

Test: Structural Mechanics in Module 1
Unit of Study code: 2020-000-43

Date: 02/10/2020

Start of the exam: 13:45

End of the exam: 15:30

End of the exam for students with additional time: 15:55

Examiner: ing. G.H.Snellink

GRADING

- ✓ Weight of this test is 50% of the grade for Structural Mechanics.
- ✓ Within this test:
 - Assignment 1: 35%
 - Assignment 2: 30%
 - Assignment 3: 35%

INSTRUCTIONS & PRACTICALITIES

- ✓ All Rules of Order for Testing as established by the examination board apply to this test. In case of inconsistencies with other rules set by the examiner, the Rules of Order for Testing prevail.
- ✓ Please read each question carefully before answering the question.
- ✓ All paper work has to be handed in at the end of the test, including scrap paper.
- ✓ For answering questions, use the attached working sheets. If one sheet is not enough, use the sheet with the text "Continuation of ..." in the top right corner. Enter the number of the question where this sheet is the follow-up. Do not make different tasks on the same working sheet.
- ✓ The elaboration of the question must be clear to all (i.e. the teacher). Show what you do, so use a lot of drawings, sketches, or schemas.

RESOURCES ALLOWED TO BE USED DURING THE EXAM:

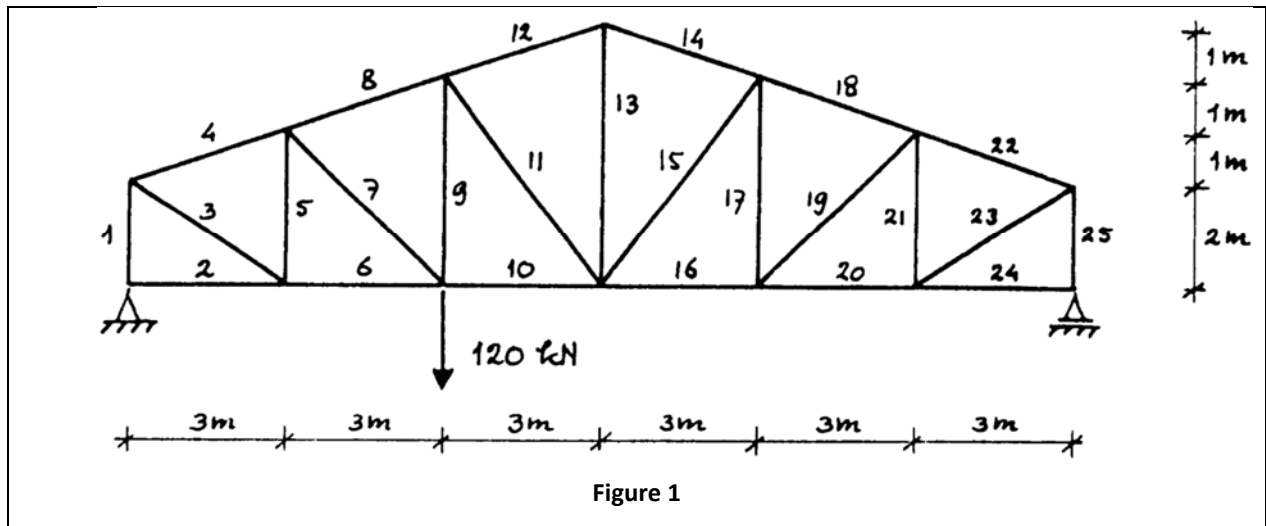
You are allowed to use the following resources during the exam (i.e. any other resource not list here is not allowed to be used during the exam):

- ✓ (graphical) calculator (not a communication device),
- ✓ pen or/and pencil,
- ✓ eraser,
- ✓ ruler.

Assignment 1. (35 %) Truss

Given:

A truss with the characteristic load of $F=120\text{kN}$ has members with a genuine cross section of 1000mm^2 (see Figure 1). The used material is steel $E=200\,000\text{N/mm}^2$



Question:

- Determine the internal forces in members 10, 11 and 12

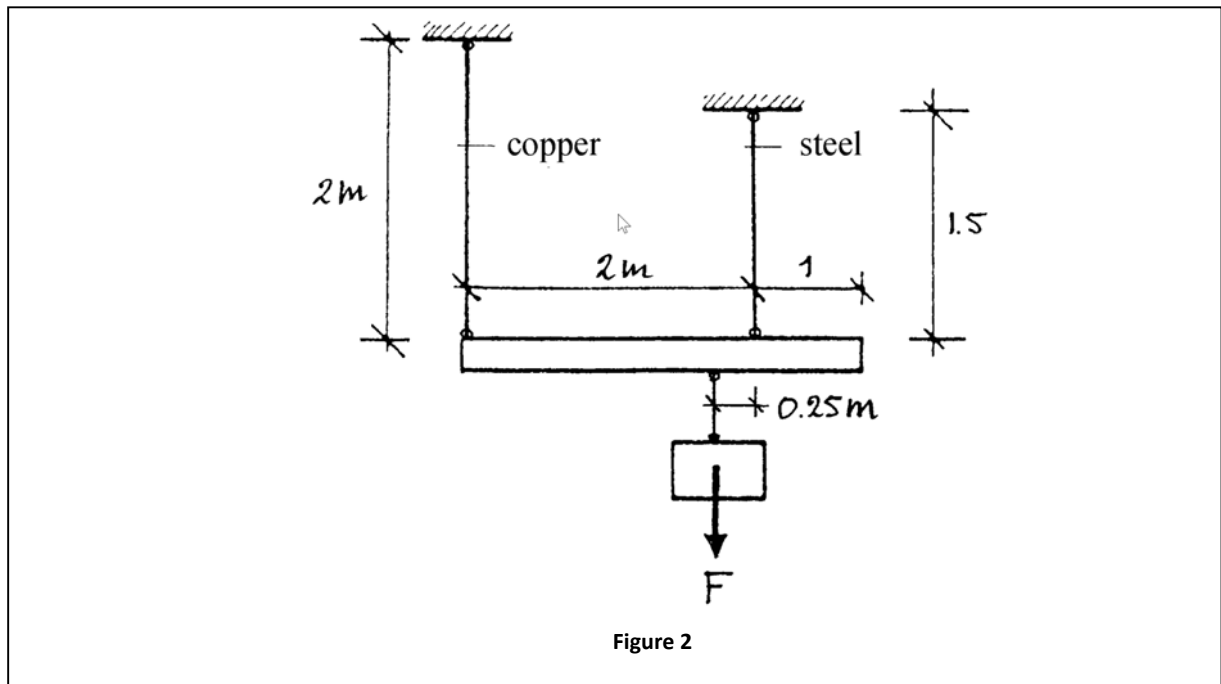
Assignment 2. (30 %) Axial extension.

Given:

A rigid beam with a dead weight of 3 kN/m is suspended from two bars, one made of copper, the other of steel. Before connecting the to the bars, the lower ends of the bars are at the same level. A load F also suspended from the beam as indicated.

For the copper bar: $E_c = 120 \text{ GPa}$ and $A_c = 100 \text{ mm}^2$

For the steel bar: $E_s = 210 \text{ GPa}$ and $A_s = 200 \text{ mm}^2$



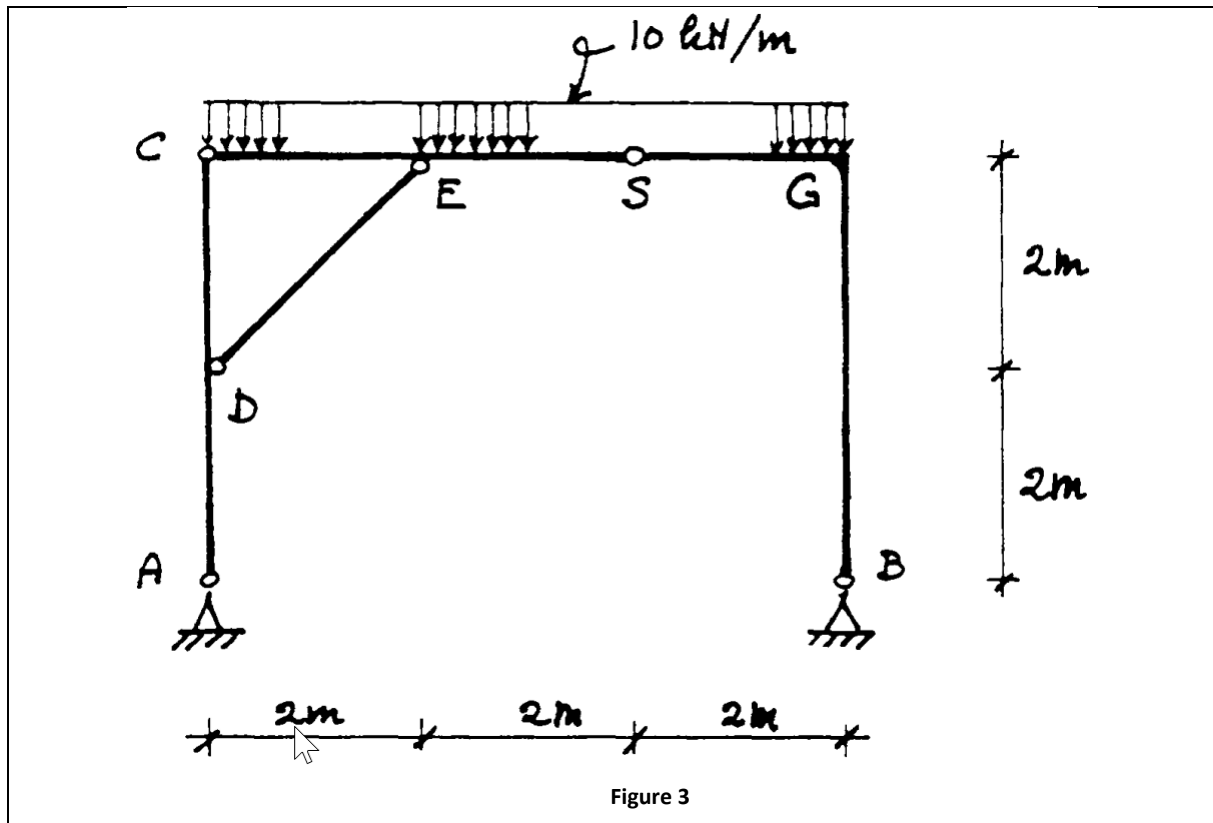
Questions:

- Determine the magnitude of the load F for which the beam remains horizontal.
- Determine the associated drop of the beam.
- Determine the associated stress in the copper bar.

Assignment 3. (35 %) Support reactions

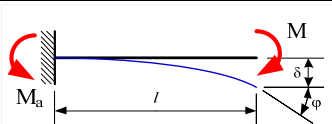
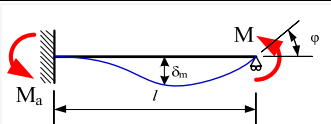
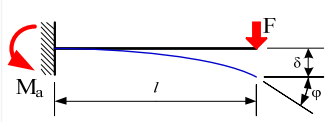
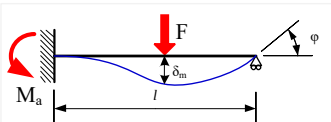
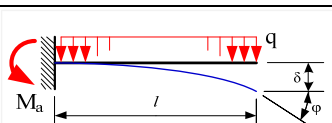
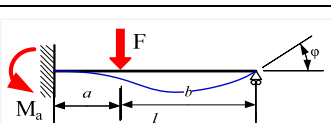
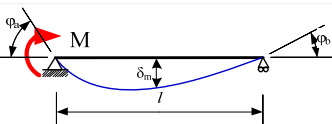
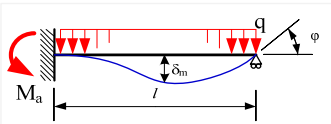
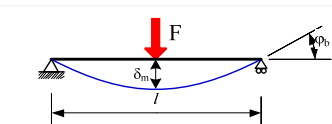
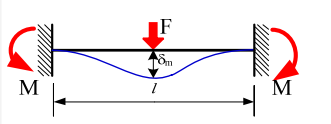
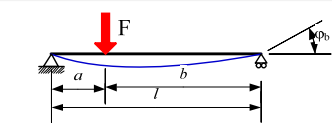
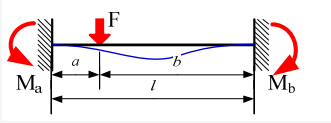
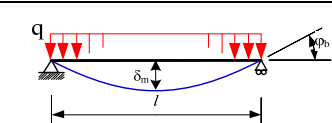
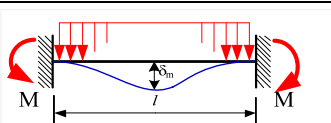
Given:

A shored three-hinged frame with a uniformly distributed full load of 10 kN/m on the girder is given.

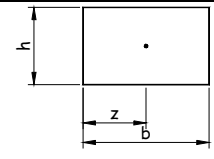
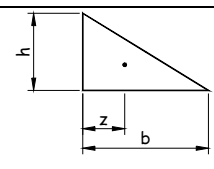
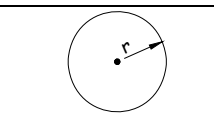
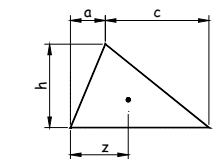
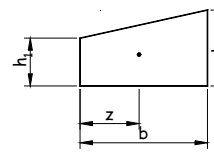
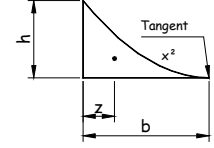
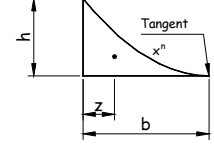
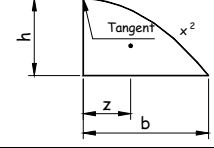
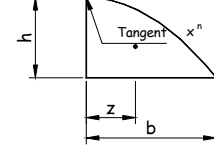
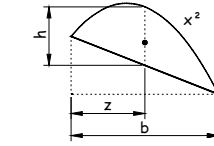


Questions:

- a) Determine the internal forces of member DE.

Index M = midden	Moment	Rotation	Deflection:			Moment	Rotation	Deflection:
	$M_a = M$	$\varphi = \frac{Ml}{EI}$	$\delta = \frac{1}{2} \frac{Ml^2}{EI}$			$M_a = \frac{1}{2} M$	$\varphi = \frac{1}{4} \frac{Ml}{EI}$	$\delta_M = \frac{1}{32} \frac{Ml^2}{EI}$
	$M_a = Fl$	$\varphi = \frac{1}{2} \frac{Fl^2}{EI}$	$\delta = \frac{1}{3} \frac{Fl^3}{EI}$			$M_a = \frac{3}{16} Fl$ $M_M = \frac{5}{32} Fl$	$\varphi = \frac{1}{32} \frac{Fl^2}{EI}$	$\delta_M = \frac{7}{768} \frac{Fl^3}{EI}$
	$M_a = \frac{1}{2} ql^2$	$\varphi = \frac{1}{6} \frac{ql^3}{EI}$	$\delta = \frac{1}{8} \frac{ql^4}{EI}$			$M_a = \frac{Fb(l^2 - b^2)}{2l^2}$	$\varphi = \frac{1}{4} \frac{Fa^2b}{EI}$	
		$\varphi_a = \frac{1}{3} \frac{Ml}{EI}$ $\varphi_b = \frac{1}{6} \frac{Ml}{EI}$	$\delta_M = \frac{1}{16} \frac{Ml^2}{EI}$			$M_a = \frac{1}{8} ql^2$	$\varphi = \frac{1}{48} \frac{ql^3}{EI}$	$\delta_M = \frac{1}{192} \frac{ql^4}{EI}$
	$M_M = \frac{1}{4} Fl$	$\varphi = \frac{1}{16} \frac{Fl^2}{EI}$	$\delta_M = \frac{1}{48} \frac{Fl^3}{EI}$			$M = \frac{1}{8} Fl$ $M_M = \frac{1}{8} Fl$		$\delta_M = \frac{1}{192} \frac{Fl^3}{EI}$
		$\varphi_a = \frac{1}{6} \frac{Fab(l+b)}{EI}$ $\varphi_b = \frac{1}{6} \frac{Fab(l+a)}{EI}$				$M_a = \frac{Fab^2}{l^2}$ $M_b = \frac{Fa^2b}{l^2}$		
	$M_M = \frac{1}{8} ql^2$	$\varphi = \frac{1}{24} \frac{ql^3}{EI}$	$\delta_M = \frac{5}{384} \frac{ql^4}{EI}$			$M = \frac{1}{12} ql^2$ $M_M = \frac{1}{24} ql^2$		$\delta_{mM} = \frac{1}{384} \frac{ql^4}{EI}$

Area+point of gravity

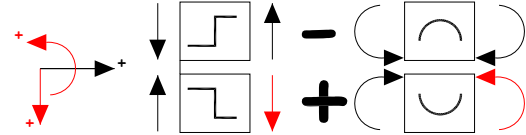
	$A = bh$ $z = \frac{1}{2}b$ $I = \frac{1}{12}bh^3$
	$A = \frac{1}{2}bh$ $z = \frac{1}{3}b$ $I = \frac{1}{36}bh^3$
	$A = \pi r^2$ $I = \frac{1}{4}\pi r^4$
	$A = \frac{1}{2}(a+c)h$ $z = \frac{1}{3}(2a+c)$
	$A = \frac{1}{2}b(h_1+h_2)$ $z = \frac{1}{3}b \frac{h_1+2h_2}{h_1+h_2}$
	$A = \frac{1}{3}bh$ $z = \frac{1}{4}b$
	$A = \frac{1}{n+1}bh$ $z = \frac{1}{n+2}b$
	$A = \frac{2}{3}bh$ $z = \frac{3}{8}b$
	$A = \frac{n}{n+1}bh$ $z = \frac{1}{2} \frac{(n+1)}{(n+2)}b$
	$A = \frac{2}{3}bh$ $z = \frac{1}{2}b$

Tangent=parallel to x-axis

Section properties

$A =$	$S_z =$	$I_{zz} =$
$\int_A dA$	$\int_A z dA$	$\int_A z^2 dA$

Sign conventions



Displacement rule of Steiner

$$I_{\bar{z}\bar{z}} = I_{zz(\text{eigen})} + \bar{z}_c^2 A$$

Basic equation for bending

$$Q(x) = -EIw''''$$

$$V(x) = -EIw'''$$

$$M(x) = -EIw''$$

$$EI\varphi(x) = -EIw'$$

$$\Delta\varphi = \varphi_B - \varphi_A = \int_A^B \frac{M}{EI} dx$$

$$\Delta w = w_B - w_A = -\varphi_{A(x_B-x_A)} - \int_A^B \frac{M}{EI} (x_B - x) dx$$

Normal tension

$$\sigma(z) = \sigma_{N(z)} + \sigma_{M(z)} = \frac{N}{A} + \frac{M \cdot z}{I}$$

Extension by axial force

$$EA_x u'_x = N_x = \int -q_x dx$$

$$EA_x u_x = \int N_x dx = \iint -q_x d^2x$$

Prismatic bar & constant force

$$\Delta l = \frac{Nl}{EA}$$

Unit load method

$$\Delta = \sum \frac{F_u \cdot F_l \cdot l}{E \cdot A}$$

Shear stress

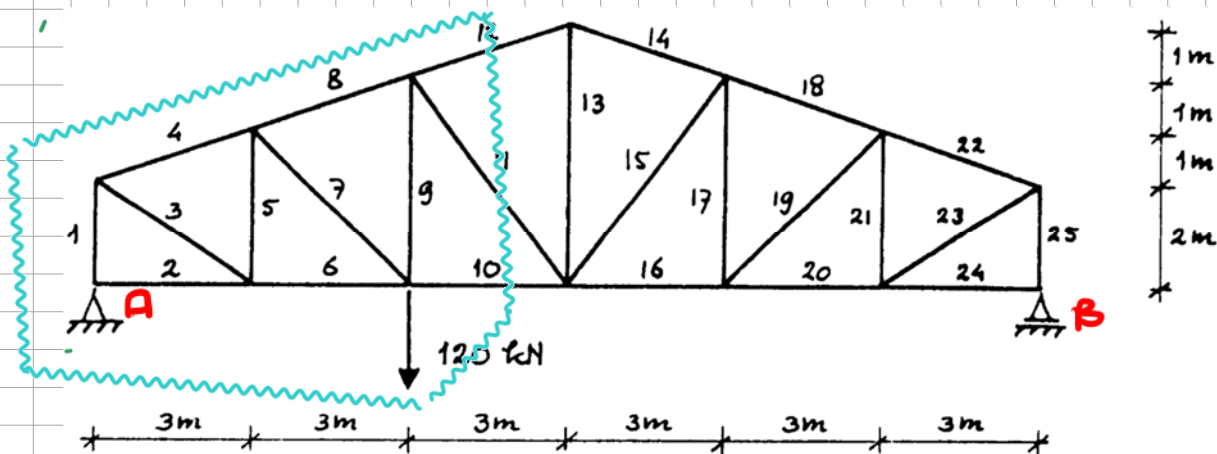
$$s_x^a = -\frac{V_z S_z^a}{I_{zz}} \quad \tau = -\frac{V_z S_z^a}{I_{zz} b^a}$$

Combinend stresses

$$\sigma_{x1} = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi - \tau_{zx} \sin 2\varphi$$

$$\sigma_{z1} = \frac{\sigma_x + \sigma_z}{2} - \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi + \tau_{zx} \sin 2\varphi$$

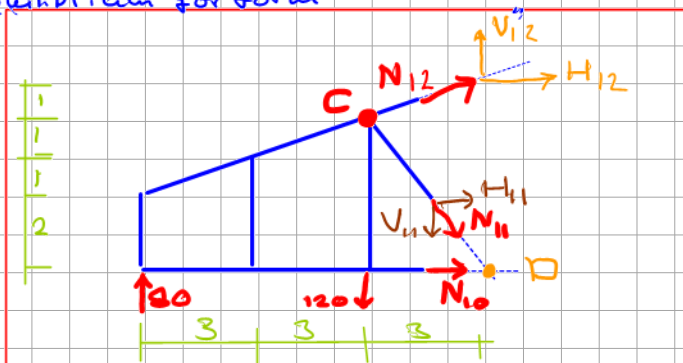
$$\tau_{x1z1} = \tau_{xz} \cos 2\varphi + \frac{\sigma_x - \sigma_z}{2} \sin 2\varphi$$



Question: Internal forces bar 10, 11, 12

Key activities: - set up equilibrium for cloud ($\sum V=0, \sum H=0, \sum M=0$)
- set up equilibrium for total

Models:



Solution:

$$\sum M_A = 0 \quad -120 \times 12 + V_B \times 36 = 0$$

$$V_B = 40 \text{ kN}$$

$$\sum V = 0 \quad -V_A - V_B + 120 = 0$$

$$-V_A - 40 + 120 = 0$$

$$V_A = 80 \text{ kN}$$

$$\sum H = 0 \quad H_A = 0$$

$$\sum M_C = 0 \quad -80 \times 6 + 4 \times N_{10} = 0$$

$$-480 + 4 N_{10} = 0$$

$$N_{10} = \frac{480}{4} = 120 \text{ kN}$$

$$\sum M_D = 0 \quad -80 \times 9 + 120 \times 3 - H_{12} \times 5 = 0$$

$$-720 + 360 - 5 H_{12} = 0$$

$$-5 H_{12} = 360$$

$$H_{12} = -72 \text{ kN}$$

$$\text{Ratio } H_{12} : V_{12} = 3 : 1$$

$$V_{12} = \frac{1}{3} H_{12} = \frac{1}{3} \times -72 = -24 \text{ kN}$$

$$N_{12} = -\sqrt{24^2 + 72^2} = -75.9 \text{ kN}$$

$$\sum V = 0 \quad -80 + 120 + V_{12} - V_{11} = 0$$

$$-80 + 120 + (-24) - V_{11} = 0$$

$$+16 - V_{11} = 0$$

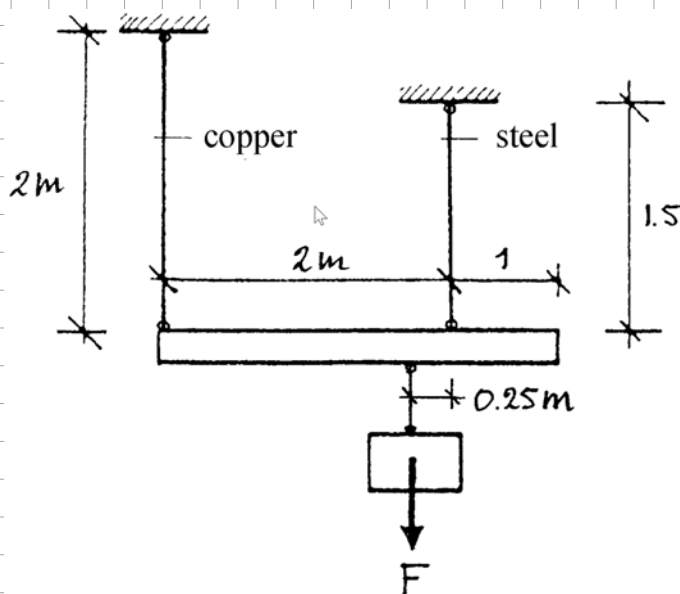
$$V_{11} = +16 \text{ kN}$$

$$\text{Ratio } H_{11} : V_{11} = 3 : 4$$

$$H_{11} = \frac{3}{4} V_{11} = \frac{3}{4} \times 16 = 12 \text{ kN}$$

$$N_{11} = -\sqrt{16^2 + 12^2} = +20 \text{ kN}$$

Answers: $N_{10} = +120 \text{ kN}$, $N_{11} = +20 \text{ kN}$, $N_{12} = -75.9 \text{ kN}$

Questions:

- Determine the magnitude of the load F for which the beam remains horizontal.
- Determine the associated drop of the beam.
- Determine the associated stress in the copper bar.

Key activities:

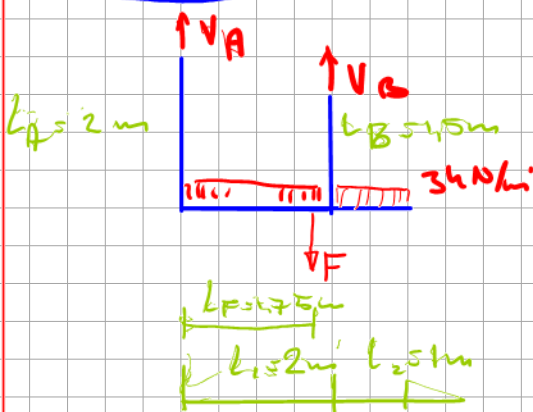
horizontal $\Rightarrow$ extension
steel = extension copper.

$$\Delta L_s = \Delta L_c$$

$$\Delta L = \frac{N \cdot L}{E \cdot A} \quad (a, b)$$

$N \Rightarrow$ out of equilibrium.

$$(c) \Rightarrow G = \frac{N}{A}$$

ModelSolving.

$$\sum V = 0 \quad -V_A + V_B + F + q \cdot (L_1 + L_2) = 0$$

$$-V_A + V_B + F + 3 \cdot (2 + 1) = 0$$

$$V_A + V_B - F = 9 \text{ kN} \quad (1)$$

$$\sum M_A = 0 \quad +V_B \cdot L_1 - F \cdot L_F - \frac{1}{2} q (L_1 + L_2)^2 = 0$$

$$+2V_B - F \cdot 2.25 - \frac{1}{2} \cdot 3 \cdot (2 + 1)^2 = 0$$

$$+2V_B - \frac{3}{4} F = 13.5 \text{ kNm} \quad (2)$$

$$\Delta L_A = \Delta L_B$$

$$\Delta L_A = \frac{V_A \cdot L_A}{E_s \cdot A_s} = \frac{[V_A \cdot 1000] \cdot 2000}{120.000 \cdot 100} = \frac{20V_A}{12}$$

$$\frac{5}{6} V_A$$

$$\Delta L_B = \frac{V_B \cdot L_B}{E_s \cdot A_s} = \frac{[V_B \cdot 1000] \cdot 1500}{210.000 \cdot 200} = \frac{7.5V_B}{84}$$

$$\frac{1}{28} V_B$$

$$\frac{5}{6} V_A = \frac{1}{28} V_B$$

$$V_A = \frac{6}{28} V_B = \frac{3}{14} V_B \quad (3)$$

(5) in (1)

$$\frac{3}{14} V_B + V_B - F = 9 \text{ kN}$$

$$\frac{17}{14} V_B - F = 9$$

$$F = \frac{17}{14} V_B - 9 \quad (4)$$

$$\begin{aligned} \textcircled{3} \& \textcircled{4} \text{ in } \textcircled{2} \quad +2 \times V_B - \frac{7}{4} (1\frac{7}{4} V_B - 9) &= 13.5 \text{ kNm} \\ +2V_B - 1\frac{7}{8} V_B + \frac{63}{4} &= 13.5 \\ -\frac{1}{8} V_B &= -2.25 \end{aligned}$$

$$V_B = 2.25 \times 8 = 18 \text{ kN} \quad \textcircled{5}$$

$$\textcircled{5} \text{ in } \textcircled{3} \quad V_A = \frac{3}{4} \times 18 = \frac{27}{2} = 13.5 \text{ kN} \quad \textcircled{6}$$

$$\textcircled{5} \text{ in } \textcircled{4} \quad F = 1\frac{7}{4} \times 18 - 9 = \frac{30}{2} = 15 \text{ kN}$$

$$\begin{aligned} \Delta L_B &= \frac{V_B \times L_B}{E_B \times A_B} = \frac{18 \times 10^3 \times 1500}{210.000 \times 200} = 0.642 \text{ mm} \\ \Delta L_A &= \frac{V_A \times L_A}{E_C \times A_C} = \frac{13.5 \times 1000 \times 2000}{120.000 \times 100} = 0.642 \text{ mm} \end{aligned} \quad \left. \vphantom{\begin{aligned} \Delta L_B \\ \Delta L_A \end{aligned}} \right\} \text{check!}$$

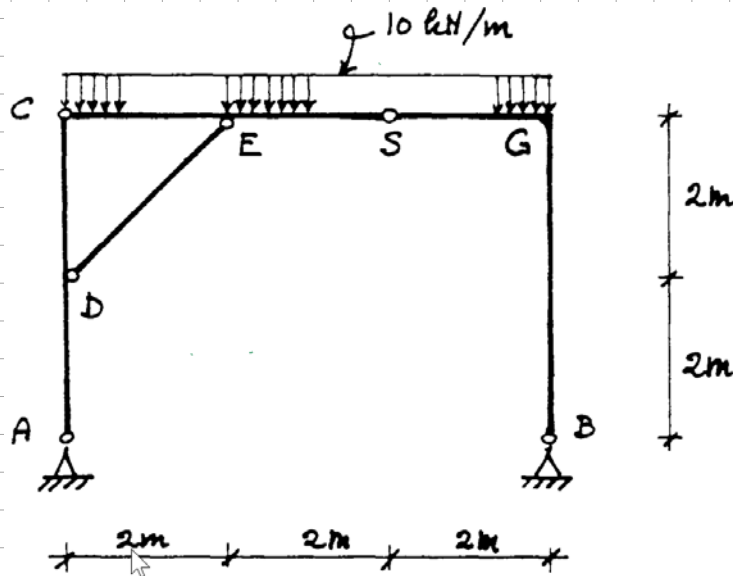
$$\sigma_c = \frac{N}{A} = \frac{V_A}{A_c} = \frac{13.5}{100} = 0.135 \text{ N/mm}^2$$

or

$$\begin{aligned} \sigma_c &= E_c \times \epsilon_c = 120.000 \times 0.000321 = 38.57 \text{ N/mm}^2 \\ \epsilon_c &= \frac{\Delta L_A}{L_A} = \frac{0.642}{2000} = 0.000321 \end{aligned}$$

Answers:

- A: $F = 15 \text{ kN}$
- B: $\Delta L = 0.642 \text{ mm}$
- C: $\sigma_c = 38.6 \text{ N/mm}^2$



Question:
internal forces DE

Key activities

per cloud:

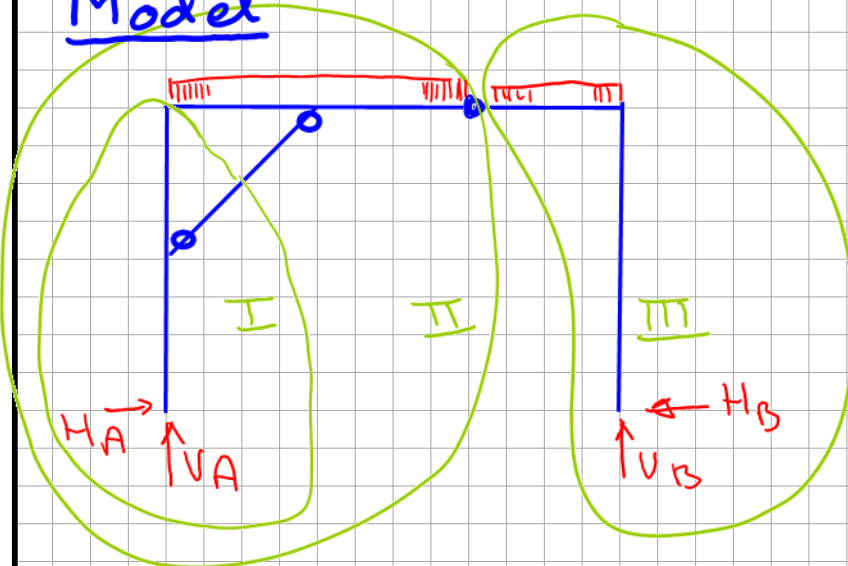
$\sum V = 0$

$\sum H = 0$

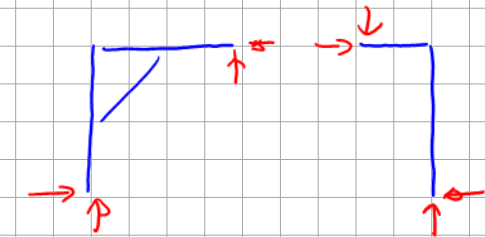
$\sum M = 0$

bar DE: pendulum bar.

Model



Cloud I:
5 unknown
3 equations
→ unsolvable



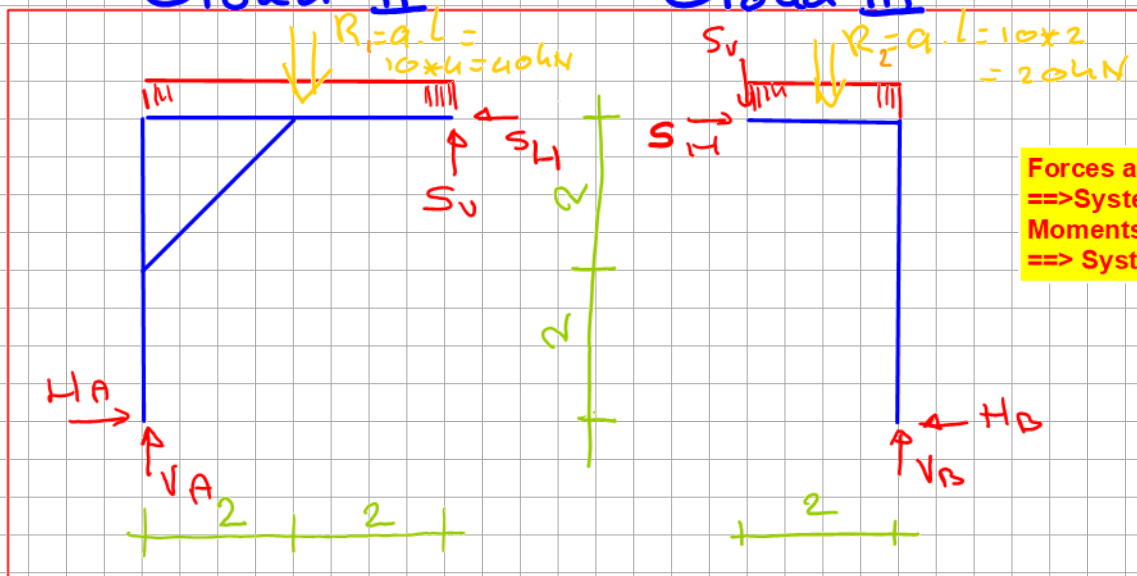
Cloud II & III

6 unknowns
6 equations
→ solvable.

Solving.

Cloud II

Cloud III



Forces at S in same direction
⇒ System Error!
Moments at A or B
⇒ System Error!

$$\sum H=0 \quad H_A - S_H = 0$$

$$\sum V=0 \quad -V_A + R_1 - S_V = 0$$

$$\sum M_A = 0 \quad -R_1 \cdot 2 + S_V \cdot 4 + S_H \cdot 4 = 0$$

$$-80 + 4S_V + 4S_H = 0$$

$$\sum H=0 \quad S_H - H_B = 0$$

$$\sum V=0 \quad S_V + R_2 - B = 0$$

$$\sum M_B = 0 \quad +S_V \cdot 2 - S_H \cdot 4 + R_2 \cdot 1 = 0$$

$$2S_V - 4S_H + 20 = 0$$

$$4S_H = 80 - 4S_V = 2S_V + 20$$

$$60 = 6S_V$$

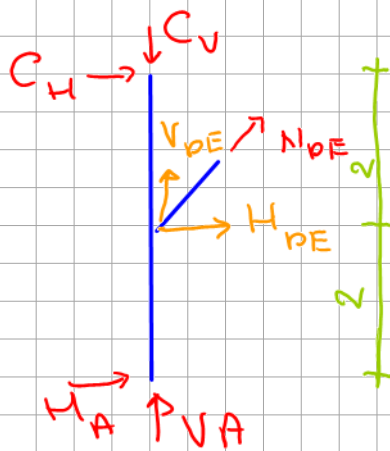
$$S_V = 10 \text{ kN}$$

$$S_H = \frac{80 - 4 \cdot 10}{4} = 10 \text{ kN}$$

$$H_A = S_H = 10 \text{ kN}$$

$$V_A = R_1 - S_V = 40 - 10 = 30 \text{ kN}$$

Cloud I



Moments at A or C
=> System Error!

$$\sum V=0 \quad C_V - V_A - V_{DE} = 0$$

$$\sum H=0 \quad C_H + H_{DE} + H_A = 0$$

$$\sum M_C = 0 \quad H_{DE} \cdot 2 + H_A \cdot 4 = 0$$

$$H_{DE} : V_{DE} = 1:1 \Rightarrow H_{DE} = V_{DE}$$

$$H_{DE} \cdot 2 + 10 \cdot 4 = 0$$

$$H_{DE} = \frac{-40}{2} = -20 \text{ kN}$$

$$V_{DE} = -20 \text{ kN}$$

$$N_{DE} = -20\sqrt{2} \text{ kN} (-28,28)$$

Answer

Internal forces in bar DE:

$$N_{DE} = -28,3 \text{ kN}$$

$$V_{DE} = 0$$

$$M_{DE} = 0$$

Name:..... Course: Structural Mechanics in Module 1

Studentnumber:..... Date: 02/10/2020

Test 1

Continuation of

1

Use this sheet for ONE assignment only!

Name:..... Course: Structural Mechanics in Module 1

Studentnumber:..... Date: 02/10/2020

