

Tag : Toetsen/21-22/C1A.21-22[v1].Test
Course : **Intro to Math + Calculus 1A**
Date : Friday October 29th, 2021
Time : 13:45 – 15:45

Motivate all your answers.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Final answer

On the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

1. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Let $P = (1, 1, 0)$, $Q = (1, 1, 1)$, and $R = (2, -1, 2)$.

Define the vectors $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$.

- (a) [1 pt] Calculate $\mathbf{u} \times \mathbf{v}$.
(b) [2 pt] Calculate the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

2. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Let $P = (1, -1, 4)$, $Q = (3, 2, 3)$ be two points in $\mathbb{R}^3$.

- (a) [2 pt] Find parametric equations for the line l_1 going through P and Q .
(b) [2 pt] Find a simplified component equation for the plane through P and perpendicular to the line $l_2 : x = 1 - t, y = 3 - 2t, z = 2 - t$.

3. Define the function $f: \mathbb{R} \rightarrow \mathbb{R}$ as follows:

$$f(x) = \sqrt{x^2 \cos x + 3x^2}$$

- (a) [3 pt] Calculate $\lim_{x \rightarrow 0^+} \frac{f(x)}{x}$ and $\lim_{x \rightarrow 0^-} \frac{f(x)}{x}$, and conclude that $\lim_{x \rightarrow 0} \frac{f(x)}{x}$ does not exist.
- (b) [2 pt] Show that f is not differentiable at 0.

4. Define the function $f: \mathbb{R} \rightarrow \mathbb{R}$ as follows:

$$f(x) = x^4 - 2x^2$$

- (a) [2 pt] Find all critical points of f .
- (b) [2 pt] Find the absolute extreme values of f on the interval $[-1, 2]$, and specify where the extreme values are attained (that is: specify all x_0 for which $f(x_0)$ is a maximum or minimum value).

5. [3 pt] Let

$$f(x, y) = \frac{x^3 + 2xy^2}{\sqrt{x^2 + y^2}}$$

Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist? Explain your reasoning.

6. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

The equation for the tangent plane to the graph of the function

$$f(x, y) = 3 + \frac{x^2}{16} + \frac{y^2}{9}$$

is given as $3x - 4y + 6z = 6$ at the point (a, b, c) , where $c = f(a, b)$.

- (a) [2 pt] Find the point (a, b, c)
- (b) [1 pt] Calculate the linearization of f at $(-4, 3)$.

Total: 22 points