

Exam: Vector Calculus for BMT, CE, CSE, ME

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2024/2025, Q-1A

Give a suitable explanation of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

Question 1. Consider the vector field $\mathbf{F}(x, y) = (x + y)\mathbf{i} - (x + y)\mathbf{j}$, and the points $A = (2, 1)$ and $B = (-1, -2)$. The curve C_1 is the line segment from A to B , and the curve C_2 is the part of the parabola $x + y^2 = 3$ from B to A (note the order!).

- (a) Calculate the work done by $\mathbf{F}$ when moving an object from A to B along C_1 .
- (b) Calculate the flow of $\mathbf{F}$ along C_2 .
- (c) The (closed) curve C is made up of C_1 followed by C_2 . Sketch the curve C and calculate the flux of $\mathbf{F}$ outward across C without computing line integrals.

Question 2. The surface S is the portion of the paraboloid $z = x^2 + y^2$ between the planes $z = 2$ and $z = 4$.

- (a) Make a clear sketch of the surface S .
- (b) Calculate the area of S using a parametrization of S .

Question 3. The surface S is the portion of the plane $x + y + z = 3$ in the first octant (hence $x \geq 0$, $y \geq 0$, $z \geq 0$). Consider the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + z\mathbf{j} - xy\mathbf{k}$.

- (a) Find a parametrization of S .
- (b) Use the parametrization found in (a) to calculate the flux of $\mathbf{F}$ across S in the direction of the unit normal $\mathbf{n}$ on S which has a negative $\mathbf{k}$ component.
- (c) Calculate the circulation of $\mathbf{F}$ around the boundary C of the surface S . The orientation of the curve C is clockwise when viewed from above, i.e., from positive $\mathbf{k}$ -direction. Use a Theorem (Green, Stokes or Gauss) to find this circulation.

Points:

Q1.	(a) 2	Q2.	(a) 1	Q3.	(a) 1
	(b) 3		(b) 3		(b) 3
	(c) 2				(c) 3

Total: $18 + 2 = 20$ points