

Exam: Vector Calculus for BMT, CE, CSE, ME

October 2nd, 2020 , 13.45 - 15.45 hrs

Give a suitable explanation of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

1. Consider the vector field $\mathbf{F}(x, y) = (x^2 + y^2)\mathbf{i} + 2xy\mathbf{j}$.
 - a) Let C be the curve consisting of the line segments from $(0, 0)$ to $(0, 2)$, followed by the line segment from $(0, 2)$ to $(1, 2)$. Evaluate $I = \int_C \mathbf{F} \cdot d\mathbf{r}$.
 - b) Let C be arbitrary. Show that the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is path independent.
 - c) The region R is given by the inequalities $0 \leq x \leq 1$ and $2x^2 \leq y \leq 2x$. Use Green's theorem to calculate the outward flux of $\mathbf{F}$ across the boundary C of R .
2. Consider the circle of radius a in the xz -plane with center $(b, 0, 0)$, $0 < a < b$. The torus T is a surface obtained by revolving this circle around the z -axis.
 - a) Sketch T in both the xz -plane and in the xyz -space.
 - b) Determine a parametrization of T . **Hint:** Use 2 angles.
 - c) Compute the surface area of T using the parametrization from b).
3. Let S be the portion of the elliptic paraboloid $z = \frac{x^2}{4} + \frac{y^2}{9}$ below the plane $z = 1$. Let C denote the boundary curve of S .
Compute the counterclockwise circulation as viewed from above of the vector field $\mathbf{F}(x, y, z) = (9xz + 2y)\mathbf{i} + (2x + y^2)\mathbf{j} + (-2y^2 + 2z)\mathbf{k}$ along C using an integration theorem (Green, Gauss or Stokes).

Points:

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|------|---|------|-----|----|-----|
| 1.a) | 3 | 2.a) | 1 | 3. | 5.5 |
| b) | 1 | b) | 1.5 | | |
| c) | 3 | c) | 3 | | |

Total: $18 + 2 = 20$ points