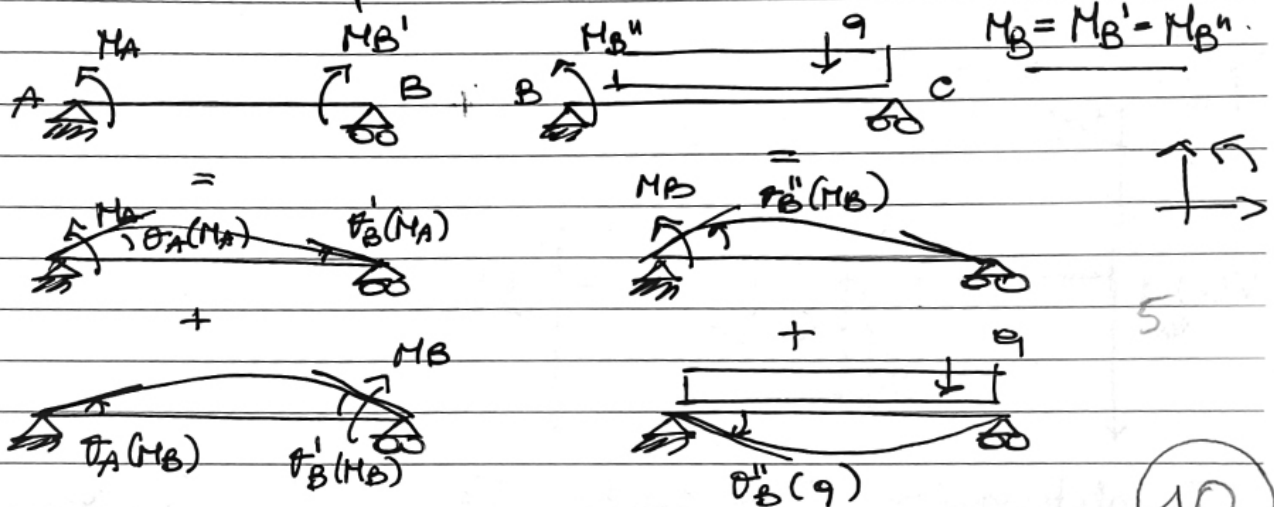
$$q = 20 \text{ kN/m}$$

$$l = 8 \text{ m}$$



- ① The beam is statically indeterminate of 2<sup>nd</sup> degree as I have 4 reaction forces/moments for 2 independent equilibrium equations.

I show one possible solution method:



compatibility equations:

$$\theta_{A, \text{TOT}} = 0 \quad \theta_A(M_A) + \theta_A(M_B) = 0$$

$$\theta_{B, \text{TOT}} = \theta_{C, \text{TOT}} \quad -\theta_B'(M_A) - \theta_B'(M_B) = \theta_B''(M_B) - \theta_B''(q)$$

From forget-me-not

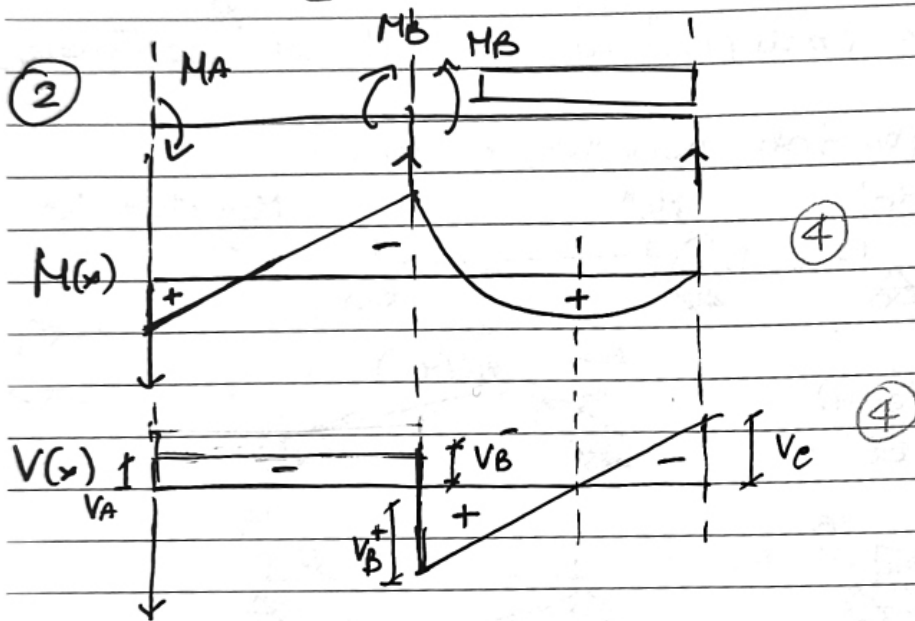
$$\frac{1}{3} \frac{M_A l}{EI} + \frac{1}{6} \frac{M_B l}{EI} = 0 \rightarrow M_A = -\frac{M_B}{2}$$

$$-\frac{1}{6} \frac{M_A l}{EI} - \frac{1}{3} \frac{M_B l}{EI} = \frac{1}{3} \frac{M_B l}{EI} - \frac{q l^3}{24 EI} \quad (5)$$

$$+\frac{1}{12} \frac{M_B l}{EI} - \frac{1}{3} \frac{M_B l}{EI} = \frac{1}{3} \frac{M_B l}{EI} - \frac{q l^3}{24 EI}$$

$$\frac{7}{12} \frac{M_B l}{EI} = -\frac{q l^3}{24 EI} \rightarrow M_B = \frac{q l^2}{14} = \frac{20 \text{ kN/m} \times 8^2 \text{ m}^2}{14} = 91,4 \text{ kNm}$$

$$M_A = -\frac{M_B}{2} = -45,7 \text{ kNm}$$



To determine the forces  $V_A$ ,  $V_B$ ,  $V_C$ , I can isolate the beam parts.

(10)

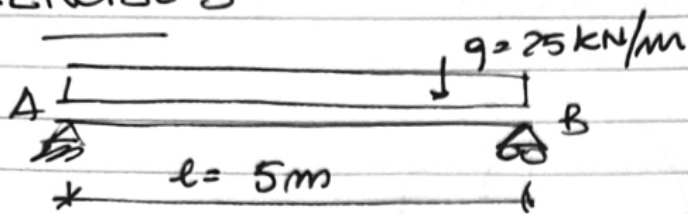
$$V_A = -\frac{M_A}{l} + \frac{M_B}{l} = 5,7 \text{ kN} \uparrow$$

$$V_B = -V_A = 5,7 \text{ kN} \downarrow$$

$$V_B^+ = \frac{q l}{2} + \frac{M_B}{l} = 91,4 \text{ kN} \uparrow$$

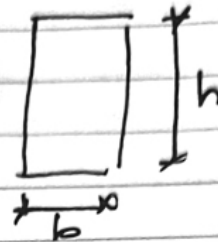
$$V_C = \frac{q l}{2} - \frac{M_B}{l} = 68,6 \text{ kN} \uparrow$$

### EXERCISE 3



$$q = 25 \text{ kN/m}$$

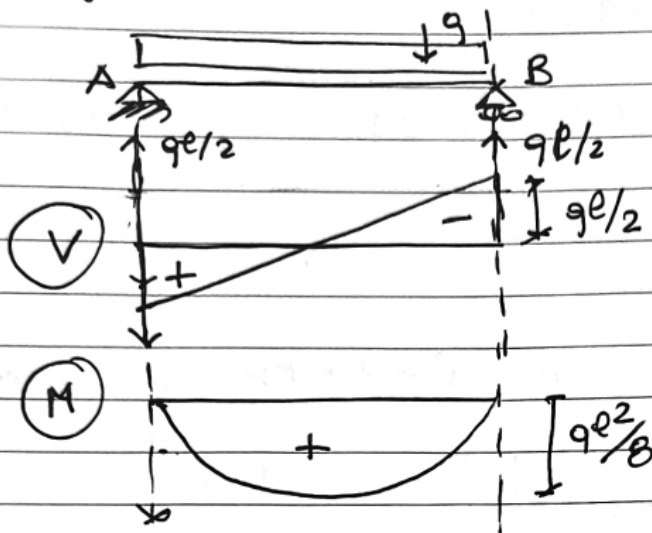
$$l = 5 \text{ m}$$



$$b = 250 \text{ mm}$$

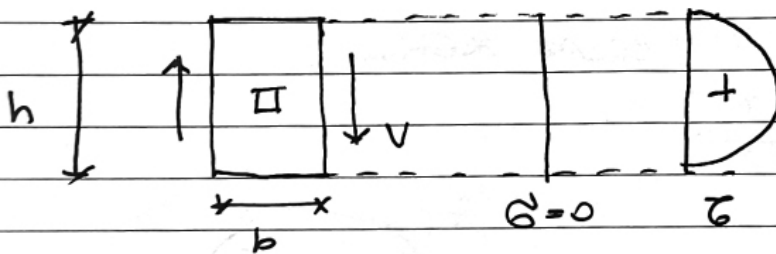
$$h = 400 \text{ mm}$$

a) the diagrams of shear load and bending moment are:



The most critical points are at the supports where  $V = V_{\max}$  and in the middle where  $M = M_{\max}$ .

b) Let's take point A, where  $V = V_{\max}^+ = 62,5 \text{ kN}$



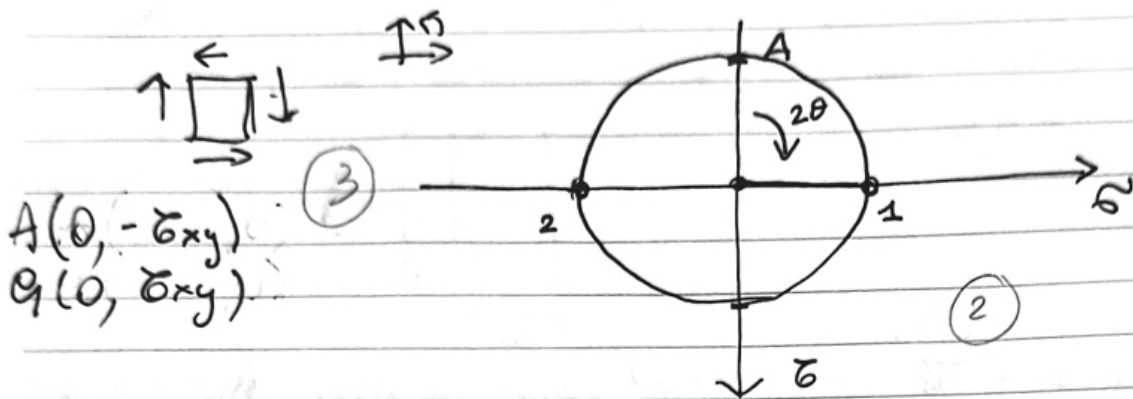
$\tau_{\max}$  in the middle of the cross section:

$$\tau_{\max} = 1,5 V / bh = 1,5 \times 62,5 \text{ kN} / (0,25 \times 0,4) \text{ m}^2 = 937 \text{ kN/m}^2$$

c) Principal stress directions and principal stresses in the middle of cross section.

Since the element is subjected to pure shear, the principal stresses

The material element is subjected to pure shear:



$$A(0, -\tau_{xy})$$

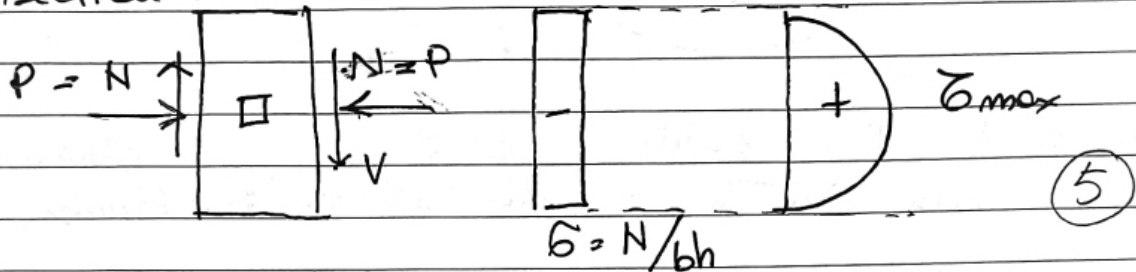
$$C(0, \tau_{xy})$$

$$\sigma_c = 0 \quad R = \tau_{xy} = 625 \text{ kN/m}^2$$

principal stresses:  $\sigma_{1/2} = \sigma_c \pm R = \pm 937 \text{ kN/m}^2$

principal directions:  $\pm 45^\circ$

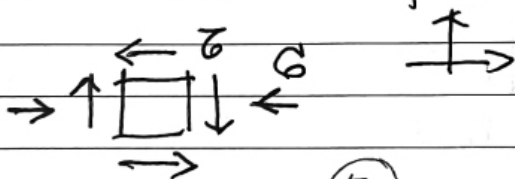
d) a compressional force  $P = 100 \text{ kN}$  added,  $N(x) = P$  cross section in A:



$$\tau_{max} = 937 \text{ kN/m}^2$$

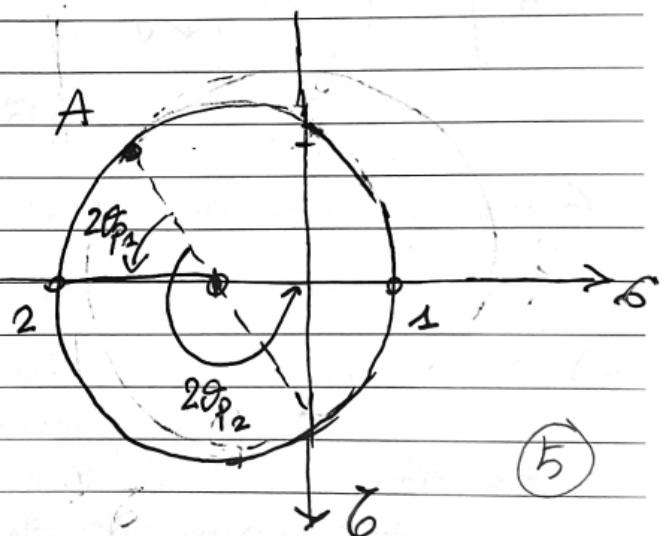
$$\tau = 100 \text{ kN} / (0,25 \times 0,4) \text{ m}^2 = 1000 \text{ kN/m}^2$$

In the middle of the cross section:



$$A(-\sigma, -\tau)$$

$$C(0, +\tau)$$



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$$\sigma_c = \frac{\sigma_x + \sigma_y}{2} = -500 \text{ kN/m}^2$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{1000}{2}\right)^2 + 937^2} = 1062 \text{ kN/m}^2$$

$$\sigma_{1/2} = \sigma_c \pm R = -500 \pm 1062 = +562 \text{ kN/m}^2$$

$$\phantom{\sigma_{1/2} = \sigma_c \pm R = -500 \pm 1062 = } -1562 \text{ kN/m}^2$$

↓ principal stresses

principal directions:

(5)

$$\theta_{p1} = \frac{1}{2} \arctan \left( \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} \right) = \frac{1}{2} \arctan \left( \frac{-937}{-1000/2} \right)$$

$$= \sim 31^\circ$$

$$\theta_{p2} = 31^\circ + 90^\circ = 121^\circ$$

c) case 1: no compression load

$$\sigma_{1/2} = \pm 937 \text{ kN/m}^2$$

$$\theta_{p1/2} = \pm 45^\circ$$

case 2: WITH compression load

$$\sigma_{1/2} = +562 ; -1562 \text{ kN/m}^2$$

$$\theta_{p1/2} = \sim 30^\circ / 120^\circ$$

The presence of compressional load makes the principal stress of compression (-) much higher and makes the principal direction tilt.

(5)