

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 4 July, 2025

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2425-Q4-RESIT

Calculus 2

Resit

Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠

Final answer questions

Report all your answers on the answer form.

1. The second-order Taylor polynomial generated by the function $f(x, y) = \ln(2x - 4y + 1) + 5$ at the point $(0, 0)$ is [3 pt]

$$P_2(x, y) = A + 2x + By + Cx^2 + 8xy - 8y^2,$$

where A , B and C are constants. Find the values of A , B and C .

Only numbers are accepted as answers.

2. The equation $(x^2 + y^2)^2 = 2x^3 + 4y^3$ describes a smooth curve $\mathcal{C}$. The point $P(\frac{3}{2}, -\frac{1}{2})$ belongs to $\mathcal{C}$. Around P , the equation defines y implicitly as a differentiable function of x . [3 pt]
Compute the value of $\frac{dy}{dx}$ at the point P .

3. The function $f(x, y) = x^2y + \cos(x)\cos(y)$ has gradient

$$\nabla f(x, y) = (2xy - \sin(x)\cos(y))\mathbf{i} + (x^2 - \cos(x)\sin(y))\mathbf{j}.$$

- (a) Compute the discriminant (Hessian) of f at the critical point $(0, 0)$. [1 pt]

- (b) At the critical point $P(0, 3\pi)$, the discriminant (Hessian) of f is $\Delta(0, 3\pi) = 1 + 6\pi$. [1 pt]

Determine whether f has a local maximum, a local minimum or a saddle point at P .

4. Rewrite the integral $\int_1^3 \int_{(x-1)^2+2}^{2x} (x-1)^2 dy dx$ using the order of integration $dx dy$. [3 pt]
Do not compute the integral.

5. Let R be the region in the first octant, bounded sideways by the planes $x = 0$ and $y = 0$, below by the cone $z = \frac{1}{\sqrt{3}} \cdot \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 7$. [3 pt]
Express the volume of R as an iterated triple integral in spherical coordinates, using the order of integration $d\rho d\phi d\theta$. Do not compute the integral.

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6. Compute the Jacobian $J(u, v) = \frac{\partial(x, y)}{\partial(u, v)}$ of the following transformation:

[2 pt]

$$\begin{cases} u = \frac{5}{2}x - y \\ v = 5x \end{cases} \quad \text{i.e.} \quad \begin{cases} x = \frac{1}{5}v \\ y = -u + \frac{1}{2}v \end{cases}$$

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

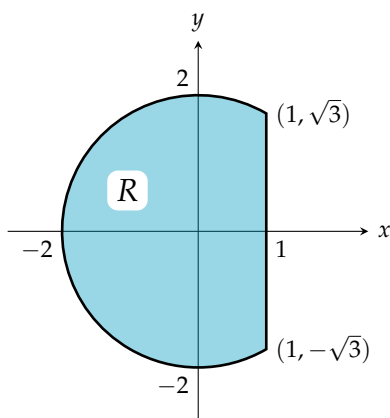
7. The line $x = 1$ divides the disk $x^2 + y^2 \leq 4$ in two parts; let R be the part containing the origin $(0, 0)$, including the boundary (see figure below).

[5 pt]

Find the absolute maximum and minimum values attained on R by the function

$$f(x, y) = (x^2 + y^2)e^x.$$

Clearly specify at which points those values are attained.



8. The function $f(x, y) = x + 4y + 5$ is constrained to the ellipse of equation $(x - 1)^2 + 2y^2 = 4$.

(a) Use the method of Lagrange multipliers to determine the absolute maximum and minimum values attained by the function f on the given ellipse. Clearly specify at which point(s) these values are attained.

[4 pt]

(b) Consider now the function $h(x, y) = \arctan(x + 4y + 5)$ and a point P on the ellipse.

[1 pt]

What is the relationship between the level curve of f passing through P and the level curve of h passing through P ?

9. Compute the average value of the function

[5 pt]

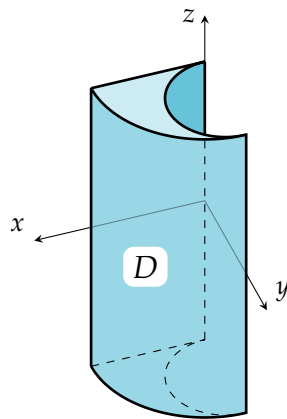
$$f(x, y) = \begin{cases} x + y & \text{if } y \geq x \\ 6x & \text{if } y < x \end{cases}$$

on the square region S of the plane with vertices $(1, 1)$, $(3, 1)$, $(3, 3)$ and $(1, 3)$.

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- 10.** The solid object D , shown in the figure below, is bounded above and below by the horizontal planes $z = 3$ and $z = -3$, and sideways by the plane $y = 0$ and the cylinders $x^2 + y^2 = 4$ and $x^2 + (y - 1)^2 = 1$. Only points with non-negative x - and y -coordinates belong to D . The density of D is described by the function $\delta(x, y, z) = 1 + z^2$, and its total mass is $M = 12\pi$. The center of mass of D is $(\bar{x}, \frac{16}{3\pi} - 1, \bar{z})$.

- (a) Express the mass M of D as a triple integral in cylindrical coordinates. [1.5 pt]
 (Its value is given, so you do not need to compute it.)
- (b) Express the first moment M_{yz} of D as a triple integral in Cartesian coordinates, then compute $\bar{x}$. [2.5 pt]
- (c) Explain why you can use an argument by symmetry to determine that $\bar{z} = 0$. [1 pt]



Total: 36 pt

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

A

Only for sorting.

Answer form

Calculus 2 – Resit – Friday 4 July, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the blanks:

$A =$ $B =$ $C =$

Question 2. $\frac{dy}{dx}$ at P is:

Question 3(a). The discriminant (Hessian) $\Delta(0,0)$ is:

Question 3(b). Mark your answer with an X.

☐ local maximum ☐ local minimum ☐ saddle point

Question 4. Fill in the blanks:

$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} dx dy$

Question 5. Fill in the blanks:

$\text{Vol}(R) = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} d\rho d\phi d\theta$

Question 6. The Jacobian $J(u,v)$ is:

Question 7. Write a full calculation/argumentation clearly in the box.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]