

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

A

Only for sorting.

Answer form

Calculus 2 - Resit - Friday 4 July, 2025

Full name in BLOCK LETTERS

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Student number

Programme

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CSE

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Other:

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. Fill in the blanks:

$$A = \boxed{5}$$

$$B = \boxed{-4}$$

$$C = \boxed{-2}$$

Question 2. $\frac{dy}{dx}$ at P is:

$$\boxed{3/16}$$

Question 3(a). The discriminant (Hessian) $\Delta(0,0)$ is:

$$\boxed{1}$$

Question 3(b). Mark your answer with an X.

☐

local maximum

☒

local minimum

☐

saddle point

Question 4. Fill in the blanks:

$$\int_{\boxed{2}}^{\boxed{6}} \int_{\boxed{y/2}}^{\boxed{1+\sqrt{y-2}}} \boxed{(x-1)^2} dx dy$$

Question 5. Fill in the blanks:

$$\text{Vol}(R) = \int_{\boxed{0}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{\pi/3}} \int_{\boxed{0}}^{\boxed{\sqrt{7}}} \boxed{\rho^2 \sin \varphi} d\rho d\phi d\theta$$

Question 6. The Jacobian $J(u,v)$ is:

$$\boxed{1/5}$$

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

B

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Question 1. Fill in the blanks:

$$A = \boxed{6} \quad B = \boxed{-2} \quad C = \boxed{-8}$$

Question 2. $\frac{dy}{dx}$ at P is:

$$\boxed{16/3}$$

Question 3(a). The discriminant (Hessian) $\Delta(0,0)$ is:

$$\boxed{1}$$

Question 3(b). Mark your answer with an X.



local maximum



local minimum



saddle point

Question 4. Fill in the blanks:

$$\int_{\boxed{3}}^{\boxed{12}} \int_{\boxed{y/3}}^{\boxed{1+\sqrt{y-3}}} \boxed{(x-1)^2} dx dy$$

Question 5. Fill in the blanks:

$$\text{Vol}(R) = \int_{\boxed{0}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{\pi/6}} \int_{\boxed{0}}^{\boxed{\sqrt{5}}} \boxed{\rho^2 \sin \varphi} d\rho d\phi d\theta$$

Question 6. The Jacobian $J(u,v)$ is:

$$\boxed{1/3}$$

Question 7. Write a full calculation/argumentation clearly in the box.

Interior: $\nabla f = \langle 2xe^x + (x^2+y^2)e^x, 2ye^x \rangle = \langle (x^2+2x+y^2)e^x, 2ye^x \rangle$

$$\nabla f = \langle 0, 0 \rangle \text{ iff } \begin{cases} (x^2+2x+y^2)e^x = 0 \\ 2ye^x = 0 \end{cases} \text{ iff } \begin{cases} x^2+2x+y^2 = 0 \\ y = 0 \end{cases} \text{ iff } \begin{cases} x(x+2) = 0 \\ y = 0 \end{cases}$$

So the critical points are (0,0) and (-2,0), which is not in the interior of R .

Boundary

- The arc parametrizes as $r(t) = \langle 2\cos t, 2\sin t \rangle$, $t \in [\frac{\pi}{3}, \frac{5\pi}{3}]$.

Restriction of f : $g(t) = f(r(t)) = 4e^{2\cos t}$

$$g'(t) = -8\sin t \cdot e^{2\cos t}$$

$g'(t) = 0$ iff $\sin t = 0$ iff $t = \pi$ (in the domain),
giving the critical point (-2,0).

- On the line $\langle 1, y \rangle$ for $y \in [-\sqrt{3}, \sqrt{3}]$:

Restriction of f : $h(y) = f(1, y) = e \cdot (1+y^2)$

$$h'(y) = 2 \cdot e \cdot y$$

$$h'(y) = 0 \text{ iff } y = 0,$$

giving the critical point (1,0).

- The endpoints of both curves, (1, $\sqrt{3}$) and (1, $-\sqrt{3}$) are also candidates.

Evaluation: $f(0,0) = 0$, $f(-2,0) = 4/e^2$, $f(1,0) = e$, $f(1, \pm\sqrt{3}) = 4e$,

So the maximum is $4e$, attained at $(1, \sqrt{3})$ and $(1, -\sqrt{3})$,
and the minimum is 0 , attained at $(0,0)$.

Question 8(a). Write a full calculation/argumentation clearly in the box.

Constraint function: $g(x,y) = (x-1)^2 + 2y^2 - 4$

The critical points are solutions to the Lagrange system:
$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 0 \end{cases}$$

$$\text{i.e. } \begin{cases} 1 = 2\lambda(x-1) \\ 4 = 4\lambda y \\ (x-1)^2 + 2y^2 = 4 \end{cases} \quad \text{i.e. } \begin{cases} 1 = 2\lambda(x-1) \\ 1 = \lambda y \\ (x-1)^2 + 2y^2 = 4. \end{cases}$$

Note that $\lambda \neq 0$ (otherwise the second equation gives $1=0$), so the system becomes:

$$\begin{cases} x-1 = 1/2\lambda \\ y = 1/\lambda \\ 1/4\lambda^2 + 2/\lambda^2 = 4 \end{cases} \quad \text{i.e. } \begin{cases} x = 1 + 1/2\lambda \\ y = 1/\lambda \\ \lambda^2 = 9/16 \rightarrow \lambda = \pm 3/4 \end{cases}$$

$$\lambda = 3/4 \text{ gives } (x,y) = (1 + 2/3, 4/3) = (5/3, 4/3)$$

$$\lambda = -3/4 \text{ gives } (x,y) = (1 - 2/3, -4/3) = (1/3, -4/3)$$

$$f(5/3, 4/3) = 5/3 + 16/3 + 5 = 12$$

$$f(1/3, -4/3) = 1/3 - 16/3 + 5 = 0$$

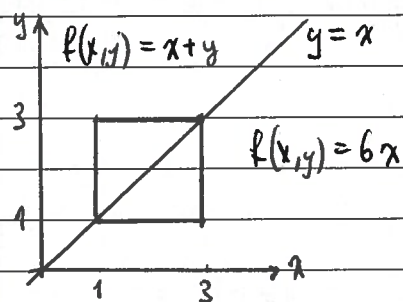
So the maximum is 12, attained at $(5/3, 4/3)$,
and the minimum is 0, attained at $(1/3, -4/3)$.

Question 8(b). Write a full calculation/argumentation clearly in the box.

They are the same curve.

Question 9. Write a full calculation/argumentation clearly in the box.

Figure (not required):



$$\text{Area}(S) = 2^2 = 4$$

$$\text{avg}_S(f) = \frac{\iint_S f(x,y) dA}{\text{Area}(S)} = \frac{1}{4} \iint_S f(x,y) dA$$

$$\iint_S f(x,y) dA = \int_1^3 \int_x^3 (x+y) dy dx + \int_1^3 \int_1^x 6x dy dx =$$

$$= \int_1^3 \left[xy + \frac{1}{2} y^2 \right]_x^3 dx + \int_1^3 [6xy]_1^x dx =$$

$$= \int_1^3 \left(3x + \frac{9}{2} - x^2 - \frac{1}{2} x^2 \right) dx + \int_1^3 (6x^2 - 6x) dx = \textcircled{*}$$

$$= \int_1^3 \left(\frac{9}{2} x^2 - 3x + \frac{9}{2} \right) dx = \left[\frac{3}{2} x^3 - \frac{3}{2} x^2 + \frac{9}{2} x \right]_1^3 =$$

$$= \frac{3}{2} [x^3 - x^2 + 3x]_1^3 = \frac{3}{2} (27 - 9 + 9 - 1 + 1 - 3) =$$

$$= \frac{3}{2} \cdot 24 = 36,$$

$$\text{so } \text{avg}_S(f) = \frac{1}{4} \cdot 36 = 9$$

* Alternatively: the first integral computes to 8, and the second one to 28.

Question 10. Write a full calculation/argumentation clearly in the box.

(a) D in cylindrical coordinates: $\theta \in [0, \pi/2]$, $z \in [-3, 3]$.

Bounds for r : the lower is $x^2 + (y-1)^2 = 1$ i.e. $r^2 - 2r\sin\theta + 1 = 1$
i.e. $r = 2\sin\theta$, the higher is $x^2 + y^2 = 4$ i.e. $r = 2$.

$$\Rightarrow M = \int_0^{\pi/2} \int_{2\sin\theta}^2 \int_{-3}^3 (1+z^2) \cdot r \, dz \, dr \, d\theta.$$

$$(b) M_{yz} = \int_0^2 \int_{\sqrt{1-(y-1)^2}}^{\sqrt{4-y^2}} \int_{-3}^3 x(1+z^2) \, dz \, dx \, dy =$$

$$= \int_0^2 \int_{\sqrt{1-(y-1)^2}}^{\sqrt{4-y^2}} x \left[z + \frac{1}{3} z^3 \right]_{-3}^3 \, dx \, dy =$$

$$= 24 \int_0^2 \int_{\sqrt{1-(y-1)^2}}^{\sqrt{4-y^2}} x \, dx \, dy = 12 \int_0^2 (4-y^2-1+(y-1)^2) \, dy =$$

$$= 12 \int_0^2 (4-2y) \, dy = 12 [4y - y^2]_0^2 = 12(8-4) = 12 \cdot 4 = 48.$$

$$\bar{x} = \frac{M_{yz}}{M} = 48/12\pi = 4/\pi.$$

(c) D is symmetric with respect to the xy -plane, and the density is too (i.e. $\delta(x, y, z) = \delta(x, y, -z)$).