

**Test: Structural Mechanics in Module 1**  
**Unit of Study code: 2020-000-43**

*Date: 1-11-2021*

*Start of the exam: 8:45*

*End of the exam: 10:30*

*End of the exam for students with additional time: 10:55*

*Examiner: ing. G.H.Snellink*

**GRADING**

- ✓ Weight of this test is 50% of the grade for Structural Mechanics.
- ✓ Within this test:
  - Assignment 1: 30%
  - Assignment 2: 35%
  - Assignment 3: 35%

**INSTRUCTIONS & PRACTICALITIES**

- ✓ All Rules of Order for Testing as established by the examination board apply to this test. In case of inconsistencies with other rules set by the examiner, the Rules of Order for Testing prevail.
- ✓ Please read each question carefully before answering the question.
- ✓ All paper work has to be handed in at the end of the test, including scrap paper.
- ✓ For answering questions, use the attached working sheets. If one sheet is not enough, use the sheet with the text "Continuation of ..." in the top right corner. Enter the number of the question where this sheet is the follow-up. Do not make different tasks on the same working sheet.
- ✓ The elaboration of the question must be clear to all (i.e. the teacher). Show what you do, so use a lot of drawings, sketches, or schemas.

**RESOURCES ALLOWED TO BE USED DURING THE EXAM:**

You are allowed to use the following resources during the exam (i.e. any other resource not listed here is not allowed to be used during the exam):

- ✓ (graphical) calculator (not a communication device),
- ✓ pen or/and pencil,
- ✓ eraser,
- ✓ ruler.

**Assignment 1. (30 %)      Cross-section properties**

Given:

A block with a square cross-section of  $600 \times 600 \text{ mm}^2$  has a mass of  $360 \text{ kg}$  and is loaded at the middle of one of its sides by a compressive force of  $36 \text{ kN}$ . The stress distribution under the block is linear. Use  $g = 10 \text{ N/kg}$  for the gravitational field strength.

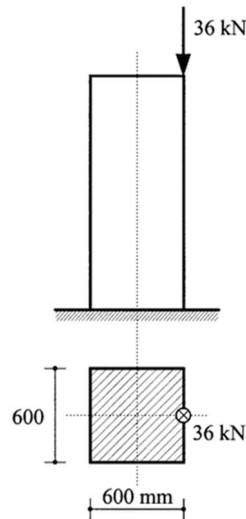


Figure 1

Question:

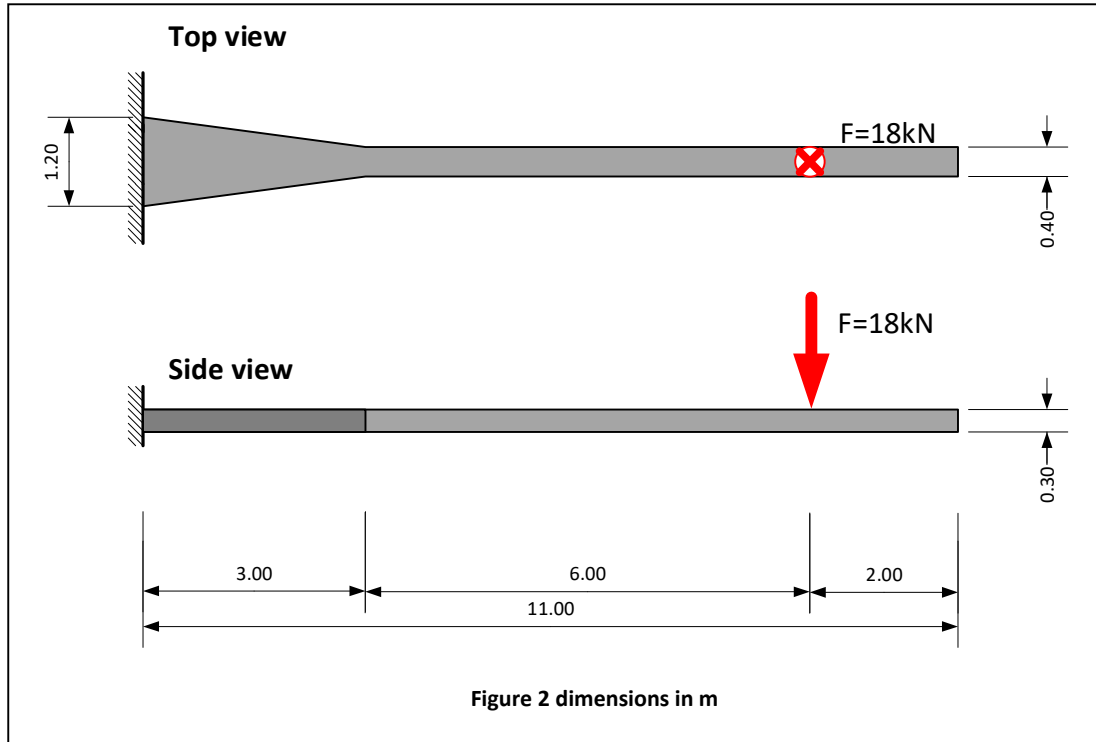
- Determine and plot the stress distribution under the block if tensile and compressive stresses are allowed.
- Determine and plot the stress distribution under the block if tensile stresses are not allowed.

Motivate your answers.

**Assignment 2. (35 %) Moment Area method**

Given:

$E=20\,000\text{ N/mm}^2$ .



Questions:

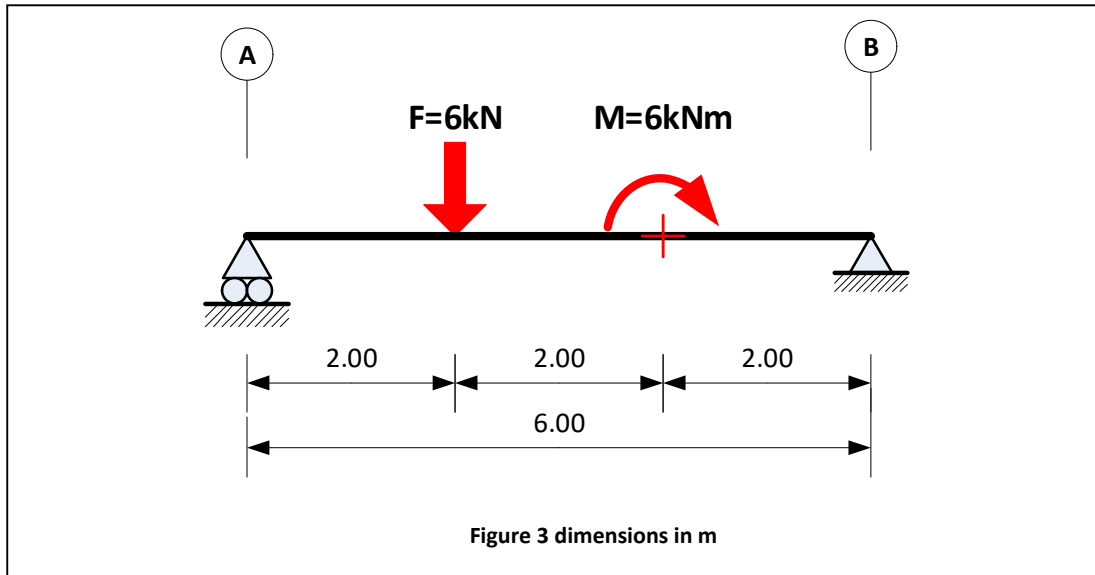
- Plot shear force and moment diagram.
- Determine location and magnitude of the maximum deflection using the Moment Area method.

Motivate your answers.

**Assignment 3. (35 %) McAuley's method**

Given:

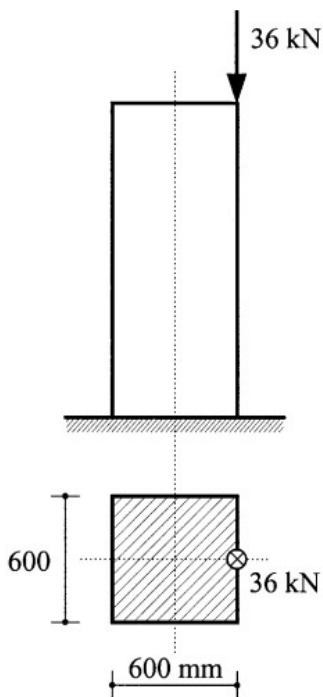
$EI=2\,000\text{kNm}^2$ .



Question:

- Determine support reactions.
- Determine location and magnitude of the maximum deflection using McAuley's method (hint:  $x_{\max}=\langle 2,4 \rangle$ )

Motivate your answers.

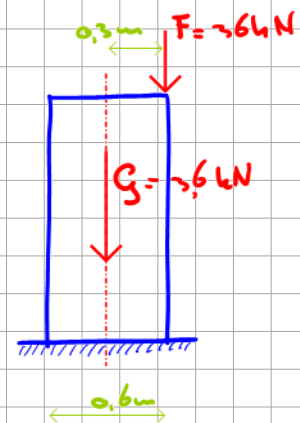
Question:

Determine &amp; plot stress distribution under the block.

a → with tensile stress allowed

b → with no tensile stress allowedPart A:Key Activities:Model:

$$\begin{aligned}\sigma &= \sigma_N + \sigma_M \\ \sigma_N &= N/A \\ \sigma_M &= \frac{M \cdot e}{I} \\ M &= F \cdot e \\ N &= F + G \\ I &= \frac{1}{12} b h^3 \\ A &= b \cdot h \\ e &= \frac{1}{2} h\end{aligned}$$

Solving:

$$e = \frac{1}{2} h = 0.3 \text{ m}$$

$$A = 0.6 \times 0.6 = 0.36 \text{ m}^2$$

$$I = \frac{1}{12} \times 0.6 \times 0.6^3 = 0.0108 \text{ m}^4$$

$$N = 36 + 36 = 72 \text{ kN}$$

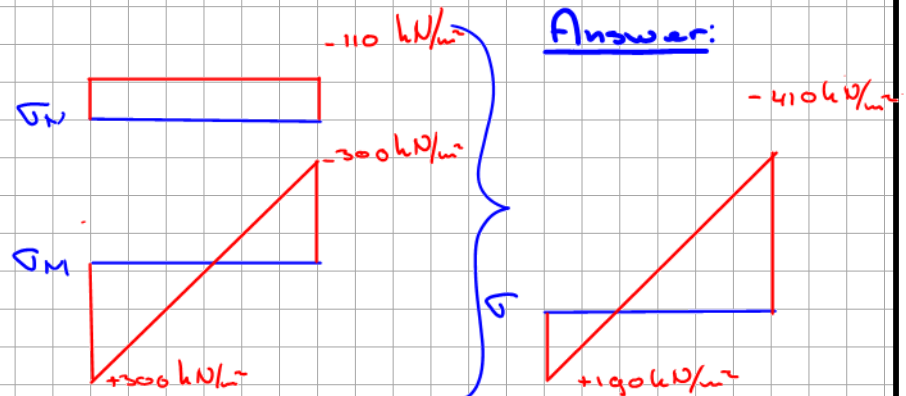
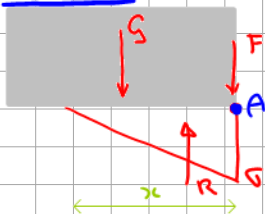
$$M = 0.3 \times 36 = 10.8 \text{ kNm}$$

$$\sigma_N = \frac{72}{0.36} = 200 \text{ kN/m}^2$$

$$\sigma_M = \frac{10.8 \times 0.3}{0.0108} = 300 \text{ kN/m}^2$$

$$\sigma_N + \sigma_M = 200 + 300 = 500 \text{ kN/m}^2$$

$$\sigma_N - \sigma_M = 200 - 300 = -100 \text{ kN/m}^2$$

Answer:Part BModel:Key Activities:

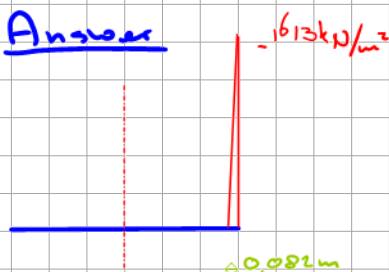
$$\begin{aligned}\sum M_A &= 0 & G \cdot \frac{1}{2} h - R \cdot \frac{1}{2} x &= 0 \\ \sum V &= 0 & G + F - R &= 0 \\ R &= \frac{1}{2} x \cdot b \cdot \sigma\end{aligned}$$

Solving:

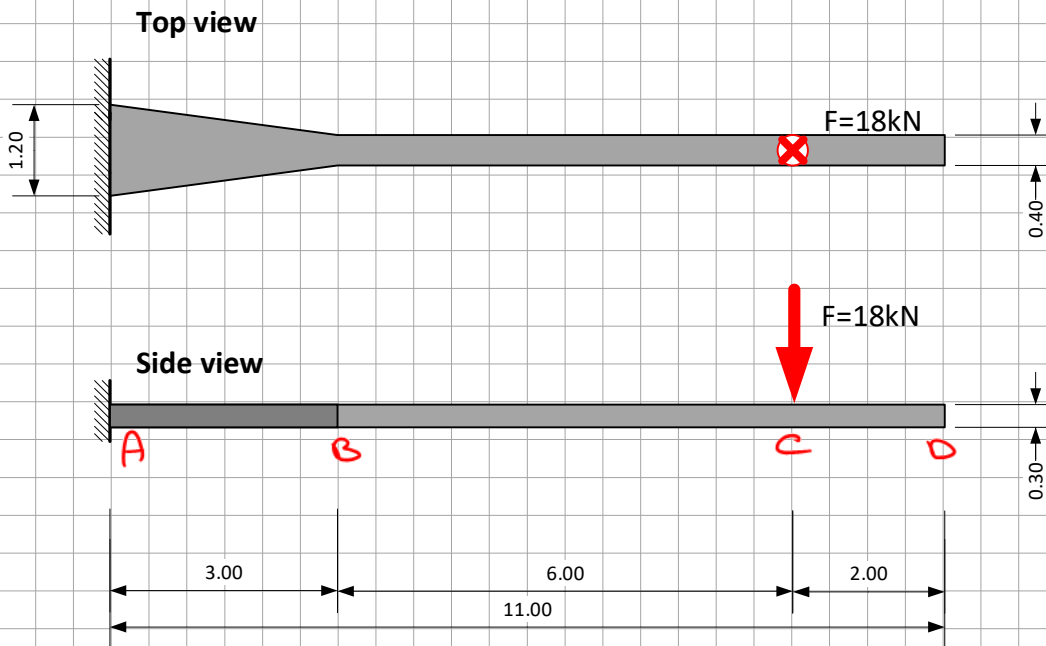
$$R = G + F = 36 + 36 = 72 \text{ kN}$$

$$x = \frac{G \cdot \frac{1}{2} h}{\frac{1}{2} R} = \frac{\frac{1}{2} \cdot 36 \cdot 0.6}{\frac{1}{2} \cdot 72} = 0.082 \text{ m}$$

$$\sigma = \frac{R}{\frac{1}{2} x b} = \frac{72}{\frac{1}{2} \cdot 0.082 \cdot 0.6} = 1613 \text{ kN/m}^2$$

Answer





**Questions:** a) Share force & Moment diagram  
b) magnitude & location maximum deflection with Moment-Area-method

**Key Activities:**

$$\sum H = 0 \quad \sum V = 0 \quad \sum M = 0$$

$$\sum H = 0$$

$$H_A = 0$$

$$\sum V = 0$$

$$-V_A + 18 = 0 \Rightarrow V_A = 18 \text{ kN}$$

$$\sum M_A = 0$$

$$-18 \times 9 + M_A = 0 \quad M_A = 162 \text{ kNm}$$

$$M_B = 18 \times 6 = 108 \text{ kNm}$$

**Model:**



**Answers**

**V-diagram**



**M-diagram**



**Part B**

**Key activities:**  $v = \frac{\sum A_m}{EI} \times z$

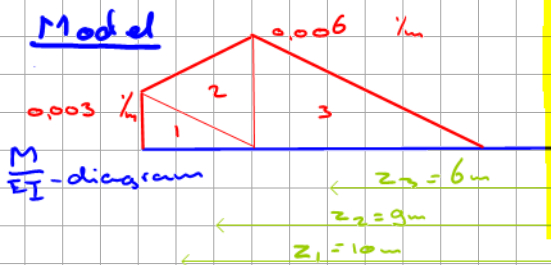
$$I = \frac{1}{12} b h^3$$

$$I_A = \frac{1}{12} \times 1.2 \times 0.3^3 = 0.0027 \text{ m}^4$$

$$I_B = \frac{1}{12} \times 0.4 \times 0.3^3 = 0.0009 \text{ m}^4$$

$$E = 20.000 \text{ N/mm}^2 = 20 \times 10^6 \text{ kN/m}^2$$

**Model**



**Solution:**

$$A: \frac{M}{EI} = \frac{-162}{0.0027 \times 20 \times 10^6} = 0.003 \text{ [1/m]}$$

$$B: \frac{M}{EI} = \frac{-108}{0.0009 \times 20 \times 10^6} = 0.006 \text{ [1/m]}$$

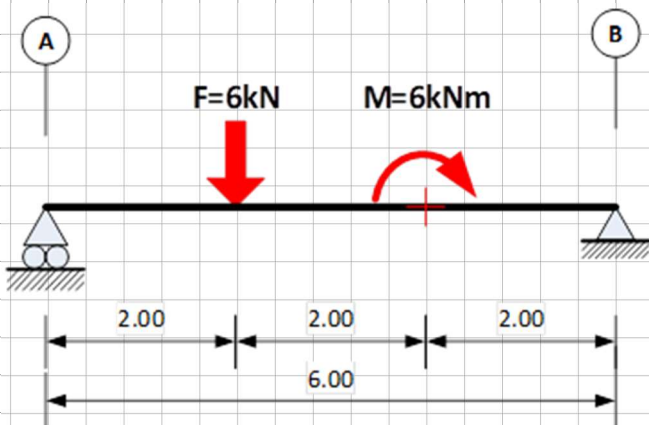
**Answer**

- Maximum deflection at outermost right side ( $x = 11 \text{ m}$ )

- Maximum deflection is  $0.234 \text{ m}$  (downwards).

|   | $\frac{M}{EI}$ | b | h     | A      | z  | $Az$  |
|---|----------------|---|-------|--------|----|-------|
| 1 | $\frac{1}{2}$  | 3 | 0.003 | 0.0045 | 10 | 0.045 |
| 2 | $\frac{1}{2}$  | 3 | 0.006 | 0.0090 | 9  | 0.081 |
| 3 | $\frac{1}{2}$  | 6 | 0.006 | 0.0180 | 6  | 0.108 |
|   |                |   |       |        |    | 0.234 |



Questions:

- support reactions
- magnitude & location maximum deflection with Macaulay's method

Part A  
ModelKey Activities

$$\begin{aligned}\sum H &= 0 & H_A &= 0 \\ \sum M_B &= 0 & -V_A \cdot 6 + 6 \cdot 4 - 6 &= 0 \\ & & V_A &= 3 \text{ kN} \\ \sum V &= 0 & -V_A + 6 - V_B &= 0 \\ & & V_B &= 6 - V_A = 6 - 3 = 3 \text{ kN}\end{aligned}$$

Answers:

$$\begin{aligned}V_A &= 3 \text{ kN} \\ V_B &= 3 \text{ kN}\end{aligned}$$

Part B  
ModelKey Activities:  $-EI \delta(x) = \int EI \phi(x) dx = \int \int M(x) dx^2$ 

See FGMN!

$$\sum M(x) = 0 \rightarrow x(x-0)' + 6 \cdot (x-2)' - 6(x-4)' - 3(x-6)' + M_x = 0$$

Macaulay's function:

$$M(x) = +3x[x-0]' - 6x[x-2]' + 6[x-4]' + 3[x-6]'$$

$$EI \phi(x) = +\frac{3}{2}x[x-0]^2 - \frac{6}{2}x[x-2]^2 + 6[x-4]' + \frac{3}{2}[x-6]^2 + C_1$$

$$-EI \delta(x) = +\frac{3}{6}x[x-0]^3 - \frac{6}{6}x[x-2]^3 + \frac{6}{2}[x-4]^2 + \frac{3}{6}[x-6]^3 + C_1 x + C_2$$

Boundary Conditions:  $\delta(0) = 0$  &  $\delta(6) = 0$ 

$$-EI \delta(0) = +\frac{1}{2}[0-0]^3 - 1 \cdot [0-2]^3 + 3[0-4]^2 + \frac{1}{2}[0-6]^3 + C_1 \cdot 0 + C_2 = 0 \Rightarrow C_2 = 0$$

$$\begin{aligned}-EI \delta(6) &= +\frac{1}{2}[6-0]^3 - 1 \cdot [6-2]^3 + 3[6-4]^2 + \frac{1}{2}[6-6]^3 + C_1 \cdot 6 = 0 \\ &= +108 - 64 + 12 + 0 + C_1 \cdot 6 = 0 \\ &\Rightarrow C_1 = -56/6 = -28/3 \approx -9.33\end{aligned}$$

max deflection at  $\phi = 0$ 

$$EI \phi(x) = +\frac{3}{2}x[x-0]^2 - \frac{6}{2}x[x-2]^2 + 6[x-4]' + \frac{3}{2}[x-6]^2 - \frac{28}{3} = 0$$

if between 2 & 4  $\Rightarrow$ 

$$\begin{aligned}EI \phi(x) &= \frac{3}{2}(x-0)^2 - 3(x-2)^2 + 0 + 0 - \frac{28}{3} = 0 \\ \frac{3}{2}x^2 &- 3(x^2 - 4x + 4) - \frac{28}{3} = 0 \\ \frac{3}{2}x^2 &- 3x^2 + 12x - 12 - \frac{28}{3} = 0 \\ -\frac{3}{2}x^2 &+ 12x - \frac{64}{3} = 0\end{aligned}$$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x_1 &= \frac{-12 + \sqrt{12^2 - 4 \cdot (-\frac{3}{2}) \cdot (-\frac{64}{3})}}{2 \cdot (-\frac{3}{2})} = \frac{8}{3} = 2.667 \text{ m} \quad \text{valid (between 2, 4)} \\ x_2 &= \frac{-12 - \sqrt{12^2 - 4 \cdot (-\frac{3}{2}) \cdot (-\frac{64}{3})}}{2 \cdot (-\frac{3}{2})} = \frac{16}{3} = 5.333 \text{ m} \quad \text{not valid.}\end{aligned}$$

$$-EI \delta(\frac{8}{3}) = +\frac{1}{2} \left[ \frac{8}{3} - 0 \right]^3 - 1 \times \left[ \frac{8}{3} - 2 \right]^3 + 3 \left[ \frac{8}{3} - 4 \right]^2 + \frac{1}{2} \left[ \frac{8}{3} - 6 \right]^3 - \frac{28}{3} \cdot \frac{8}{3}$$

$$9,4815 - 0,2963 - 24,8889 = -15,7037$$

$$\delta(\frac{8}{3}) = \frac{-15,7037}{-2000} = 0,00785 \text{ m} = 7,85 \text{ mm.}$$

Name:..... Course: Structural Mechanics in Module 1

Studentnumber:..... Date: 1-11-2021

Test 2:

Continuation of

1

Use this sheet for ONE assignment only!



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