

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 13 June, 2025

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2425-Q4-EXAM

## Calculus 2

### Exam

### Instructions

This exam contains 10 exercises. You shall use the provided *answer form* to submit your answers.

- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
- For exercises 7–10, you are required to write down a **full calculation and argumentation**.

You will hand in your answer form only. **Any text outside the answer form will not be considered.**

If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.

Do not write with red pen or pencil. Do not use correction fluid or tape.

**⚠ THE USE OF ELECTRONIC DEVICES IS NOT ALLOWED. ⚠**

### Final answer questions

**Report all your answers on the answer form.**

1. Compute the directional derivative of  $f(x, y, z) = 6e^x \cos(yz)$  at the point  $(0, 0, 0)$  in the direction of  $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ . [3 pt]

2. The function  $f(x, y) = 3 \sin(xy) - x^2 + y^2$  has gradient:

$$\nabla f(x, y) = \langle 3y \cos(xy) - 2x, 3x \cos(xy) + 2y \rangle.$$

- (a) Compute  $f_{xy}(0, 0)$ . [1 pt]
- (b) Determine whether  $f$  has a local maximum, a local minimum, or a saddle point at  $(0, 0)$ . [2 pt]

3. Let  $f(x, y)$  be a differentiable function defined on  $\mathbb{R}^2$ , and let  $P_2(x, y)$  be the second-order Taylor polynomial generated by  $f$  at  $(a, b)$ . [2 pt]

**Only one** of the following statements is **always true**; identify it.

- ☐ A If  $(a, b)$  is a critical point of  $f$ , then  $P_2(a, b) = 0$ .
- ☐ B The point  $(a, b)$  is a critical point of  $P_2(x, y) - f(x, y)$ .
- ☐ C If  $f(x, y)$  is always positive, then  $P_2(x, y)$  is always positive.
- ☐ D  $P_2(x, y)$  has no critical points.
- ☐ E If  $f$  has a local minimum at  $(a, b)$ , then  $P_2(x, y)$  is always positive.

4. Let  $R$  be the region in the  $xy$ -plane bounded above by the parabola  $y = 4 - (x - 4)^2$  and below by the  $x$ -axis. Rewrite [3 pt]

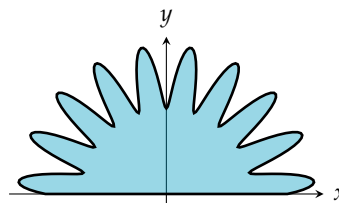
$$\iint_R (y - 4) \, dA$$

as iterated double integrals:

- (a) using the order of integration  $dydx$ .
- (b) using the order of integration  $dx dy$ .

Continues on the following page.

5. The region  $R$  of the plane, shaded in the figure, is bounded by the  $x$ -axis and by the curve whose polar equation is  $r = \sqrt{4 + 2\sin(19\theta)}$ . Compute its area.



[3 pt]

6. Below is a list of points whose  $(x, y, z)$ -coordinates are given. Among those, find all the points that belong to the cone whose equation in spherical coordinates is  $\phi = \frac{3\pi}{4}$ .

[2 pt]

- ☐ A  $(-1, \sqrt{3}, -2)$      
 ☐ B  $(1, -\sqrt{3}, -2)$      
 ☐ C  $(-1, -\sqrt{3}, 2)$      
 ☐ D  $(1, \sqrt{3}, 2)$   
☐ E  $(-\sqrt{2}, \sqrt{2}, 4)$      
 ☐ F  $(\sqrt{2}, -\sqrt{2}, 4)$      
 ☐ G  $(-\sqrt{2}, \sqrt{2}, -4)$      
 ☐ H  $(\sqrt{2}, -\sqrt{2}, -4)$

## Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

*Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.*

7. Find the absolute maximum and minimum values attained by the function

[5 pt]

$$f(x, y) = \frac{x^2 - y^2 - 2}{x^2 + y^2 + 1}$$

on the closed, bounded region  $D$  given by the disk  $x^2 + y^2 \leq 4$ .

Clearly specify at which points these values are attained.

8. Use the method of Lagrange multipliers to find the absolute maximum and minimum values attained by the function  $f(x, y) = y^2 - (x - 2)^2$  on the ellipse  $(x - 2)^2 + 2(y - 3)^2 = 8$ .

[5 pt]

Clearly specify at which points these values are attained.

9. Consider the following transformation:

[5 pt]

$$\begin{cases} u = \frac{y}{x} \\ v = x + y \end{cases} \quad \text{or, equivalently,} \quad \begin{cases} x = \frac{v}{u+1} \\ y = v - \frac{v}{u+1} \end{cases}.$$

Use it to compute  $\iint_R \left(\frac{y}{x} + 1\right)^2 dA$ , where  $R$  is the region bounded by the lines:

$$y = 2x, \quad y = 3x, \quad y = -x + 4, \quad y = -x + 6.$$

10. The solid brick  $B$  is bounded by the planes  $x = -1$ ,  $x = 1$ ,  $y = -1$ ,  $y = 1$ ,  $z = -3$  and  $z = 3$ . Its mass is  $M = 160$ , and its density is described by the function:

$$\delta(x, y, z) = x^2 + y^2 - 2x - 4y + 6.$$

- (a)** Determine the first moments  $M_{xy}$  and  $M_{yz}$  of  $B$  about the coordinate planes. [3 pt]  
**(b)** Compute the average value of  $\delta(x, y, z)$  over  $B$ . [1 pt]  
**(c)** Let  $a$  be the average value of  $\delta(x, y, z)$  over  $B$  (its exact value does not matter). [1 pt]  
 A second brick  $C$  has the same shape as  $B$ , but its density is constant and equal to  $a$ .  
 Do  $B$  and  $C$  share the same center of mass? Explain your answer.

**Total: 36 pt**

|   |   |   |   |   |   |   |   |   |    |          |
|---|---|---|---|---|---|---|---|---|----|----------|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\Sigma$ |
|   |   |   |   |   |   |   |   |   |    |          |

Do not fill in this table.

A

Only for sorting.

# Answer form

Calculus 2 – Exam – Friday 13 June, 2025

Full name in BLOCK LETTERS \_\_\_\_\_

Student number \_\_\_\_\_

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: \_\_\_\_\_

**⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠**

Question 1. The directional derivative is:

Question 2(a).  $f_{xy}(0,0) =$

Question 2(b). Mark your answer with an X.

☐ local maximum ☐ local minimum ☐ saddle point

Question 3. Mark the correct option with an X. ↓ Use this space for corrections. ↓

☐ A ☐ B ☐ C ☐ D ☐ E

Question 4(a). Rewrite the integral using the order of integration  $dydx$ :

Question 4(b). Rewrite the integral using the order of integration  $dx dy$ :

Question 5. The area of  $R$  is:

Question 6. Mark all and only the correct options with an X. ↓ Use this space for corrections. ↓

☐ A ☐ B ☐ C ☐ D ☐ E ☐ F ☐ G ☐ H

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]