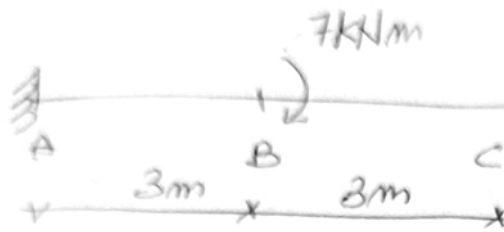


EXERCISE 2 [25 pt]

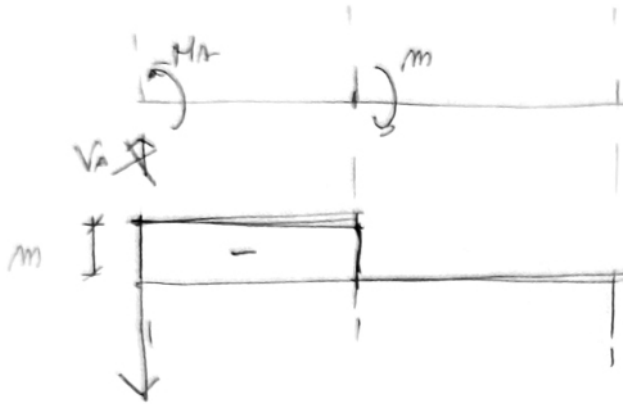
EXAM 4/7/2025



$$l = 3m$$

$$m = 7 \text{ kNm}$$

a)



$$\sum F_V = F_A = 0 \quad (2)$$

$$\sum M_A = M_A - m = 0$$

$$\Rightarrow M_A = m = 7 \text{ kNm}$$

$$M^{(I)} = -m = \text{const} \quad (3)$$

$$M^{(II)} = 0$$

$$\theta^{(I)} = \frac{1}{EI} \int M^{(I)} dx = \frac{1}{EI} \int -m dx = \frac{1}{EI} [-mx + c_1]$$

$$v^{(I)} = \int \theta^{(I)} dx = \frac{1}{EI} \left[-\frac{m}{2} x^2 + c_1 x + c_2 \right] \quad (5)$$

$$\theta^{(II)} = \frac{1}{EI} \int M^{(II)} dx = \frac{1}{EI} \int 0 dx = \frac{1}{EI} c_3$$

$$v^{(II)} = \int \theta^{(II)} dx = \frac{1}{EI} [c_3 x + c_4]$$

Boundary conditions: $\theta^{(I)}(x=0) = 0 \rightarrow c_1 = 0$

$$v^{(I)}(x=0) = 0 \rightarrow c_2 = 0$$

(4)

Continuity conditions: $\theta^{(I)}(x=l) = \theta^{(II)}(x=l)$

$$v^{(I)}(x=l) = v^{(II)}(x=l)$$

$$\theta^{(I)}(x=l) = \theta^{(II)}(x=l) \quad \frac{1}{EI} [-ml] = \frac{1}{EI} c_3 \rightarrow c_3 = -ml$$

$$v^{(I)}(x=l) = v^{(II)}(x=l) = \frac{1}{EI} \left[-\frac{m}{2} l^2 \right] = \frac{1}{EI} \left[-ml^2 + c_4 \right] \rightarrow c_4 = \frac{ml^2}{2}$$

Equations

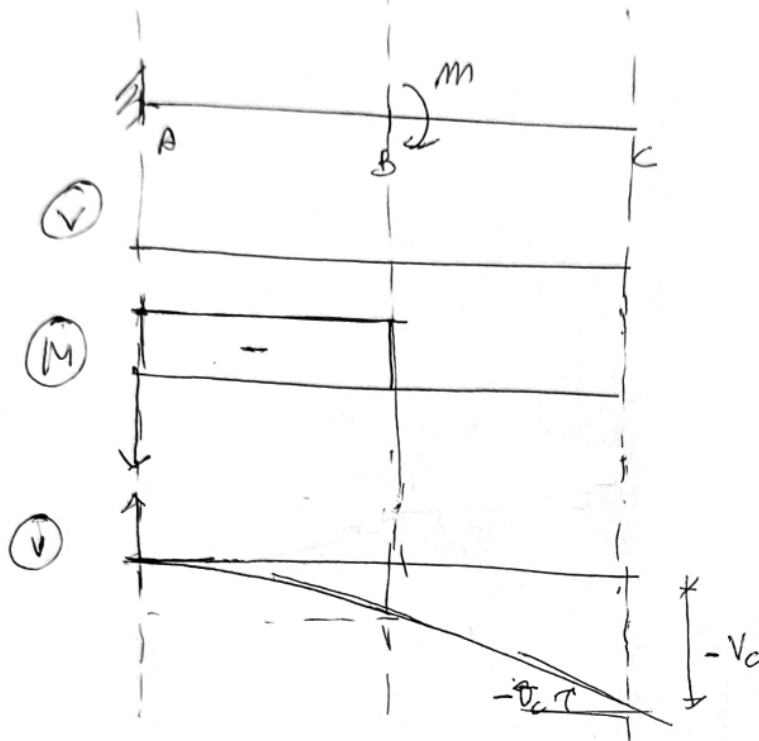
$$\vartheta^{(I)} = -\frac{mx}{EI} = -\frac{7x}{EI}$$

$$v^{(I)} = -\frac{mx^2}{2EI} = -\frac{7}{2} \frac{x^2}{EI} \quad (2)$$

$$\vartheta^{(II)} = -\frac{ml}{EI} = -\frac{21}{EI}$$

$$v^{(II)} = \frac{1}{EI} \left[-mex + \frac{ml^2}{2} \right] = \frac{1}{EI} [-21x + 345]$$

b)



(5)

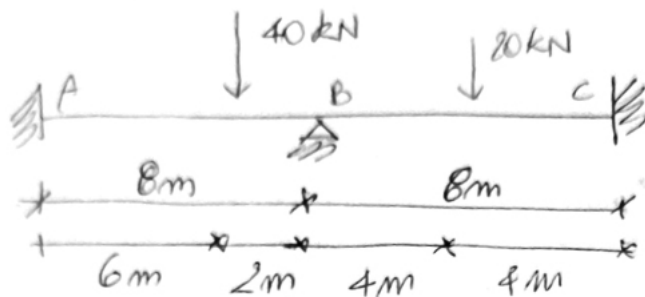
c) in C:

$$\vartheta^{(II)} (x=2l) = -\frac{21}{EI} \quad (3)$$

$$v^{(II)} (x=2l) = \frac{1}{EI} [-21 \times 6 + 345] = -\frac{1 \times 94,5}{EI}$$

Also possible via forget-me-not-

EXERCISE 2 [35pt]



$$F_1 = 40 \text{ kN}$$

$$F_2 = 20 \text{ kN}$$

$$l_1 = 6 \text{ m}$$

$$l_2 = 2 \text{ m}$$

$$l_3 = 4 \text{ m}$$

$$l = 8 \text{ m}$$

a)

• Stiffness factors:

$$K_{BA} = \frac{4EI}{e}$$

$$K_{BC} = \frac{4EI}{e}$$

$$\sum K = K_{BA} + K_{BC} = \frac{8EI}{e}$$

(3)

• Distribution factors

$$\mu_{BA} = \frac{K_{BA}}{\sum K} = \frac{1}{2}$$

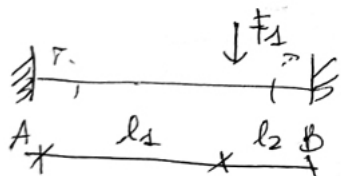
$$\mu_{BC} = \frac{K_{BC}}{\sum K} = \frac{1}{2}$$

(2)

• Carry over factors = $M_{AB} = \frac{1}{2} M_{BA}$

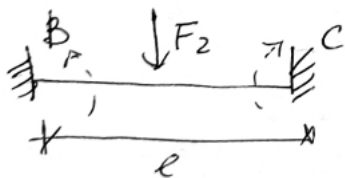
$$M_{CB} = \frac{1}{2} M_{BC}$$

• Fixed-end moments



$$M_{AB} = \frac{F_1 l_2^2 l_1}{l^2} = \frac{40 \times 2^2 \times 6}{8^2} = +15 \text{ kNm}$$

$$M_{BA} = -\frac{F_1 l_1^2 l_2}{l^2} = -\frac{40 \times 6^2 \times 2}{8^2} = -45 \text{ kNm}$$



$$M_{BC} = \frac{F_2 l}{8} = +20 \text{ kNm}$$

$$M_{CB} = -20 \text{ kNm}$$

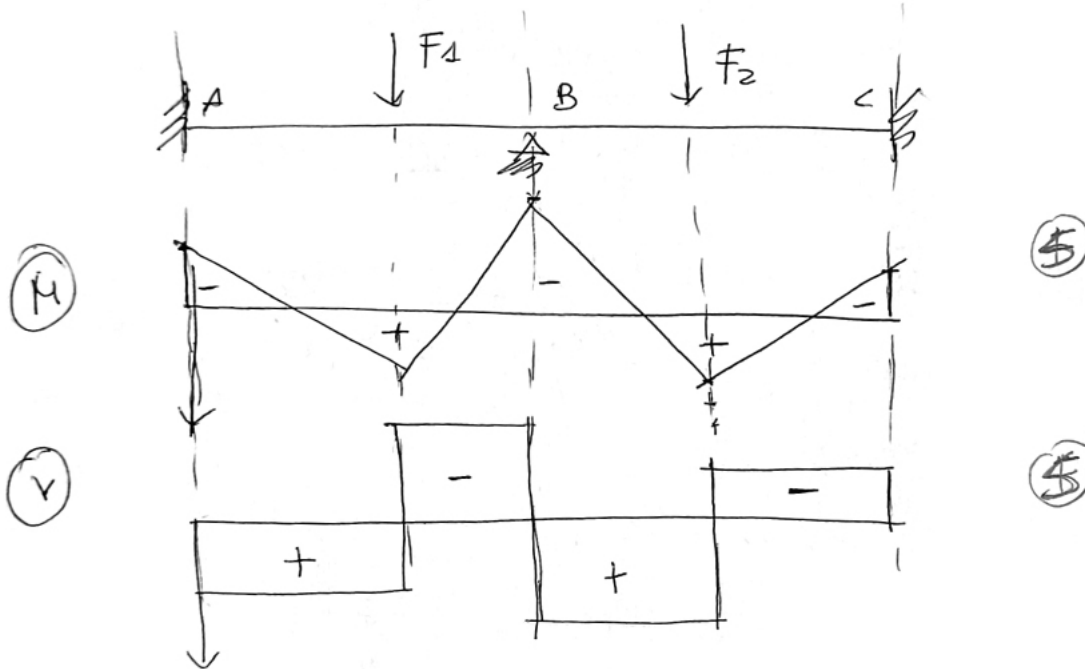
(5)



	A	B		C
		BA	BC	
μ		$1/2$	$1/2$	
COF	$1/2$			$1/2$
FEM	+25	-45	+20	-20
$-\Delta M$		+25		
DIST		$\swarrow$	$\searrow$	
CO		12,5	12,5	
	6,25			6,25
ΣM	21,25	-32,5	+32,5	-13,75

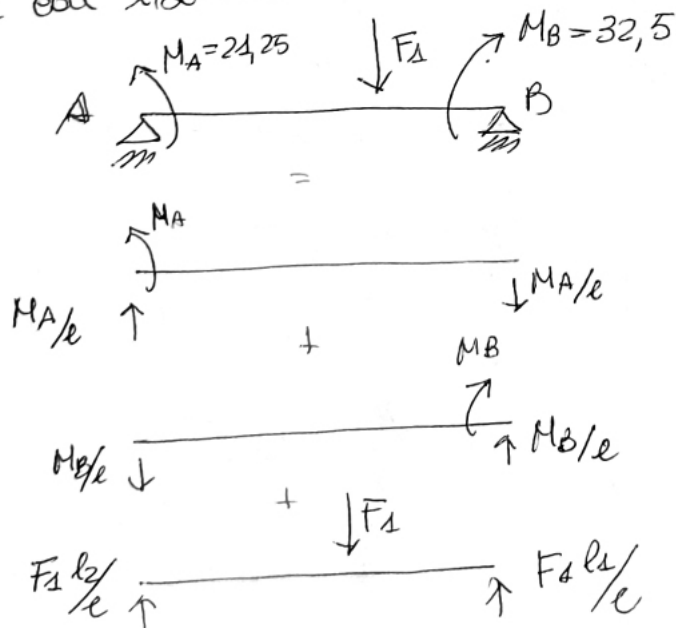
(5)

b)



e) I can isolate the beam AB :

(10)



$$V_A = M_A/e - M_B/e + F_1 l_2/e$$

$$= (21,25 - 32,5 + 40 \times 2)/8$$

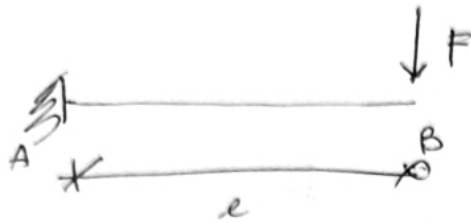
$$= 8,6 \text{ kN}$$

$$V_B^{(-)} = -M_A/e + M_B/e + F_1 l_1/e$$

$$= (-21,25 + 32,5 + 40 \times 6)/8$$

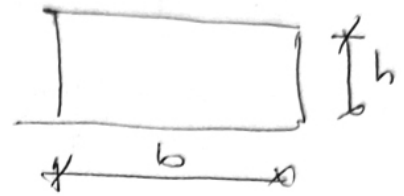
$$= 34,4 \text{ kN}$$

EXERCISE 3 [40pt]



$$l = 6 \text{ m}$$

$$F = 15 \text{ kN}$$



$$b = 600 \text{ mm}$$

$$h = 300 \text{ mm}$$

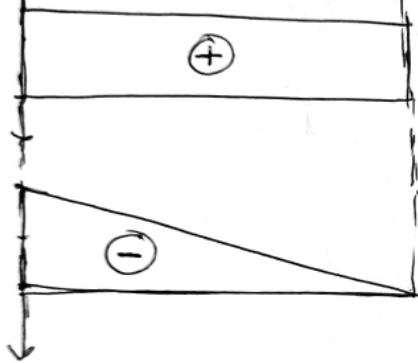
a)



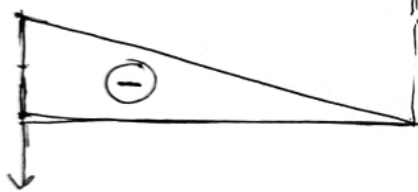
$$\sum F_V = V_A - F = 0 \Rightarrow V_A = F = 15 \text{ kN}$$

$$\sum M_{(A)} = M_A + Fl = 0 \Rightarrow M_A = Fl = 90 \text{ kNm}$$

(V)



(M)



(5)

The most critical sections are at ~~where~~ A and B, where V and M are extremes:

$$\text{in A} = V_A = 15 \text{ kN}$$

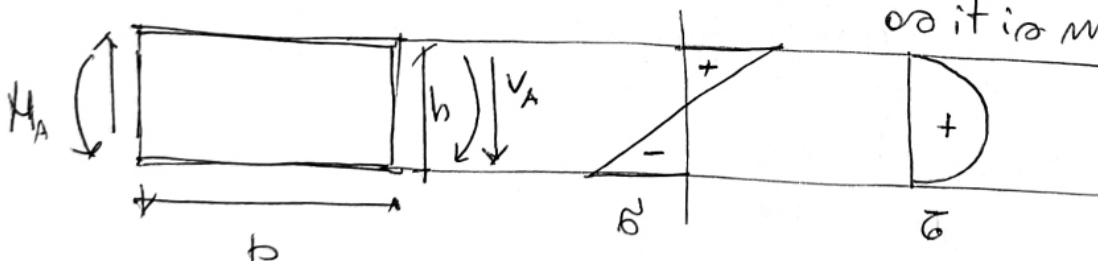
$$M_A = 90 \text{ kNm}$$

$$\text{in B} : V_A = 15 \text{ kN}$$

$$M_B = 0$$

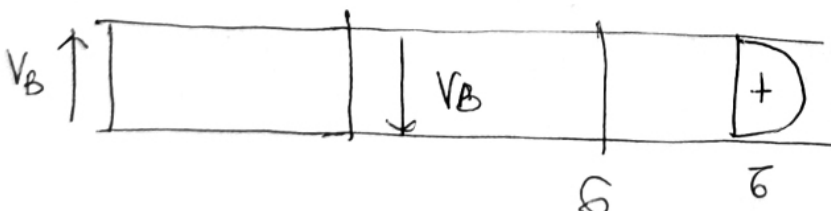
(good to analyze also B, even if $M_A = 0$)
so it is minimum

b) in A:



in B: ~~60 kNm~~

(5)



e). in A = top section:

$$\sigma_{A, \text{top}} = + \frac{M_A h/2}{I}, \quad \tau_{A, \text{top}} = 0$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} \times 600 \times \left(\frac{800}{2}\right)^3 = 1,35 \times 10^9 \text{ mm}^4 = 1,35 \times 10^{-3} \text{ m}^4$$

$$\sigma_{A, \text{top}} = \frac{90 \text{ kNm} \times 10^3 \text{ mm/m} \times 400 \text{ mm}}{1,35 \times 10^9 \text{ mm}^4} = 26,7 \times 10^3 \text{ kN/m}^2$$

(5)

• in A, middle section:

$$\sigma_{A, m} = 0, \quad \tau_{A, m} = 1,5 \text{ V/A}$$

(2)

$$\tau_{A, m} = \frac{15 \text{ kN}}{300 \times 600 \text{ mm}^2} \times 1,5 = 125 \text{ kN/m}^2$$

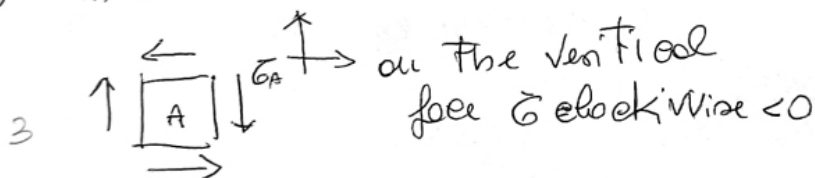
• in B:

• top section: $\sigma_{B, T} = 0, \quad \tau_{B, T} = 0$

(2)

• middle section: $\sigma_{B, m} = 0, \quad \tau_{B, m} = \tau_{A, m} = 125 \text{ kN/m}^2$

2) • middle A:



$$A = (0, \tau_{A, m})$$

$$B = (0, -\tau_{A, m})$$

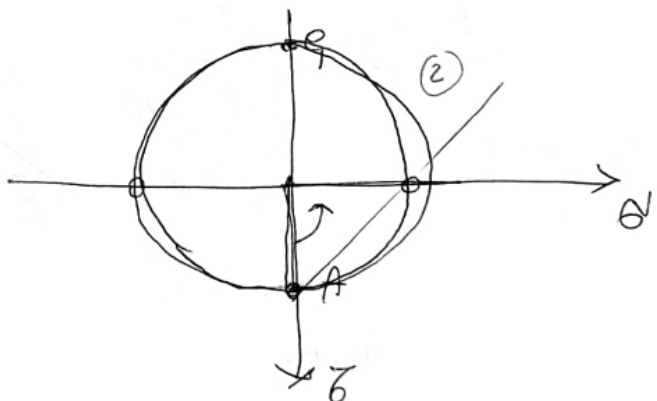
$$\sigma_c = 0$$

$$\tau = \tau_{A, m} = 125 \text{ kN/m}^2$$

$$\sigma_{1/2} = \pm 125 \text{ kN/m}^2$$

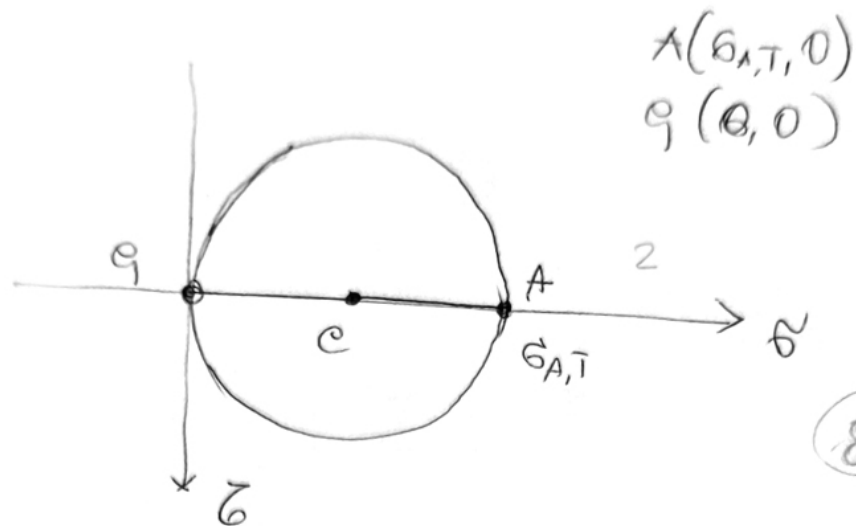
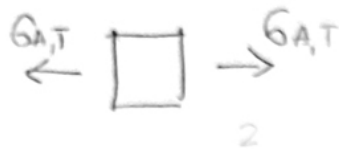
$$\theta_{1/2} = \pm 45^\circ$$

(8)



• middle B: exactly the same as middle A -

2) Top A :



$$\sigma_c = \frac{\sigma_{A,T}}{2} = \frac{20 \times 10^3 \text{ kN/m}^2}{2} = \underline{10 \times 10^3 \text{ kN/m}^2}$$

$$R = \underline{10 \times 10^3 \text{ kN/m}^2}$$

$$\sigma_{1/2} = \underline{10 \times 10^3 \text{ kN/m}^2} \quad \text{and} \quad \underline{0}$$

$$\theta_{1/2} = \underline{0} \quad \text{and} \quad \underline{90^\circ}$$

• top B : $\sigma_{B,T} = \tau_{B,T} = 0$

So $\sigma_{1/2} = 0$ $\theta_{1/2} = \text{n.a.}$

3) when moving from A to B (from the fixed support to the free-end of the beam) the following happens:

- on the top of the beam principal directions go from $0/90^\circ$ to not defined as value of principal stresses decreases to 0, ($\sigma = \tau = 0$)
- in the middle principal stresses stay constant.
- normal stresses are $\gg$ tangential stresses.

cracks expected -

