

UNIVERSITY OF TWENTE.

Traffic Operations
201100005

Examination

21 June 2018
13:45-16:45

Questions	Points
START	10
PART I	50
Question 1 (a: 4, b: 4, c: 4, d: 4, e: 2, f: 2)	20
Question 2 (a: 5, b: 5, c: 5)	15
Question 3 (a: 8, b: 7)	15
PART II	40
Question 4	10
Question 5 (a: 4, b: 6)	10
Question 6 (a: 4, b: 6, c: 4, d: 6)	20
Total	100

The exam consists of 6 questions. Points are listed in the table above. There are two parts and you are asked to use separate sheets for each part.

Good luck.

Eric van Berkum and Tom Thomas

1. [20 points] Imagine a truck drivers strike in France on a 3-lane motorway from time $t = t_0$. For the road you may assume a triangular fundamental diagram with a capacity of 2000 veh/h/lane, a critical density of 25 veh/km/lane and a jam density of 125 veh/km/lane. Suppose there is no traffic jam at $t < t_0$, and a constant demand of 5000 veh/h. Assume the truck drivers drive at 30 km/h, not allowing any vehicle to pass.
- (a) [4 points] Calculate is the maximum flow on the road behind the trucks?
 - (b) [4 points] Sketch the traffic flow operations in a space-time diagram. Pay attention to the direction of the shockwave, and note the propagation speeds of the other waves.
 - (c) [4 points] After a while, the trucks leave the road. Sketch again the traffic operations in the space time diagram. Do this until the traffic jam is solved (if applicable), or until the traffic states propagate linearly if demand does not change.
 - (d) [4 points] Calculate the speed at which the tail of the queue propagate backwards?
 - (e) [2 points] In the plot of question 2, draw the trajectory of the vehicle arriving at the tail of the queue at the moment the strike ends (Use a different colour or style than the shockwaves).
 - (f) [2 points] What is the speed of the vehicle in the jam?

2. [15 points] Suppose there are two classes of vehicles, fast vehicles driving 120 km/h and slow vehicles moving at 90 km/h. At the entrance to the road ($x=0$), an equal number of each type of vehicles is measured under a total flow of 1000 veh/h (so 500 veh/h each).

- (a) [5 points] Calculate the density on the road. Base your answer on the density of the slow vehicles and the fast vehicles.
- (b) [5 points] Calculate the space mean speed.
- (c) [5 points] Calculate the time mean speed.

3. [15 points] Helly's car-following model is given by the following equations:

$$a = \alpha \Delta v + \beta(s - s^*)$$

$$s^* = x_0 + vT$$

In this equation, a is the acceleration, v denotes the speed, Δv the difference in speed between the leader and its predecessor, s the spacing. Parameters in the model are α , β , x_0 and T . The interpretation of the parameters, and the meaning of s is left to the students. For reasons of simplicity, the time dependence and reaction time are ignored in this question.

- (a) [8 points] Derive the equation of the fundamental diagram, $q = q(k)$, from the above equation for the congested branch. Hint: use the relation $v = v(s)$ in the derivation.
- (b) [7 points] Suppose the following parameters are found to fit the traffic best: $\alpha=0.3\text{s}^{-1}$; $\beta=0.08\text{s}^{-2}$; $x_0 = 20\text{m}$; $T = 1\text{s}$. (In this line, s means second and m meter). Besides the car-following part, also a free speed is required to derive a full fundamental diagram. This is found to be 30 m/s . Compute the capacity of the road in veh/h

Part II

Please write your derivations and answers for Part II on separate paper sheets.

4. [10 pts] Consider the following 6 data points (or items): [1,3], [2,4], [3,4], [5,6], [6,9] and [8,7]. Plot these points in a graph. Use Euclidean distance to cluster these items according to the **nearest** and **farthest** neighbour method. Provide the clusters after each iteration. For both methods, which clusters would you distinguish and why? Which method would you prefer in this case and why?

5. [10 pts] For 5 days, the volumes were measured on a highway during the morning and evening rush hour. The outcomes are given in the following table.

	Day 1	Day 2	Day 3	Day 4	Day 5
Morning	3022	2978	2901	2776	3011
Evening	2828	2898	2955	2696	2867

- (a) [4 pts] Suppose that you use a 2-sigma (two times the standard deviation) significance level. Are morning and evening rush hour volumes significantly different? Explain your answer.
- (b) [6 pts] Assume that the random variation in the rush hour intensity due to variation in the process is normally distributed with variance equal to the expectation value of the volume. Use a Chi-square test to test whether the volumes in the *morning* rush hour are constant over the days. Use as significance level $\alpha = 0.01$. Explain all steps that lead to your answer.

6. [20 pts] Students were asked to estimate the capacity C of highway sections using $q - v$ (intensity - velocity) diagrams. They fitted a straight line through the data-points in the non-congested part of the $q - v$ diagram, so that $v = b * q + a$, and came up with the model: $C = -0.7*(a - v_C)/b$, with:

- $v_C > 0$: the velocity at capacity in km/h
- $a > 0$: the average velocity in free flow in km/h. Note $a > v_C$
- $b < 0$: the average negative slope between v and q
- $err(a)$: the (standard) random error in a
- $err(b)$: the (standard) random error in b

- (a) [4 pts] Express the error in C due to the error in a as function of $err(a)$, and express the error in C due to the error in b as function of $err(b)$.
- (b) [6 pts] The relative error in a is 4% ($err(a)/a = 0.04$) and the relative error in b is 20% ($err(b)/b = 0.20$). Express the total error in C , $err(C)$, as function of a , b and v_C (i.e., put the values of the relative errors in the estimate of $err(C)$), given that the (individual) errors in a and b are negatively correlated with a correlation coefficient of -1.
- (c) [4 pts] Estimate C and $err(C)$, given that $v_C = 70$, $a = 100$ and $b = -0.005$.
- (d) [6 pts] Illustrate in the $q - v$ diagram how C and the error in C are estimated. Which error has the largest effect on the error in C ? The error in a or in b ? How does the value of v_C influence the estimate of C and $err(C)$? Explain your answers using your illustration.

T-distribution: Critical values of t

Df	Significance level						
	Two-tailed test: One-tailed test:	0.1 0.05	0.05 0.025	0.02 0.01	0.01 0.005	0.002 0.001	0.001 0.0005
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
40		1.684	2.021	2.423	2.704	3.307	3.551
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
Z		1.645	1.960	2.326	2.576	3.090	3.291

χ^2 (Chi-Squared) Distribution: Critical Values of χ^2

Df	One Tailed Test:	Significance level								
		0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.0025	0.001
1		1.64	2.07	2.71	3.84	5.02	6.63	7.88	9.14	10.83
2		3.22	3.79	4.61	5.99	7.38	9.21	10.60	11.98	13.82
3		4.64	5.32	6.25	7.81	9.35	11.34	12.84	14.32	16.27
4		5.59	6.74	7.78	9.49	11.14	13.23	14.86	16.42	18.47
5		7.29	8.12	9.24	11.07	12.83	15.09	16.75	18.39	20.51
6		8.56	9.45	10.64	12.53	14.45	16.81	18.55	20.25	22.46
7		5.80	10.75	12.02	14.07	16.01	18.48	20.28	22.04	24.32
8		11.03	12.03	13.36	15.51	17.53	20.09	21.95	23.77	26.12
9		12.24	13.29	14.68	16.92	19.02	21.67	23.59	25.46	27.83
10		13.44	14.53	15.99	18.31	20.48	23.21	25.19	27.11	29.59
11		14.63	15.77	17.29	19.68	21.92	24.72	26.76	28.73	31.26
12		15.81	16.99	18.55	21.03	23.34	26.22	28.30	30.32	32.91
13		15.58	18.90	19.81	22.36	24.74	27.69	29.82	31.88	34.53
14		18.15	19.4	21.06	23.68	26.12	29.14	31.32	33.43	36.12
15		19.31	20.60	22.31	25.00	27.49	30.58	32.80	34.95	37.70
16		20.47	21.79	23.54	26.30	28.85	32.00	34.27	36.46	39.25
17		21.61	22.98	24.77	27.59	30.19	33.41	35.72	37.95	40.79
18		22.76	24.16	25.99	28.87	31.53	34.81	37.16	39.42	42.31
19		23.90	25.33	27.20	30.14	32.85	36.19	38.58	40.88	43.82
20		25.04	26.50	28.41	31.41	34.17	37.57	40.00	42.34	45.31
21		26.17	27.66	29.62	32.67	35.48	38.93	41.40	43.78	46.80
22		27.30	28.82	30.81	33.92	36.78	40.29	42.80	45.20	48.27
23		28.43	29.98	32.01	35.17	38.08	41.64	44.18	46.62	49.73
24		29.55	31.13	33.20	36.42	39.36	42.98	45.56	48.03	51.18
25		30.68	32.28	34.38	37.65	40.65	44.31	46.93	49.44	52.62
26		31.79	33.43	35.56	38.89	41.92	45.64	48.29	50.83	54.05
27		32.91	34.57	36.74	40.11	43.19	46.96	49.64	52.22	55.48
28		34.03	35.71	37.92	41.34	44.46	48.28	50.99	53.59	56.89
29		35.14	36.85	39.09	42.56	45.72	49.59	52.34	54.97	58.30
30		36.25	37.99	40.26	43.77	46.98	50.89	53.67	56.33	59.70
40		47.27	49.24	51.81	55.76	59.34	63.69	66.77	69.70	73.40
50		53.16	60.35	63.17	67.50	71.42	76.15	79.49	82.66	86.66
60		68.97	71.34	74.40	79.08	83.30	88.38	91.95	95.34	99.61
80		90.41	93.11	96.58	101.9	106.6	112.3	116.3	120.1	124.8
100		111.7	114.7	118.5	124.3	129.6	135.8	140.2	144.3	149.4