

Answer sheet Introduction to Mathematics

September 23, 2022, 150233

Name : Solutions

Programme : _____

Student number : _____

1. (a) Write only the answer in the frame below:

$$\bigcup_{i=6}^{60} A_i = \left[\frac{1}{60}, 60 \right]$$

1. (b) Write only the answer in the frame below:

$$\bigcap_{i=6}^{60} A_i = \left[\frac{1}{6}, 6 \right]$$

2. For each of the following statements, encircle True if it is True, and False if it is False:

- (i) For all sets A : $\emptyset \in A$ is ~~True~~ / False
- (ii) For all sets A : $\emptyset \subseteq A$ is True / ~~False~~
- (iii) For all sets A, B : $\overline{A \cap B} \subseteq \overline{A \cup B}$ is ~~True~~ / False
- (iv) For all sets A, B : $\overline{A \cap B} \subseteq \overline{A \cup B}$ is True / ~~False~~

3. Give a full calculation/argumentation in the frame below:

Suppose for contradiction that the statement is false.

Then, there exist p, q for which $p+q \notin \mathbb{Q}$ and $p \in \mathbb{Q}$ and $q \in \mathbb{Q}$.

Then, there exist $a, b \in \mathbb{Z}, b \neq 0$ for which $p = \frac{a}{b}$
and there exist $c, d \in \mathbb{Z}, d \neq 0$ for which $q = \frac{c}{d}$.

Now $p+q = \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \in \mathbb{Q}$, since
 $ad+bc \in \mathbb{Z}, bd \in \mathbb{Z}$, and $bd \neq 0$.

This contradicts that $p+q \notin \mathbb{Q}$.

Therefore the original statement is true.

4. Give a full calculation/argumentation in the frame below:

Basis Step

$$\sum_{i=1}^1 i^2 = 1$$

$$\frac{1(1+1)(2 \cdot 1 + 1)}{6} = 1,$$

which is equal, so the basis step holds.

Induction Step

Suppose $\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6}$ for some $k \in \mathbb{N}$. (IH)

We will show that $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$.

$$\text{Well, } \sum_{i=1}^{k+1} i^2 = \sum_{i=1}^k i^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \text{ by IH.}$$

$$\text{This equals } \frac{(k+1)(2k^2+k)}{6} + \frac{(k+1) \cdot 6(k+1)}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6},$$

which indeed equals $\frac{(k+1)(k+2)(2(k+1)+1)}{6}$, so the

induction step holds. By mathematical induction, the statement is true for all $n \in \mathbb{N}$.

5. (a) Give a full calculation/argumentation in the frame below:

We select 5 out of 8 positions.

$$\binom{8}{5} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 8 \cdot 7 = 56.$$

5. (b) Give a full calculation/argumentation in the frame below:

We select 5 out of 8 positions and 3 other digits. For each other digit, we have 9 choices:

$$\binom{8}{5} \cdot 9^3$$

Extra writing space: Give a full calculation/argumentation in the frame below: