

Short Waves

The periodic short surface waves as we can generally observe on beaches have typical wave lengths between 1 and 1000 m and wave periods between 1 and 30 s.

On the other hand: friction forces at the seabed, which often form a dominant factor in tidal wave propagation, are small for short waves and are generally neglected.

In Coastal Engineering practice, knowledge about short waves and their behavior is of general importance for nearshore coastal management (e.g. coastal defense, harbors, coastal erosion) since they form an important driving force for sand transport processes in the coastal zone and also have a direct impact on coastal structures like dikes, harbor dams, breakwaters etc. At open sea short waves form an important factor for the construction of wind farms, oil and gas exploitation platforms and e.g. for the routing of ships in connection to weather forecasts.

Assumptions for equations

- The kinematic water viscosity is rather small ($\sim 10^{-6} \text{ m}^2/\text{s}$). As a result, the molecular diffusion is generally much smaller than the other terms in the equations such as inertia:
- Due to oscillating nature of the flow a new turbulent boundary layer will be developed at the seabed each half wave cycle after the flow changed its direction (flow reversal). Due to the small period of short surface waves the thickness w of the wave boundary layer will remain small ($w \ll$ water depth; order of magnitude: centimeters) and the turbulence induced by bottom friction will stay close to the sea bed

Irrotational flow

A flow is irrotational when the resulting angular velocity or rotation, is equal to zero

- Boundaries
- At the (fixed, horizontal) bed, $z = h$, the vertical velocity equals zero: $w = 0$ and so
- At the water surface, $z = \zeta$, the pressure is equal to the atmospheric pressure that is assumed to be zero: $p=0$ and so:

The kinematic boundary condition at the water surface, $z = \zeta$, expresses that a water particle follows the water surface, since there is no net water flux through the surface

Linear wave theory for progressive waves

Under the assumption that the waves are low with respect to their length as well as to the water depth, i.e. $H/L \ll 1$ and $H/h \ll 1$, the non-linear advection terms in the Bernoulli-equation, (1.16), can be neglected and the flow is now described by the following two linear equations:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$
$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho w} + gz = 0$$

- Note that because this is a linear theory, in both free-surface boundary conditions the values of the velocity potential at the fixed mean level $z = 0$ are used, instead of at $z = \zeta$.

General solution for progressive sinusoidal waves

$a = \frac{1}{2} H$, L = wave length, T = wave period, H/L = wave steepness. Velocity $c_w = L/T$.

$$\phi(x,t) = a \sin(\omega t - kx)$$

$$\omega = 2\pi / T$$

ω wave angular frequency

$$k = 2\pi / L$$

k wave number

$$c_w = \omega / k \rightarrow \text{velocity}$$

Substitution of this solution in the remaining boundary condition at the water surface leads to the so-called dispersion relation:

$$\omega^2 = gk \tanh(kh) \rightarrow \text{dispersion relation}$$

→ Long waves travel faster.

→ Waves travel faster in deep water.

Pressure under wave $p(x, z, t) = \underbrace{-\rho_w g z}_{\text{Still water column} \leftarrow \text{hydrostatic}} + \underbrace{\rho_w g a \frac{\cosh(k(z+h))}{\cosh(kh)}}_{\text{hydrodynamic!} \rightarrow \text{Due to the progressive wave-}} \cdot \sin(\omega t - kx)$

Approaching the bed ($z = -h$) the horizontal velocity amplitude becomes constant (shape is vertically uniform) and the vertical velocity amplitude approaches zero

$$U(z = -h) = \frac{\omega a}{\sinh(kh)}$$

$$\hat{w}(z = -h) = 0$$

Deep and shallow water approximations

In deep water the particles follow circular orbits (horizontal and vertical velocities have the same magnitude) that decrease towards the bed. In shallow water the orbits stretch in the horizontal direction (elliptic orbits), and become purely horizontal at the bed $z = h$. The horizontal orbital velocity component is vertically uniform in shallow water.

	Deep	Shallow	For full relation
L	$\frac{gT^2}{2\pi}$	$T\sqrt{gh}$	multiply deep by $\tanh(kh)$
c_w	$\frac{gT}{2\pi}$	$\sqrt{gh}$	
	$L = L_0 \tanh\left(\frac{2\pi h}{L}\right)$		$L_0 = \frac{gT^2}{2\pi}$

Non-linearity/irregularity

Wave skewness (Asimetría de la ola)

The linear wave theory as described in the previous section only holds for relatively small wave heights H in comparison with the water depth h and the wave length L . This deformation of the wave leads to wave skewness with higher but short wave crests and longer but less deep wave troughs.

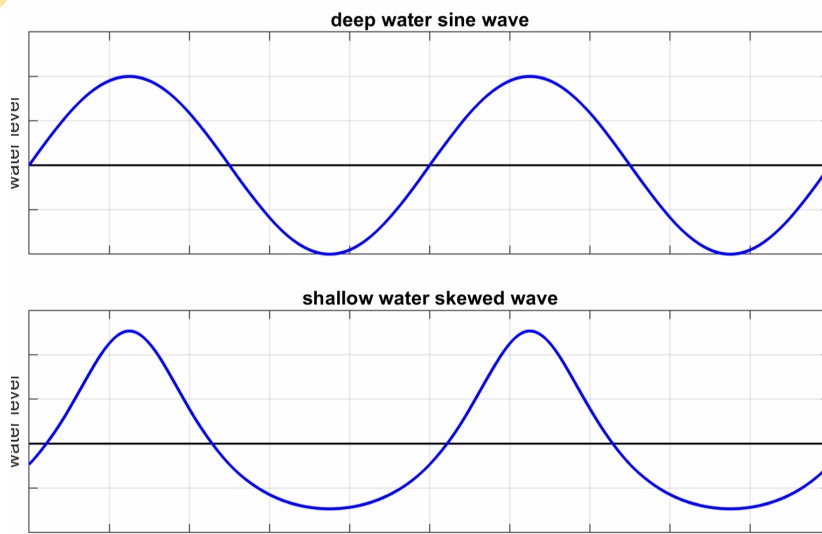
A well-known non-linear wave theory is the second-order theory of Stokes. The deformation of the wave is reflected in the increased importance of an additional higher harmonic with frequency $2!$, superimposed on the basic wave with frequency $1!$. Stokes' second-order solution for the horizontal velocity is:

$$U_1 = \frac{M L^2}{h^3} \rightarrow \text{check for usefulness of linear wave theory}$$

In equation of skewness $\hat{u}_2(z) = \frac{3}{4} k \omega a^2 \frac{\cosh[2k(h+z)]}{\sinh^4(kh)} \cdot (\max 0.25 \beta_1)$

The figure shows the increasing orbital velocities coming from deep water into shallow water and it shows that the non-linear deformation of waves start to become important at deeper water for increasing wave heights. The strong near-shore reduction of wave height and velocities is caused by

wave breaking



Tutorial 1

Describe whether the following waves are in shallow, intermediate or deep water.

- $L=100\text{m}, T=8.0\text{s}, H=5.0\text{m}, h=100\text{m}$
- $L=100\text{m}, T=8.0\text{s}, H=1.0\text{m}, h=100\text{m}$
- $L=50\text{m}, T=8.0\text{s}, H=1.0\text{m}, h=5\text{m}$
- $L=140\text{m}, T=20.0\text{s}, H=1.0\text{m}, h=5\text{m}$
- $L=600\text{km}, T=12.4\text{hrs}, H=1.0\text{m}, h=20\text{m}$
- $L=4500\text{km}, T=12.4\text{hrs}, H=1.0\text{m}, h=1000\text{m}$

Deep water		Shallow water.	
$h > 0.5L$		$h < 0.05L$	
a)	$100 > 50$ ✓		
b)	$100 > 50$ ✓		
c)	$5 > 25$ ✗	Intermediate	$2.5 > 5\text{m}$ ✗
d)	$5 > 70$ ✗		$7 > 5\text{m}$ ✓
e)	$20 > 300000$ ✗	Intermediate	$30000 > 20$ ✗
f)	$1000 > 2250000$ ✗	Intermediate	$225000 > 1000$ ✗

a. Consider a wave with a period of 10 s and a height of 4.0 m in a water depth of 25 m. Calculate the wavelength L .

$$L_0 = \frac{gT^2}{2\pi} \rightarrow L_0 = 156.13\text{m}$$

$$\frac{h}{L_0} = 0.16$$

$$\frac{h}{L_0} = 0.16 \sim \frac{h}{L} = 0.192 \rightarrow \frac{h}{0.192} = L = 130.21\text{m}$$

b. Calculate the wave number k and the wave frequency ω .

$$k = \frac{2\pi}{L} = 0.0483\text{ m}^{-1}$$

$$\omega = \frac{2\pi}{T} = 0.628\text{ s}^{-1}$$

Consider a wave with a wave height of 3.0 m and a period of 8.0 s.

a. Determine the equations for the horizontal and vertical orbital velocities and the pressure from the velocity potential, Eq. (1.41).

$$\text{Horizontal} \rightarrow u = \frac{w a \cosh[k(z+h)]}{\sinh(kh)} \cdot \sin(\omega t - kx)$$

$$\text{Vertical} \rightarrow w = \frac{w a \sinh[k(z+h)]}{\sinh(kh)} \cdot \cos(\omega t - kx)$$

Pressure:

$$\text{Considering } \underbrace{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}}_{\text{Non-linear term}} = 0; \quad \frac{\partial \phi}{\partial t} + \frac{p}{\rho w} + gz = 0; \quad \phi(x, z, t) = \frac{w a}{k} \frac{\cosh[k(z+h)]}{\sinh(kh)} \cdot \cos(\omega t - kx)$$

Non-linear term

$$-\frac{w^2 a}{k} \frac{\rho w \cosh[k(z+h)]}{\sinh(kh)} \cdot \cos(\omega t - kx) + p + \rho w g z = 0$$

Use dispersion relation.

$$P = -\rho w g z + \rho w g a \frac{\cosh[k(z+h)]}{\cosh(kh)} \cdot \cos(\omega t - kx) \quad \omega^2 = g k \tanh(kh)$$

b. Calculate the orbital velocity amplitudes and the pressure amplitudes at the seabed and at the surface in a water depth of 15 m and in a water depth of 7.0 m.

Seabed

$$\hat{u}(z=-h) = \frac{w a}{\sinh(kh)} = 0.83 \text{ m/s}$$

$$\hat{w}(z=-h) = 0$$

$$P = -\rho w g (-h) + \frac{\rho w g a}{\cosh(kh)} = 155.6 \text{ kPa}$$

$$h = 15 \text{ m}$$

$$H = 3 \text{ m}$$

$$T = 8 \text{ s}$$

$$L_0 = \frac{T^2 g}{2\pi} = 99.92 \text{ m}$$

Surface

$$\hat{u}(z=0) = \frac{w a \cosh(kh)}{\sinh(kh)} = 1.44 \text{ m/s}$$

$$\hat{w}(z=0) = w a = 1.18 \text{ m/s}$$

$$P(z=0) = \rho w g a = 14.7 \text{ kPa}$$

$$\frac{h}{L_0} = 0.15 \sim \frac{h}{L} = 0.183 \rightarrow L = 81.97 \text{ m}$$

Seabed

$$\hat{u}(z=-h) = \frac{w a}{\sinh(kh)} = 1.51 \text{ m/s}$$

$$\hat{w}(z=-h) = 0$$

$$P = -\rho w g (-h) + \frac{\rho w g a}{\cosh(kh)} = 86.8 \text{ kPa}$$

Surface

$$\hat{u}(z=0) = \frac{w a \cosh(kh)}{\sinh(kh)} = 1.917 \text{ m/s}$$

$$\hat{w}(z=0) = w a = 1.18 \text{ m/s}$$

$$P(z=0) = \rho w g a = 14.7 \text{ kPa}$$

$$\frac{h}{L_0} = 0.07 \sim \frac{h}{L} = 0.114 \rightarrow L = 61.4 \text{ m}$$

$$L_0 = 99.92 \text{ m}$$

Assume that sediment transport will occur as soon as the near-bed horizontal orbital velocity amplitude is equal to or larger than 0.50 m/s. Consider a wave with the following conditions: a wave height of 6 m and a period of 12 s. Calculate at which water depth the sediment at the bed will first start to move.

Seabed

$$\hat{u}(z=-h) = \frac{wa}{\sinh(kh)}$$

$$H = 6 \text{ m} \quad T = 12 \text{ s} \quad \rightarrow L_0 = \frac{T^2 g}{2\pi} = 229.83 \text{ m}$$

$$\hat{u} = \frac{wa}{\sinh(kh)} \quad \rightarrow \hat{u} = \frac{\frac{22.3 \text{ m}}{T}}{\sinh(kh)} \quad \rightarrow kh \approx 1.86$$

$$kh \approx 1.85 \rightarrow \frac{h}{L_0} \approx 0.28 \rightarrow h = L_0 \cdot 0.28 \rightarrow h = 62.95 \text{ m}$$

WAVE PROPAGATION

Free-surface waves will generally propagate in the direction of the wind, by which they are generated (wind waves). Especially long and high ocean waves, as generated during storms, are able to propagate over long distances (hundreds of kilometers) without losing much of their energy even after the wind died down. These swell waves can enter other seas like the North Sea with another wind climate.

In shallow nearshore areas the characteristics of the waves change strongly due to the decreasing water depth and the interaction with the seabed. In the surf zone of coastal areas and on beaches wave energy is finally dissipated during the wave breaking process.

In Civil Engineering practice knowledge about wave propagation is especially important for the management of coastal zones, coastal erosion, beach nourishment, design of coastal defense structures (groynes, breakwaters), harbor design and shipping.

The total energy density for a surface wave E_w is the sum of the kinetic and the potential energy density:

$$E_w = E_p + E_k = \frac{1}{8} \rho w g H^2 \quad [\text{J/m}^2]$$

ENERGY FLUX

Waves transport energy horizontally by work (W) done by the pressure in the direction of wave propagation. This horizontal transfer through a vertical plane in a time interval t is equal to the dynamic part of pressure, p_d , times the distance moved in that interval (in the x -direction, $x = u \tilde{t}$):

The mean energy flux for one wave follows after integration of F_w over one wave period T :

From linear wave theory $F_w = c_g E_w$

and the wave celerity; $c_g = n_w \cdot c_w \leq c_w$

$$c_g = n_w \cdot c_w \leq c_w$$

the wave group celerity. A wave front, transporting energy into still water, propagates with this

group velocity (Figure 2.2). Hence, individual waves propagate faster than a wave front!

WAVE GROUP

In reality, a wave field consists of different frequencies ω_i and wave lengths k_i and therefore generally shows an irregular pattern

$$c_g = \frac{c_\omega}{2} \left[1 + \frac{2kh}{\sinh(2kh)} \right] = n_\omega \cdot c_\omega$$

From dispersion relation $c_g = n_\omega c_\omega$ in shallow water $n_\omega = 1$

SHOALING

Waves that propagate into shallow coastal waters are affected by the reduced water depth. Here we consider first the simplest (one-dimensional) problem of waves that propagate in the direction

Energy dissipation

For calculations in the breaker zone near the coastline (beach) the energy flux balance is therefore generally provided with a (negative) source term, representing the dissipation of wave energy (D_w). Due to bottom friction and wave breaking energy losses will take place especially in shallow water.

Wave breaking

The latter phenomenon ('white capping') generally takes place when waves grow strongly in height during storm conditions and become very steep (i.e. large height/length-; H/L -ratio). Miche (1944) proposed the following expression for the maximum wave steepness above which waves will break:

$$\left(\frac{H}{L} \right)_{\max} \approx 0.14 \tanh(kh)$$

The most common type of wave breaking is observed at beaches, also during more moderate wave conditions, and is caused by the reducing water depth.

Different types of breakers can be distinguished, depending on parameters as beach slope (slope angle), wave height and wave period (or offshore wave length, L_0).

The following simple model for wave height change in the breaker zone can be used as a first approximation:

$$H(x) = y_w \cdot h(x) \quad (2.34)$$

$$H(x) = y_w \cdot h(x)$$

with breaking parameter $w = 0.5 \sim 0.8$ for spilling breakers. The location of the breaker point can be found by combining the shoaling model (offshore of the breaker point) with the breaker model (onshore of the breaker point)

with spilling breakers for $\xi < 0.5$, plunging breakers for $0.5 < \xi < 3.0$ and surging/collapsing breakers for $\xi > 3.0$ (Figure 2.6).

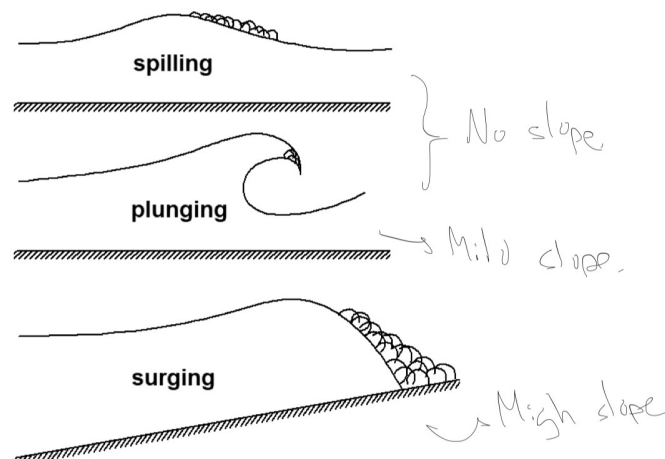


Figure 2.6: Wave breaking types.

Wave propagation in two horizontal directions: obliquely-incident waves

Wave refraction

Waves that approach a uniform coast under an angle (obliquely-incident waves) will change their direction of wave propagation more normal to the shoreline when the water depth decreases. This is caused by the decreasing wave propagation velocity as a result of the decreasing water depth ($c_w = \sqrt{g h}$ in shallow water). The nearshore part of a wave crest, being in more shallow water, decelerates earlier than the more offshore part (deeper water), see Figure 2.8. Based on geometrical considerations the following general law for wave refraction (Snell's law)

From conservation of energy

Assuming that no energy is transported through wave rays, the wave energy flux between two wave rays (distance b) is conserved from deep water to shallow water with a the constant distance between the two wave rays in y -direction. Then:

$$E_w n_w c_w \cos \theta_w = E_{w0} n_{w0} c_{w0} \cos \theta_{w0} \quad E_w = H^2$$
$$\frac{H}{H_0} = \sqrt{\frac{n_{w0} c_{w0}}{n_w c_w}} \cdot \sqrt{\frac{\cos \theta_{w0}}{\cos \theta_w}} = K_{sh} \cdot K_r$$

This shows that both wave shoaling and refraction lead to changes in wave height.

- With parallel depth contours, the distance b between wave rays increases into the direction of the coast due to refraction ($\cos \theta_w > \cos \theta_{w0}$), hence the wave height decreases ($K_r < 1$); in shallow water this decrease is somewhat compensated by shoaling ($K_{sh} > 1$).
- Depending on the shape of the depth contours in the nearshore area wave rays may diverge or converge (see Figure 2.11). Especially converging wave rays may lead to heavy wave attack and a need for a severe coastal protection measures (e.g. dikes).
- The wave angle θ_w reduces with decreasing water depth. However, it remains > 0 , also on the beach. It will lead to a wave-driven current and sediment along the coast (alongshore transport or littoral drift).

Diffraction

Wave diffraction is the process of wave energy transfer in directions other than the wave propagation direction. It can be explained from Huygens' principle according to which an advancing wave front can be considered as a sequence of wave sources, each of which radiates energy in a circular pattern.

Figure 2.12 gives a basic example of wave diffraction behind a breakwater in constant water depth. It shows energy transfer along the wave crest towards the sheltered region between the breakwater. The wave energy (wave height) decreases along the wave crest on the right-hand side of curve A. The wave height reduction can be determined using a diffraction coefficient:

$$K_d(x, y) = \frac{H(x, y)}{H_{\text{offshore}}}$$

with $K_d = 1$ for curve A. In case of variable water depths, diffraction occurs simultaneously with shoaling and refraction.

QUESTIONS

Assume a coastal profile with depth contours parallel to the coast with a deep water wave train propagating to the coast as shown in Figure 2.13.

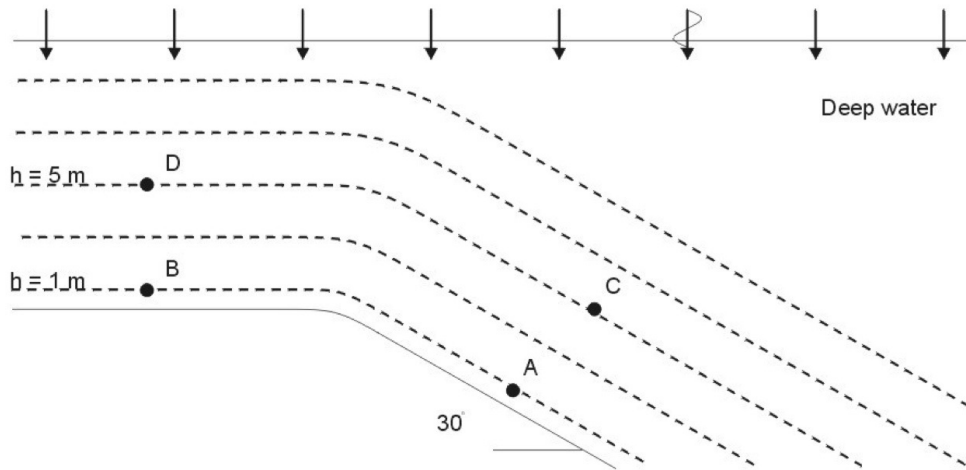


Figure 2.13: Coastal profile with parallel depth contours.

a. Describe the following processes: shoaling, refraction and diffraction.

$$\frac{H}{H_0} = \sqrt{\frac{n_{w0} \cdot c_{w0}}{n_w \cdot c_w}} \quad \text{— Shoaling is the increase of wave height given the reduction of speed due to the change of water depth.}$$

Refraction → refraction is the reduction of wave height as the waves approach the coast, given it's reduction in angle within the wave and the coast. As, the energy is conserved and the length within wave rays as well, this reduction is given for the change in angle.

Diffraction → Diffraction is the obstruction of the wave (towards the propagation direction) while dissipates the energy of the wave to different sides than the coast.

b. Sketch the wave propagation by plotting the track of the wave rays starting at the black arrows.

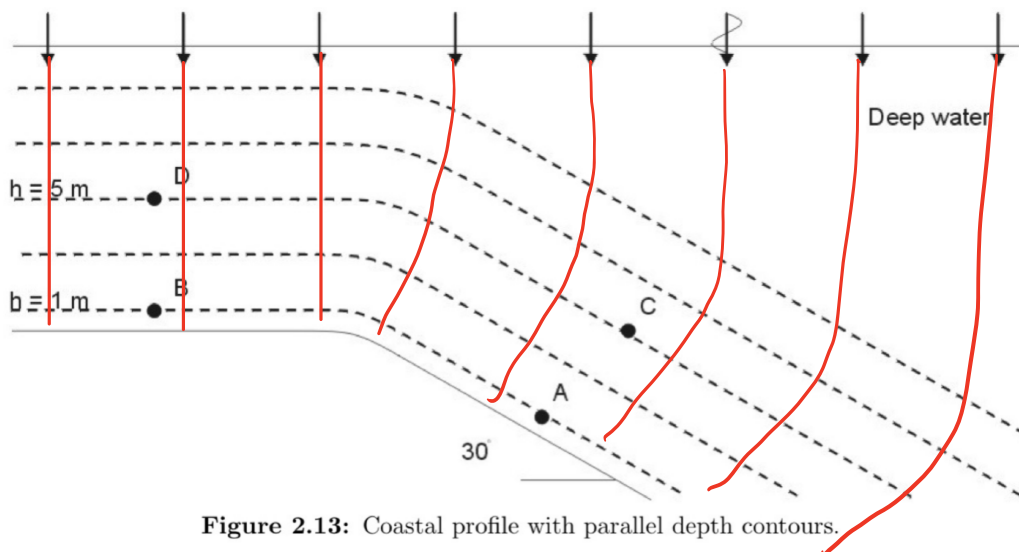


Figure 2.13: Coastal profile with parallel depth contours.

c. Which quantity remains constant (outside the breaker zone) when a wave train approaches the shoreline?

The wave energy flux between two wave rays

$$F_{wb} = E_w \cdot c_g \cdot b = E_w \cdot c_w \cdot n_w \cdot b.$$

d. Explain the differences between the energy flux (F_w) and the energy (E_w). What are the units?

The wave energy density (E_w) is the kinetic and potential energy density per unit horizontal area

$$E_w = \frac{1}{8} \rho g H^2 \rightarrow \left[\frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}^2 \right] \rightarrow \left[\frac{\text{J}}{\text{m}^2} \right]$$

The wave energy flux (per unit width) is the transport of this energy with a velocity equivalent to c_g in the propagation direction.

$$F_w = E_w \cdot c_g = E_w \cdot n_w \cdot c_w \left[\frac{\text{J}}{\text{m}^2} \cdot \frac{\text{m}}{\text{s}} \right] \rightarrow \left[\frac{\text{J}}{\text{s}} \cdot \frac{1}{\text{m}} \right] \rightarrow \left[\frac{\text{W}}{\text{m}} \right]$$

e. Derive an expression for the wave height in shallow water (outside the breaker zone) as a function of the parameters at deep water and known parameters in shallow water. Explain how the parameters can be calculated.

$$\frac{H}{H_0} = \sqrt{\frac{0.5 \cdot \frac{gT}{2\pi}}{\sqrt{gh}}} \cdot \sqrt{\frac{\cos(\theta_{w0})}{\cos(\theta_w)}}$$

$$\frac{\sin \theta_{w0}}{c_{w0}} = \frac{\sin \theta_w}{c_w}$$

$$\hookrightarrow \frac{\sin \theta_{w0} \cdot \sqrt{gh}}{\frac{gT}{2\pi}} = \sin \theta_w$$

$$H = \sqrt{\frac{g^{1/2} \cdot T}{4\pi h^{1/2}}} \cdot \sqrt{\frac{|\cos \theta_{w0}|}{\cos \left(\sin^{-1} \left(\frac{\sin \theta_{w0} \cdot 2\pi \cdot h^{1/2}}{g^{1/2} \cdot T} \right) \right)}} \cdot H_0$$

$$\hookrightarrow \sin^{-1} \left(\frac{\sin \theta_{w0} \cdot 2\pi \cdot h^{1/2}}{g^{1/2} \cdot T} \right) = \theta_w$$

The parameters are the period of the wave.

• The water depth at which the wave height is required.

• The angle at which the wave approaches the coast.

• The characteristics can be known from wave buoy measurements.

Assume the following offshore wave conditions: wave height 2 m, wave period 8 s. In addition it is assumed that the bottom friction can be neglected.

f. Describe in a qualitative sense how the wave heights vary along the depth contour $h = 5$ m.

As the wave approaches the coast the wave height will be lowered due to refraction (considering it stays at the same water depth) given that the angle within the wave and coast is smaller $\theta_w < \theta_{w0}$

• At the left waves are just governed by shoaling.

• At the bend waves are concentrated (refraction) leading to higher wave height.

g. Determine where the breaker line is located (in terms of water depth). What is the wave height at the breaker line? Use $w = 0.6$ for the breaker index.

h. Compute the wave heights at A, B, C and D.

i. Calculate the loss of energy between the points D and B.

g-h-i computed together.
 $H_0 = 2\text{m}$; $T = 8\text{s}$

D $h = 5\text{m}$
 B $h = 1\text{m}$

C $h = 5\text{m} \rightarrow \theta_{wo} = 30^\circ$
 A $h = 1\text{m} \rightarrow \theta_{wo} = 30^\circ$

Table	h	θ_{wo}	$L_0 [\text{m}] \left[\frac{gT^2}{2\pi} \right]$	h/L_0	h/L	$L [\text{m}]$	kh	M	$E_w \left[\frac{\text{J}}{\text{m}} \right]$	$\Delta E_w \left[\frac{\text{J}}{\text{m}} \right]$
A	1	30°	99.92	0.01	0.04	25	0.283	0.6	442	
B	1	0°	99.92	0.01	0.04	25	0.283	0.6		
C	5	30°	99.92	0.05	0.094	53.2	0.592	1.91		
D	5	0°	99.92	0.05	0.094	53.2	0.592	2.05	5153.32	
							2.16		5721.2	

$\left. \begin{array}{l} 442 \\ 5153.32 \\ 5721.2 \end{array} \right\} \rightarrow -5280$
(i)

Breaker.

$$c_g = \frac{1}{2} \left(1 + \frac{2kh}{\sinh(2kh)} \right) \cdot c_w \sim c_w = \frac{L}{T}$$

(h)

Looking for breaker line at *(h)*

$M = 4$

Assuming it occurs in shallow water.

$$H_0 \sqrt{\frac{m_w c_{wo}}{m_w c_{wl}}} = 4 \cdot h \rightarrow h = 3.45 \text{ so height for A and B is given by breaker index } \boxed{9}$$

Always check if shallow water behaviour is not.

WAVE-DRIVEN CURRENTS

Short free surface waves not only generate orbital flows, but on a larger (wave-averaged) time-scale they also lead to mean currents and mean water-level variations especially in the shallow coastal zone. However, due to non-linear effects waves generate in reality an excess flow of mass (mass flux) and momentum (radiation stresses) with principal components in the direction of wave propagation.

In the cross-shore direction this wave-induced force is generally balanced by a slope of the mean water surface (wave set-up and set-down). In the alongshore direction this force is balanced by shear forces associated with mean (wave-driven) currents (alongshore current). Alongshore currents are generally larger than the cross-shore currents and responsible for the so-called littoral drift of water and sediment along a coast.

Finally, the full hydrodynamic problem of tide-, wind- and wave-induced flows is shortly described by

discussing the structure of (numerical) 2DH wave and flow models. Flow computations for the mouth of the Scheldt estuary, on the border of The Netherlands and Belgium, are shown as an example of a model application.

MASS TRANSPORT DUE TO WAVES

Mass transport due to waves, also known as Stokes drift, is an intriguing and practically relevant phenomenon (Figure 3.1). It can be understood from an Eulerian and a Lagrangian perspective.

The Eulerian perspective is looking at fluid motion that focuses on specific locations in space through which the fluid flows as time passes.

The Lagrangian perspective is looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time. This can be visualized as sitting in a boat and drifting down a river.

To start with the Eulerian perspective: image a velocity instrument just below the wave trough. In case of a progressive wave it will measure onshore velocities as the wave crest passes (assuming waves that advance towards the coast), and offshore velocities under the wave trough. The net (wave-averaged) velocity will be zero. Hence, the net velocity is non-zero and onshore-directed in the region between the wave trough and crest.

Then the Lagrangian perspective: under progressive waves water particles move along elliptic orbits (orbital flow, see Chapter 1), which are generally not completely closed (Figure 3.2). This can be explained by the fact that the horizontal velocity on the top of the orbit (under the wave crest) is slightly higher than at the bottom of the orbit (under the wave trough) and water particles will therefore experience a net resulting displacement in the direction of the wave propagation (Lagrangian wave drift also known as Stokes drift).

This drift is a non-linear phenomenon which cannot be explained by linear wave theory. Averaging of the (sinusoidal) horizontal orbital flow velocity (based on linear wave theory, see Eq. 1.45) over one wave period at a fixed level above the sea bed leads to a zero mean (time-averaged) velocity!

Although wave drift of water particles is a non-linear phenomenon, the total net water flux M_w associated with the wave drift can be approximated well using linear wave theory. Integrating the mass flow through an element with height dz over the water depth (from sea bed to water surface) and over one wave period leads to an expression for the total mean wave-induced mass flux per unit of width M_w

The depth-averaged mass transport velocity is:

$$\langle \bar{u} \rangle = \frac{E_w}{\rho_w p_w h} = \frac{g M^2}{8 c_w h} \approx \frac{1}{8} \cdot \frac{\sqrt{g} M^2}{h^{1.5}}$$

The magnitude of this mean velocity associated with the Stokes drift is always small in comparison to the horizontal orbital flow velocity

$$\frac{\langle \bar{u} \rangle}{\bar{u}_b} = \frac{1}{4} \frac{M}{h} \cdot \cosh(kh) = \frac{1}{4} \frac{M}{h} \ll 1$$

↳ In shallow water.

In general, for waves propagating in an arbitrary direction the mass flux can be considered as a vector with two components in the horizontal x-y plane. The Eulerian drift takes fully place between the wave trough and crest level and is zero in the rest of the water column. The Lagrangian drift increases with distance above the bed and extends up to the still water level.

RADIATION STRESSES AND WAVE FORCE

Short progressive waves exert a net wave-averaged force on the fluid mass in which they propagate. This wave force can lead to additional water level variations (see Figure 3.6) and/or to wave-driven mean flows.

In the direction of the wave propagation this momentum flux consists of a component associated with the horizontal water velocity and a component associated with the water pressure.

The momentum per unit volume associated with the horizontal orbital velocity is $\rho \tilde{u}$, so its contribution to the flux of momentum across a vertical element z is $\rho \tilde{u}^2$. Similarly the pressure p contributes to the momentum flux through this element z .

In the direction of the wave propagation the principal component of the radiation stress is defined as the mean, i.e. wave-averaged, value of the total momentum flux in the water column in the presence of waves minus its value in the absence of waves (excess momentum flux). Similarly as for the mass flux $\rho \tilde{u}$ (in stead of $\tilde{u}$) is integrated over the water column and integrated over one wave period. The contribution of the pressure in the absence of waves (hydrostatic pressure, p_0) is subtracted in order to obtain the excess momentum flux:

Normal to the direction of wave propagation only the pressure contributes to the total mean momentum flux:

$$S_{yy} = S_{xx,2} + S_{xx,3} = \left(n - \frac{1}{2}\right) E_w$$

Radiation stress is a tensor. The first subscript refers to the plane on which it acts, and the second subscript to the direction in which it acts. Essentially, radiation stress is the net force, due to wave action, that the water exerts on a control area (Figure 3.7), just like pressure.

In a steady and spatially uniform wave field with constant wave height, period and direction over a horizontal sea bed, the radiation stresses acting on a water column element x - y have the same magnitude but opposite direction at both sides of the element and therefore do not lead to a net resulting force on the element. A resulting net wave force only appears in spatially non-uniform situations with varying wave characteristics and/or water depth. This wave force vector R , in the direction of the wave propagation (Figure 3.8), directly follows from the horizontal gradients of the radiation stresses:

$$R_x = -\frac{\partial S_{xx}}{\partial x} - \frac{\partial S_{yx}}{\partial y}$$
$$R_y = -\frac{\partial S_{xy}}{\partial x} - \frac{\partial S_{yy}}{\partial y}$$

Since the bed shear stress is non-linear in the velocity, the oscillatory motion of waves will lead to an enhancement of the wave-mean bed shear stress

Normally incident waves: set-down and set-up and cross-shore currents

In this section cross-shore currents and water-level variations are described as generated by normally-incident waves, i.e. incoming waves with wave crests parallel to the coast line.

Due to the uniformity of the coast and the seabed no variations along the coast can occur: $\frac{\partial}{\partial y} = 0$, so we consider a 1D flow in the cross-shore x -direction. Since the coast is a closed boundary no water can pass through this boundary, which means that the mean water discharge $h q_{xi}$ as well as the depth-averaged current velocity $h U_i$ equal zero at this boundary. Continuity of water then requires that this is also true at all other cross-shore x -positions:

$$\frac{\partial \langle \eta \rangle}{\partial x} = \frac{\partial \langle \tau_b \rangle}{\partial x} = 0 \rightarrow \langle \eta \rangle = \langle \tau_b \rangle = 0.$$

$$\text{Gives to } g \frac{\partial \langle \zeta \rangle}{\partial x} = \frac{R_x}{\rho_w h} - \frac{\langle \tau_{bx} \rangle}{\rho_w h}$$

Although the bed shear stress is an important parameter for sediment transport, it is of less importance for the description of the cross-shore flow and will be neglected in the following sections. Here this equation will be applied in order to show that wave-height variations along the cross-shore profile due to shoaling (outside the surf zone) and wave breaking (inside the surf zone) lead to different behavior of the mean water level.

SET DOWN

This relation shows that the mean water level drops outside the surf zone, which is referred to as (wave) "set-down". For shallow water $\sinh(2kh) \sim 2kh$ and the set-down can be estimated using:

$$\langle \zeta \rangle = -\frac{1}{16} \frac{H^2}{h} \quad \text{for normal} \quad \langle \zeta \rangle = -\frac{1}{8} \cdot \frac{kH^2}{\sinh(2kh)}$$

which shows that the set-down increases with increasing wave height and decreasing water depth.

SET UP

Inside the surf zone, the wave force R_x is relatively large due to the strongly decreasing wave height (energy dissipation due to wave breaking). If we approximate the wave height in the surf zone by: $H_{br} = Y_{wh}$

$$\Delta \langle \zeta \rangle = \frac{3}{8} \gamma_w H_{br}$$

This shows the mean water level rises in the surf zone, known as (wave) "set-up".

WAVE-DRIVEN CURRENTS

Although the depth-averaged mean current velocity $U = 0$ for the situation of a closed uniform coast, the mean current is not equal to zero at each level above the sea bed.

Note that in the field the depth-averaged current is not necessarily equal to zero, as water could potentially also flow in the alongshore direction.

Outside the surf zone (non-breaking waves), the onshore-directed Stokes drift is compensated by an offshore-directed return flow. This return flow or undertow typically has a more or less a logarithmic vertical distribution, such that velocity is offshore-directed in the lower part of the water column and onshore-directed in the upper part.

Wave breaking leads to an onshore waves force, see Eq. (3.44). In analogy with the vertical distribution of the orbital velocities, these stresses are largest near the water surface. The depth-integrated wave force is balanced by wave set-up. The opposing pressure due to the water level gradient has a (nearly) uniform vertical distribution (Figure 3.11). This results in a seaward wave-averaged current near the seabed (undertow) and onshore flow higher in the water column.

the flow, an additional mean shear stress is generated by the interaction between the horizontal and vertical orbital velocities. This stress leads to an onshore-directed mean flow, indicated as the Longuet-Higgins (LH) 'boundary layer streaming', as it was discovered by Longuet-Higgins (1953). At

the edge of the boundary layer the analytical expression for the LH streaming is remarkably simple:

$$u_{LH} = \frac{1}{16} \frac{g^{0.5} H^2}{h^{1.5}}$$

As an example, waves with height 1 m and period 5 s travel in 5 m water depth with a celerity $c_w = 6.1$ m/s according to linear wave theory. The orbital velocity amplitude at the bed is $\hat{U} = 0.51$ m/s. The LH-streaming according to Eq. (3.51) is 0.01 m/s. This shows that the LH streaming is relatively small. Yet it can be important for sand transport as it occurs near the bed with relatively high sand concentrations.

such that the wave-mean shear stress consists of contributions due to the wave-mean eddy viscosity and velocity, and due to the interaction between the intra-wave eddy viscosity and velocity. Outside the wave boundary layer the shear stress and vertical velocity gradient are zero:

$$\langle \tau_{xz} \rangle = \rho_w \langle \nu_t \rangle \frac{\partial \langle u \rangle}{\partial z} + \rho_w \left\langle \bar{v} \frac{\partial \bar{u}}{\partial z} \right\rangle = 0$$

For skewed oscillatory flows with higher onshore velocities, the largest near-bed turbulence (high $\langle \bar{v} \rangle$) is present during the onshore half wave-cycle during which $\partial \bar{u} / \partial z > 0$. During the offshore half wave-cycle $\partial \bar{u} / \partial z < 0$, and turbulence and hence ν_t is relatively small. As a result:

$$\rho_w \langle \nu_t \rangle \frac{\partial \langle u \rangle}{\partial z} < 0$$

As the wave-mean eddy viscosity is positive, $\langle \bar{v} \rangle > 0$, this means that:

$$\frac{\partial \langle u \rangle}{\partial z} < 0$$

and hence near the bed:

$$\langle \bar{v} \rangle < 0$$

Obliquely incident waves: alongshore currents

Now we consider the more general 2DH problem of incoming waves under an arbitrary angle, or so-called obliquely-incident waves. The coast is again treated as a uniform coast with parallel straight depth contour lines of sea bed (Figure 3.12).

$$g \frac{\partial \langle \xi \rangle}{\partial x} = \frac{R_x}{\rho_w h} - \frac{\langle \tau_{bx} \rangle}{\rho_w h}$$

$$0 = \frac{R_y}{\rho_w h} - \frac{\langle \tau_{by} \rangle}{\rho_w h}$$

The cross-shore flow and wave set-up and set-down is the same as for the normally-incident waves (Section 3.4). In the longitudinal direction the wave force should be balanced by a bed shear stress, which requires a longitudinal current, i.e. a mean current $\langle \bar{u} \rangle$ along the coast. In this y-direction a mean current can develop, without being blocked by a closed boundary.

Wave force and bed shear stress

Inside the surf zone, assuming shallow water ($c_w = gh$, $n_w = 1$) and $H = wh$, the expression for the wave force can be simplified to ($E_w = 1/8 \rho_w g H^2 = 1/8 \rho_w g w h^2$):

$$R_y = - \sum_{\substack{l=1 \\ \omega_l = \omega}} \frac{\sin \theta \omega_l}{c_{w0}} \cdot \rho_w g^{1.5} \gamma_w^2 h^{1.5} \frac{\partial h}{\partial x}$$

This wave force is compensated by positive bed shear stress (see Eq. 3.70), which indicates that obliquely-incident waves drive an alongshore current.

$$\bar{\tau}_s = \frac{1}{2} \rho_w f |\bar{u}| \bar{u}$$

The velocity is time-dependent vector (both variable magnitude and direction), since it is composed of a longitudinal mean current, $h v_{bi}$, and a time-dependent near-bed orbital velocity, $u \tilde{b}(t)$. Assuming that due to refraction the orbital velocity is approximately perpendicular to the coast line and assuming that the near-bed longshore current is relatively small ($|u \tilde{b}| \gg |h v_{bi}|$), the alongshore component of the bed-shear stress can be approximated by:

Note that the current-alone bed shear stress can be expressed as $\tau_b = 12 \rho_w (v_b)^2$. The presence of waves and the additional near-bed orbital velocities thus lead to an increase of the bed-shear stress for the same mean current (bed shear stress enhancement).

In shallow water:

$$\tau_b = \frac{1}{2} \frac{\rho_w}{h} \kappa \omega = \frac{1}{2} \frac{\rho_w}{h} \gamma_w \sqrt{g h} \quad \rightarrow \quad \langle \tau_{b_y} \rangle = \frac{1}{2} \frac{\rho_w}{h} \frac{\gamma_w}{\alpha_w} \sqrt{g h} \langle V \rangle$$

Alongshore current

Substitution of Eqs. (3.79) and (3.74) into the momentum equation, Eq. (3.70), leads to the following expression for the depth-averaged alongshore current:

$$\langle V \rangle = -\frac{52}{8} \frac{\alpha_w \gamma_w}{f} g h \frac{\sin \theta w_0}{\omega_0} \frac{\partial h}{\partial x}$$

The expression shows that the alongshore current increases with the sine of deep-water wave angle. This suggests that the velocity is at maximum for shore-parallel traveling waves ($\sqrt{w_0} = 90^\circ$, $\sin \sqrt{w_0} = 1$). In case of alongshore traveling waves, there is no momentum transfer in the cross-shore direction. S_{xy} and hence $h v_i$ are at maximum for $\theta_{w_0} = 45^\circ$.

For a constant cross-shore beach slope ($\partial h / \partial x = \text{constant}$), the alongshore current velocity increases linearly with water depth h (see Figure 3.13). In reality, a more gradual transition is present between the breaker zone and the shoaling area (see Figure 3.14). Momentum exchange takes place between these areas and, moreover, due to wave irregularity waves in a wave train break at different positions (no fixed breaking point).

NUMERICAL MODELS FOR CURRENT AND WAVES

2DH coupled Wave-current modeling systems

Since water depth and currents also affect the waves (current refraction) the models should interact to reach a certain wave/current equilibrium situation (Figure 3.15). For these complicated 2D situations the set of wave and current equations are discretized and solved numerically by a computer model. *Delft3D*

The tidal flow is exiting the Western Scheldt, flowing towards the W-NW. In the surf zone along the coast of (Zeeuws-)Vlaanderen the waves drive a current to the NE, which is important for coastal erosion and sedimentation as will be discussed in Chapter

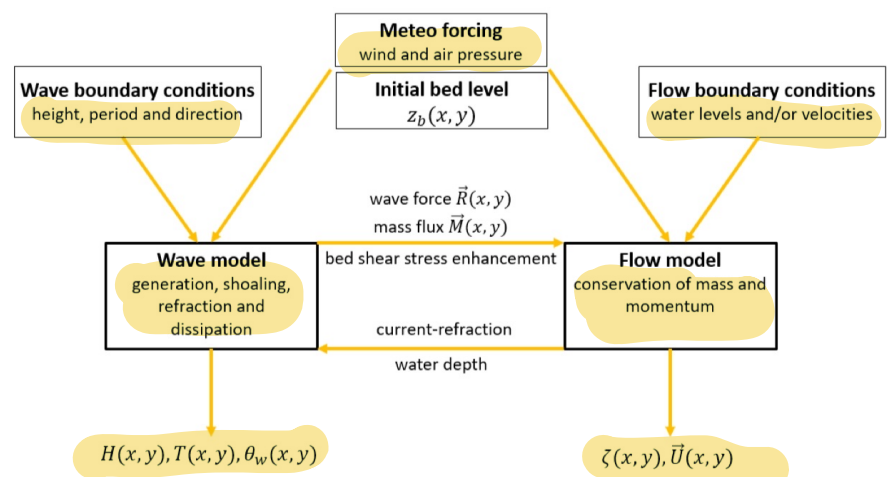


Figure 3.15: Schematic representation of wave-current coupling in a 2DH numerical model.

QUESTIONS

Consider the following wave conditions: an incoming wave with a height of 3 m and a period of 8 s (at $h = 10$ m). The water depth at the seaside is 10 m and at the shoal crest 8 m (lagoon situation only), Figure D.3.

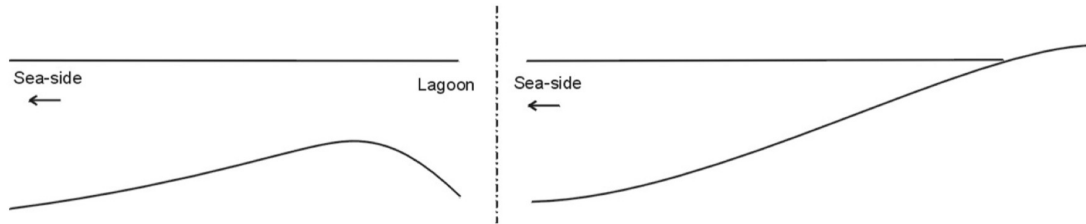


Figure 3.17: Situation 1: lagoon (left) and situation 2: beach (right).

a. Explain why there will be, in general, a net mass flux of water in a situation with short waves.

Given that there is a higher velocity at the crest compared to the underflow as it is not a equal relationship (non-linearity). As the elliptic orbits of the water particles are not completely closed



Therefore water particles experience a net resulting displacement in the direction of wave propagation.

b. Calculate the depth-integrated mass-transport (M_w) at the crest of the shoal in situation 1 (lagoon).

$M_w \rightarrow$ given by $M_w = \frac{E_w}{c_g}$ $H=3m$ $T=8s \rightarrow$ at $h=10m$

Lagoon $h=8m$ $L_0 = \frac{gT^2}{2\pi} \rightarrow L_0 = 99.92m$

$\frac{H}{\sqrt{\frac{nw \cdot c_{w0}}{nw \cdot c_{w1}}}} = H_0 \rightarrow H_0 = 3.22m$

$c_{w0} = \frac{gT}{2\pi} = 12.99 \text{ m/s}$

$E_w = \frac{1}{8} \rho w g H^2 \rightarrow E_w =$

h	h/L_0	kh	h/L	L	$H[m]$	E_w	M_w
10	0.1	0.886	0.141	70.92	3	$11 \cdot 10^3$	$1.24 \cdot 10^3$
8	0.08	0.774	0.173	65.04	3.07m	$11.55 \cdot 10^3$	$1.42 \cdot 10^3$

c. Calculate the depth-integrated mass-transport in situation 2 (beach).

$M_w \rightarrow$ lagoon $\rightarrow 1.42 \cdot 10^3 \left(\frac{kg}{m/s} \right)$

$M_w \rightarrow$ seaside $\rightarrow 1.24 \cdot 10^3 \left(\frac{kg}{m/s} \right)$

d. Calculate for both situations the depth-averaged velocity at these locations. Is this velocity of the same order as or much larger or smaller than the orbital velocity?

$\langle U \rangle = \frac{M_w}{\rho w h}$ $\sim$ lagoon $\langle U \rangle = 0.178 \text{ m/s}$ $\left| \begin{array}{l} \text{Orbital velocity near the bed,} \\ 0.17 = h = \frac{w_0}{\sinh(kh)} = 1.41 \text{ m/s.} \end{array} \right.$

$\sim$ seaside (close boundary) $\langle U \rangle = 0$

Much larger than orbital velocity.

A wave train (deep-water conditions: height of 2 m and a period of 8 s) approaches the shore. The wave crests are parallel to the shoreline. The beach slope is 1:30. Assume a breaker coefficient $Y_w = 0.6$.

ρ wave-mean momentum flux due to the pressure of waves

a. Describe radiation stress. What is the unit?

Radiation stress is the wave-mean depth integrated excess momentum flux due to the presence of waves. The unit of radiation stress is $[N/m] = [kg/s^2]$.

b. Calculate the wave height and water depth at the breaker line.

$H = 2m$ $T = 8s$ $ib = 1:30$ $Y_w = 0.6$ $H_{br} = Y_w h_{br}$

Assuming it occurs at shallow water.

$$H_0 \cdot \sqrt{\frac{0.5 \cdot g T^2}{2\pi}} = Y_w \cdot h_{br} \rightarrow$$

$2.12 \times = 2.4$ 4

$2.17 \approx 2.16$ 2.16 3.6

Approximated.

$h = 3.6$ $H = 2.16m$

for $4 - 3.9 - 3.8 - 3.7 - 3.6 - 3.5 - 3.4$
 $\frac{h}{L_0} = 0.04 \quad 0.039 \quad 0.038 \quad 0.037 \quad 0.036 \quad 0.035$

$kh \rightarrow 0.523$ 0.487

$L \rightarrow 48.19$ 45.45

c. Describe the phenomena 'set-up' and 'set-down' and explain them in physical terms.

Set-up is the increase in mean water level - it increase due to the high energy dissipation due to wave breaking

As the H decreases and Energy and Radiation stress $\downarrow$

It is compensated by increase in water level.

} Due to wave breaking.

Set-down is the decrease in mean water level.

As the $H \uparrow$ so does the energy and radiation stress $\uparrow$

It is compensated by decrease in water level.

d. Compute the wave heights and 'set-up'/'set-down' at the locations with a water depth of respectively $h = 5, 3$ and 2 m.

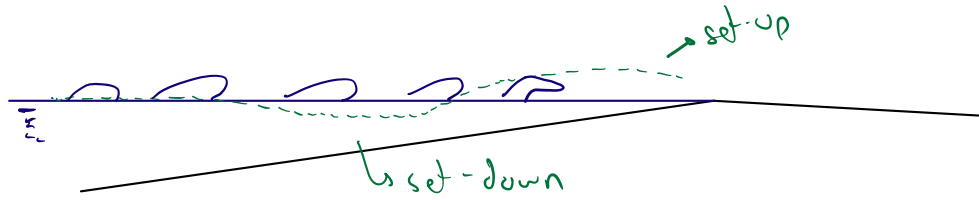
At $h = 5$ there is set-down given by $S = -\frac{1}{8} \frac{k H^2}{\sinh(2kh)}$ $\frac{h}{L_0} = 0.05$ $kh = 0.592$ $ksh = 1.023$

$L = -0.042m$

For set-up $S = \frac{3}{8} Y_w^2 \frac{\partial h}{\partial x} - S = -\frac{3}{8} Y_w^2 (h - h_b) + \langle \xi \rangle h_b \rightarrow \int_{2m}^{5m} \rightarrow 0.081$
 $\int_{3m}^{5m} \rightarrow 0.216$

From breaker line.

e. Make a sketch of the mean water level and sea bed as a function of cross-shore position.



You can use the following expression to calculate the depth-averaged alongshore current velocity:

$$\langle V \rangle = \frac{-3\rho g \gamma_w h}{2} \frac{\partial h}{\partial x} \cdot \frac{\sin 2\omega_0}{\cos \omega_0}$$

F. Calculate the alongshore current velocity.

$$\sin 2\omega_0 \text{ of } 0^\circ = 0 \quad \rightarrow \quad \langle V \rangle = 0$$

As it approaches normal to the coast, all radiation stresses are balanced by set-up and set-down. Thus, the alongshore current velocity is 0.

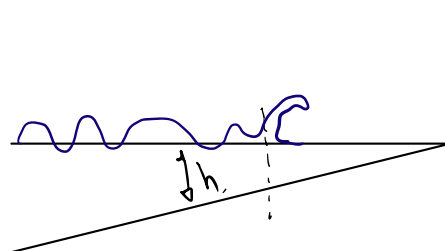
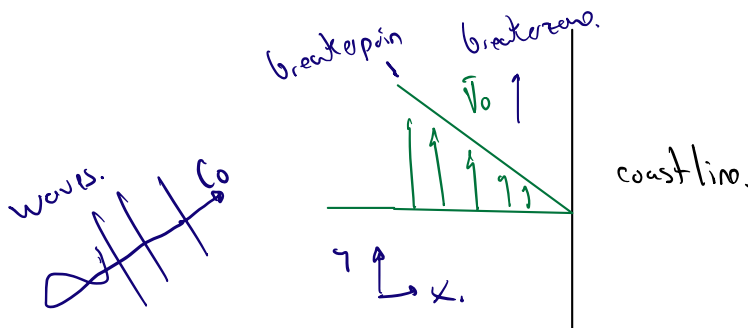
The wave train now propagates towards the coast with the wave crests at an angle of 30° with the coastline.

g. Calculate the alongshore current velocity again, and make a sketch of the cross-shore distribution.

$$\frac{\partial h}{\partial x} = \frac{1}{30} \quad \rightarrow \quad \frac{\partial h}{\partial x} = -\frac{1}{30}$$

Depth is given by $h(x) = h_b + \frac{1}{30}(x_b - x)$ (x-axis positive on-shore).

The velocities for $h = [3.6, 2.6, 1.6, 0.6, 0] = \langle V \rangle = [0.1332, 0.096, 0.06, 0.02, 0]$.



COASTAL SEDIMENT TRANSPORT

Sand transport by coastal waves and currents is highly relevant. It is the essential link between hydrodynamics and bed levels. Understanding and the ability to reliably predict net sand transport is crucial to determine the coastal impact of both natural processes and human interventions. One example are the so-called phase-lag effects, see also Section 4.5.4. Low flow velocities under small waves generate ripples on the seabed. If these are sufficiently steep, the flow cannot follow the bed anymore and separates, and vortices (eddies) are generated on the ripple flanks. These are very effective in bringing sand into suspension (Figure 4.2). If the vortices are strong enough, offshore net sand transport can be generated under shoaling waves with higher onshore velocities.

From turbulence (sub-second) and grain-particles (sub-millimeter) to the small net cross-shore sand transport rate that largely controls the response of the shoreface (cross-shore extent of tens of kilometers) to sea-level rise on time-scales of decades to centuries.

The wave bottom boundary layer is typically centimeters to a few decimeters thick, which requires a high vertical resolution to accurately measure and model sand transport.

Classification of sediment properties

Sediment grains are classified according to their diameter into clays, silts, sands, cobbles and boulders. Clays and silts are collectively called mud.

This means that the logarithm of the grain-size has an approximately normal (Gaussian) frequency distribution by weight. ϕ_n is the grain-size for which $n\%$ of the grains by mass is finer.

Density

The sediment mineralogy and the shape of the grains determine its hydraulic properties. Most sediments in European waters are primarily composed of quartz grains. They usually have a density (ρ_s) close to 2650 kg/m³ and are, very roughly, spherical. Sometimes the beach sediments contain heavy minerals such as garnet that has a density of about 4000 kg/m³. As a result, heavy minerals have a different hydraulic behavior than quartz grains.

The bulk density is the mass of many particles of the material divided by the total volume they occupy. Natural sands beds have a typical porosity of 0.4 such that $\rho_{b,dry} \hat{=} 1600$ kg/m³ and $\rho_{b,wet} \hat{=} 2000$ kg/m³. This means that 1 cubic meter of sandy bed weighs about 2 tons, and contains approx 1.6 tons of sand.

Mud has a similar density as quartz sands, which lowers with the content of organic matter. However, a muddy bed contains more water than a sandy bed. Through the process of consolidation pore water is expelled, increasing the bulk density

Settling velocity

The settling velocity (or fall velocity) of sand grains in water, w_s , is determined by their diameter and density, and the viscosity of the water.

It results from a balance between the downward gravity force, and the upward fluid drag force and buoyancy.

The drag coefficient is a function of the particle Reynolds number:

In the so-called Stokes region at the finest end of the sand range of diameters ($d \sim 62 \mu\text{m}$, $Re_p < 1$) the drag coefficient is given by $C_d = 24/Re_p$ and the expression for the fall velocity becomes:

$$w_{s,\infty} = \frac{\Delta \rho_s g d^2}{18\mu}$$

The settling velocity of typical Dutch beach sand with $d = 0.2$ mm is thus about 2 cm/s.

Angle of repose

The angle of repose, α , is the angle to the horizontal at which grains start to roll on a flat bed of

sediment which is gradually tilted. In other words, it is the steepest angle of descent to the horizontal plane to which a material can be piled without slumping. At this angle, the material on the slope face is on the verge of sliding.

If the bed is tilted by more than 30° gravity no longer pulls the top particle towards the bed particles, and it will tumble down (Figure 4.5b).

The value for r for non-cohesive sediments depends on the shape, sorting and packing of the grains. A good general value for the angle of repose is $r = 32^\circ$.

The angle of repose is also called the angle of internal friction. This is because it relates the friction force F_f to the normal force F_n . On a r -sloping bed the force balance of a sediment particle with mass m is:

$$F_f = \tan(\phi) F_n$$

BED SHEAR STRESS

The effect of the hydrodynamic forcing by waves and currents on the sediment dynamics takes place primarily through the friction they exert on the seabed. This is expressed in terms of the bed shear stress, τ_b , which is the friction force exerted by the flow per unit area of bed.

For turbulent flows the shear stress is typically modeled in an analogous way:

$$\tau = \rho_w \nu_t \cdot \frac{\partial u}{\partial z}$$

with ν_t the turbulent viscosity which is generally much larger than the kinematic viscosity.

CURRENT VELOCITY PROFILE

Bed friction slows down the flow. The part of the flow affected by bed friction is called the boundary layer. In shallow water the current boundary layer may occupy the entire depth, whereas in deep water it occupies the lower part of the water column and is overlain by water relatively unaffected by friction. ((wall roughness dominant, $Re_w > 70$))

ROUGHNESS

The total bed shear stress is made up from contributions due to i) skin friction, $\tau_{b,s}$ (grain-related roughness), ii) form drag, $\tau_{b,f}$ (bedform-related roughness), and iii) a sediment-transport contribution, $\tau_{b,t}$, caused by the momentum transfer to mobilize grains.

It is important to be aware that only the skin friction contribution acts directly on the sediment grains, and it is therefore this contribution which is used to calculate the inception of motion, bedload transport and reference concentration.

BED SHEAR STRESS COMPUTATION

Current related

The current-related bed shear stress is also often related to the depth-averaged current using the Chézy (C) or the Manning (nm) roughness coefficient:

$$\tau_{b,c} = \rho_w \bar{u}^2 \frac{g}{C^2} = \rho_w \bar{u}^2 \cdot \frac{g n^2 m}{h^{1/3}}$$

It is important to note that a higher C-value corresponds to a lower bed roughness. And that for a given nm the bed shear stress depends (weakly) on the water depth: lower bed shear stress for a larger water depth.

Wave related

Bed friction produces an oscillatory boundary layer within which the wave orbital velocity amplitude

increases rapidly from zero at the bed to the top of the boundary layer. The wave bottom boundary layer is only centimeters to a few decimeters thick, in contrast to the steady current boundary layer which may be meters or tens of meters thick.

$$\tau_{b,w} = \frac{1}{2} \rho \omega f_w U_w^2$$

The wave friction factor increases with k_s and decreases with a_w . The latter is related to the fact that wave boundaries are thicker, inducing lower shear, when they have more time to grow for waves with a larger amplitude of orbital motion

Combined waves and currents

The bed shear stress beneath combined waves and currents are enhanced beyond the values which would result from a simple linear addition of the wave-alone and current-alone stresses. This is because of a non-linear interaction between the wave and current boundary layers.

$$\tau_{b,max} = \sqrt{(\tau_{b,c} + \tau_{b,w} \cos \phi_{cw})^2 + (\tau_{b,w} \sin \phi_{cw})^2}$$

As f_w is generally larger than f_c the mean bed shear stress could even increase beyond this. The effect of the waves becomes smaller for stronger currents; it decreases with increasing a_{cw} to become zero for the current-alone case ($a_{cw} = 1$).

THRESHOLD OF MOTION

The fluid forces acting on a sand particle resting on a horizontal bed consist of a skin friction force and pressure forces (Figure 4.7). The skin friction force acts on the surface of the particles by viscous shear. The pressure force consists of drag (F_d) and lift (F_l) generated by pressure difference along the particle surface.

Particle movement will occur if the fluid forces with respect to the point of contact exceed the stabilizing moment of the submerged particle weight

For reasons of simplicity the lift force is assumed to be relatively small, and as it depends on the same variables as the drag force, its effect is accounted for through the empirical drag coefficient. the threshold motion can be expressed in terms of the Shields number:

$$\theta_{cr} = \frac{\sigma'_{r,cr}}{\Delta \rho \cdot g \cdot d}$$

SLOPING BEDS

If the bed is sloping, then gravity provides a component of force on the grain which may increase or decrease the threshold shear stress required from the flow.

For water flow up a slope with angle β , the force moment balance become

$$\theta_{cr,u} = \theta_{cr,0} \cdot \frac{\sin(\phi_r + \beta)}{\sin \phi_r}$$

and similarly for a downward slope: $\theta_{cr,d} = \theta_{cr,0} \cdot \frac{\sin(\phi_r - \beta)}{\sin \phi_r}$

For a flow up a slope the factor increases from 1 to 1.7; at maximum 70% additional force is needed to mobilize the grains.

The generic expression for the threshold of motion for sand grains on a bed sloping at angle β to the horizontal, in a flow making an angle ϕ_r to the upslope direction is:

SAND TRANSPORT PROCESSES

The sand flux is the product of the sand particle velocity and sand concentration. The horizontal sand particle velocity is almost equal the horizontal water velocity.

This is the sand transport per unit of width. It has to be multiplied with a representative width to get a volume/mass per unit of time.

The volumetric sand transport can be expressed including and excluding pore space. These are related to each other in the following manner:

Unless stated otherwise, sand transport is excluding pores in the remainder of this syllabus.

The sand transport is commonly made dimensionless in the following way:

$$\Phi_s = \frac{q_s}{\sqrt{g \Delta d^3}}$$

TRANSPORT MODES

If threshold of motion is exceeded, sand moves by bedload transport. This involves rolling, sliding and saltation of grains along the bed. Sand particles are frequently in contact with the bed and each other; intergranular forces are important. The bedload layer is usually only a few grain diameters thick ($\hat{=}$ 1 mm).

For stronger flows and/or finer particles sizes, sand is entrained off the bed and carried with the flow velocity as suspended load. For grains to remain in suspension, their settling velocity must be smaller than the turbulent velocity component that mixes sediment in the vertical direction. This turbulent velocity component is generated by bottom friction and the velocity shear, which is related to the shear velocity. This leads to a criterion for the threshold of suspension:

$$\frac{u_*}{w_s} < 1 \quad \text{— bedload}$$

$$\frac{u_*}{w_s} > 1 \quad \text{— bedload and suspended load.}$$

WAVE AND CURRENT RELATED TRANSPORT

The current-related sand transport is sand transport due to net (wave-mean) currents generated by tides, wind, density-gradients and waves (e.g. undertow due to wave breaking, see Chapter 3). The wave-related sand transport is sand transport due to the wave-induced oscillatory motion.

Note that in both cases there is no net mass transport of water. The general result for colinear waves and currents is:

$$\langle q_s \rangle = m \left[\langle u \rangle^3 + \frac{3}{2} \langle u \rangle U_0^2 + \frac{3}{2} \langle u \rangle U_0^2 + \frac{3}{4} U_0^2 U_1 \right]$$

PHASE LAG EFFECTS

The entrainment and settling of sand particles takes time. If this sand particle response time, t_s , is not small compared to the wave period, the sand concentrations may significantly lag behind the orbital velocity. The settling time depends on the sand particle stirring height, s , and fall velocity: $t_s = s/w_s$.

The sediment-laden vortex is ejected into the flow near flow reversal, to a height that scales with the ripple height, λ_r . Therefore, the importance of phase-lag effects can be expressed through the phase-lag parameter

Phase-lag effects are, in general, more important for finer sands and shorter wave periods, and in the sheet-flow regime for higher velocities that generate thicker sheet-flow layers. Figure 4.11 shows that large phase-lag effects can even result in a change of net sand flux direction. Generally, this only occurs for vortex ripples.

ACCELERATION EFFECTS

Acceleration skewness affects net sand transport through two mechanisms:

- *Higher bed shear stresses are associated with higher flow accelerations because the boundary layer has less time to develop before peak velocity is reached.*
- *High horizontal pressure gradient associated with high acceleration may also be directly responsible for the mobilisation of grains and therefore enhance net transport.*

Both promote net sand transport in the direction of the highest acceleration, i.e. typically in the onshore (positive) direction.

SAND TRANSPORT MODELLING

Later on, wave-related sand transport was included assuming a quasi-steady response of sand transport to the flow forcing; the so-called quasi-steady models. In the 1990s, the importance of unsteady effects, such as phase-lags between velocities and concentrations, was acknowledged, and so-called semi-unsteady models were developed. These three model types are also referred to as practical models or sand transport formulas, as these are relatively simple and require little computational power. These computationally expensive models are also called process-based models as the solved equations are based on first physical principles such as conservation of mass and momentum.

MODELED USING

CONCENTRATION PROFILES LIKE EXPONENTIAL - POWER LAW or ROUSE PROFILE

WAVE AVERAGE MODELING

Wave-averaged models were the first models to be developed to derive the sand transport rate under waves and currents.

0

In wave-averaged models, the total net transport is always in the direction of the mean current and the wave-related transport component is not taken into account. Due to refraction, wave generally have a dominant cross-shore component. As a result, cross-shore sand transport typically has an important wave-related component. This is less true for alongshore sand transport, and hence wave-averaged models are typically used in coastal engineering practice to compute this transport component.

CERC FORMULA

The CERC2 formula is one of the most widely-used method for the alongshore sand transport (CEM, 2006). It is a bulk formula for the cross-shore integrated alongshore sand transport in the surfzone generated by the wave-driven alongshore current also known as littoral drift.

$$Q_{sl} = 0.023 \frac{\sqrt{g}}{D} \cdot H_{sb}^{2.5} \cdot \sin(2\theta_{wb})$$

The alongshore transport according to the CERC is maximum for $\theta_{wb} = 45^\circ$, which coincides with a maximum alongshore current. Furthermore, the transport is not dependent on the grain-size. For H_s

= 1 m (typical average wave height Dutch coast), $\sqrt{wb} = 50$, $g = 9.81 \text{ m/s}^2$ and $d = 1.6$ the alongshore transport in the surfzone according to the CERC formula is $200 \cdot 10^3 \text{ m}^3/\text{year}$

VAN RIJN

Harder computational formula includes all types of transport.

QUASI-STEADY MODELS

Quasi-steady models consider the sand transport on an intra-wave scale, i.e. a time-scale much shorter than the wave period. These are based on the assumption of quasi-steadiness: sand transport responds instantaneously to the flow velocity, as if the flow is steady at each phase during the wave-cycle. In this way the wave-related sand transport is implicitly included.

(RIBBERINK)

SEMI-UNSTEADY MODELS

Semi-unsteady transport models are positioned between the quasi-steady and the unsteady models. Semi-unsteady models account for phase lags between the flow velocity and sand concentration, but they do not resolve the spatial and temporal structure of the velocity and sand concentration like unsteady models.

SANTOSS

It does not distinguish between bedload, suspended load or wave- and current-related contributions; it is a bulk formula for the total sand transport in the wave boundary layer. In general, most sand transport does indeed take place in the wave boundary layer, with the exception of cases with very fine sands, strong currents and/or breaking waves.

The division of sand load between the half cycles is determined by a phase-lag parameter similar to Eq. (4.72). In this way, unsteady phase-lag effects between velocities and concentrations, which are especially important for rippled bed and fine sand sheet-flow conditions, are accounted for. Recently-recognized effects on the net transport rate related to flow acceleration skewness and progressive surface waves are also included.

The formula is especially developed for cross-shore sand transport under wave-dominated conditions. The formula was developed using a database of 226 net transport rate measurements from large-scale oscillatory flow tunnels and a large wave flume, covering a wide range of full-scale flow conditions and uniform and graded sands with median diameter ranging from 0.13 mm to 0.54 mm

UNSTEADY MODELS

Unsteady models resolve the temporal (intra-wave) structure of the velocity and suspended sand concentration. Unsteady models are based on first physical principles (such as conservation of mass and momentum), and therefore also referred to as process-based models. Such models can be classified according to the spatial dimensions they consider:

- 1DH, one-dimensional horizontal. Model computes depth-averaged variables as function of one horizontal coordinate, usually the cross-shore direction.
- 1DV, one-dimensional vertical. Model computes variables as function of the vertical coordinate.
- 2DH, two-dimensional horizontal. Model computes depth-averaged variables as function of the cross- and alongshore position.
- 2DV, two-dimensional vertical. Model computes variables as function of the vertical coordinate and one horizontal coordinate, usually the cross-shore direction.
- 3D, three-dimensional. The full shebang.

The fact that most unsteady models still contain empiricism and are computationally (very) expensive, makes them equally (un)suitable as the more practical models described in the previous sections.

Every problem requires its own modeling approach, also known as appropriate modeling. There is just not one single model that can do it all!

QUESTIONS

Gravity influenced by grain size, shape and sediment density.

Q1 Which parameters influence a) the sediment particle fall velocity, and b) the motion of sediment particles at the bed? Give the parameters, describe their effect as well as the underlying physical mechanisms.

Grain size, density and shape.

a) The fall velocity is influenced by buoyancy, gravity and the drag force of the liquid.

The lower the drag force of the liquid the higher the fall velocity.

Buoyancy and drag push the particle upwards while gravity push it downwards.

b) The motion is influenced by the bed slope, the particle size (the higher the more speed is required). It is also influenced by the friction in the bed.

Therefore, it is influenced by the exerted shear velocity on the particles.

Bed shear stress $\rightarrow$ shear velocity $\rightarrow$ influenced by water density, bed roughness and near bed-flow velocity.

Q2 Assume the following equation for a vertical-uniform tidal current velocity: $u(t) = u_1 \sin(\omega t) + u_2 \sin(2\omega t)$ with velocity amplitudes u_1 and u_2 , $\omega = 2\pi/T$ and T the tidal period. The seabed consists of fine sediment. Therefore, the suspended sediment concentration also has a vertical-uniform distribution. At each instant, the sediment concentration is modeled to be proportional to the square of the velocity.

a. Derive an analytical expression for the time-dependent sediment transport.

$$u(t) = u_1 \sin(\omega t) + u_2 \sin(2\omega t) \quad \rightarrow \quad \omega = 2\pi/T.$$

$$c(t) = (u_1 \sin(\omega t) + u_2 \sin(2\omega t))^2 = (u_1^2 \sin^2(\omega t) + 2u_1 u_2 \sin(\omega t) \sin(2\omega t) + u_2^2 \sin^2(2\omega t))$$

$$u \cdot c = u_1^3 \sin^3(\omega t) + 2u_1^2 u_2 \sin^2(\omega t) \sin(2\omega t) + u_1 u_2^2 \sin(\omega t) \sin^2(2\omega t) + u_2^3 \sin^3(2\omega t)$$

$$C = c(t) \cdot A \rightarrow$$

$$Q_{s(t)} = \int_0^h \bar{u}(z,t) \cdot c(z,t) dt = hA \cdot [u_1^3 \sin^3(\omega t) + u_2^3 \sin^3(2\omega t) + 3u_1^2 u_2 \sin^2(\omega t) \sin(2\omega t) + 3u_1 u_2^2 \sin(\omega t) \sin^2(2\omega t)].$$

b. Derive an analytical expression for the net (tide-averaged) sediment transport.

$$\langle Q_s \rangle = \frac{1}{T} \int_0^T Q_{s(t)} dt$$

$$\langle Q_s \rangle = \frac{hA}{T} \int_0^T \left[v_1^3 \sin^3(\omega t) + v_2^3 \sin^3(2\omega t) + 3v_1^2 v_2 \sin^2(\omega t) \sin(2\omega t) + 3v_1 v_2^2 \sin(\omega t) \sin^2(2\omega t) \right] dt.$$

Integral of odd numbers from $t=0$ to $t=T$ equals zero.

$$\langle Q_s \rangle = \frac{hA}{T} \int_0^T \left[3v_1^2 v_2 \sin^2(\omega t) \sin(2\omega t) + 3v_1 v_2^2 \sin(\omega t) \sin^2(2\omega t) \right] dt$$

$\sin(2\omega t) = 2 \sin(\omega t) \cos(\omega t)$ — using,

$$\langle Q_s \rangle = \frac{hA}{T} \int_0^T \left[3v_1^2 v_2 \sin^2(\omega t) \cdot 2 \sin(\omega t) \cos(\omega t) + 3v_1 v_2^2 \sin(\omega t) \cdot 4 \sin^2(\omega t) \cos(\omega t) \right] dt$$

↳ odd terms = 0 so.

$$\langle Q_s \rangle = 0$$

c. In which direction is the net water transport, if there is any? Explain your answer.

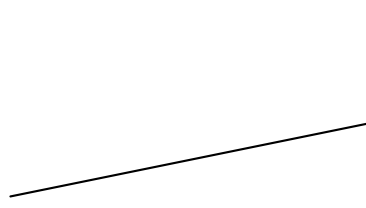
As it is being used a linear term symmetrical, there is no net water transport.

d. In which direction is the net sediment transport, if there is any? Explain your answer.

As it is being used a symmetrical transport, there is no net sediment transport.

Q3 Assume a straight, uniform North-South orientated coastline, with a constant bottom slope towards the shoreline.

a. Describe the steps and the essential formulas, required to calculate the wave-induced alongshore sediment transport rate, using the CERC-formula.



It is required to determine the wave angle and the breaker line height and depth.

Also it is essential to determine the sediment density and the breaker coefficient.

To calculate it, it is essential to calculate wave height based on shoaling and refraction

$$\frac{H}{H_0} = \sqrt{\frac{n_{w0} \cdot c_{w0}}{n_{w1} \cdot c_{w1}}} \cdot \sqrt{\frac{\cos \theta_{w0}}{\cos \theta_{w1}}}$$

b. Make a sketch of the cross-shore distribution of the alongshore sediment transport rate in the breaker zone. In reality, does this profile occur? Why (not)?

CERC formula assumes that alongshore sediment transport is proportional to the incoming alongshore component of wave energy flux. $P_l = c_g E \sin \phi$ ($\cos \phi \approx 1$)

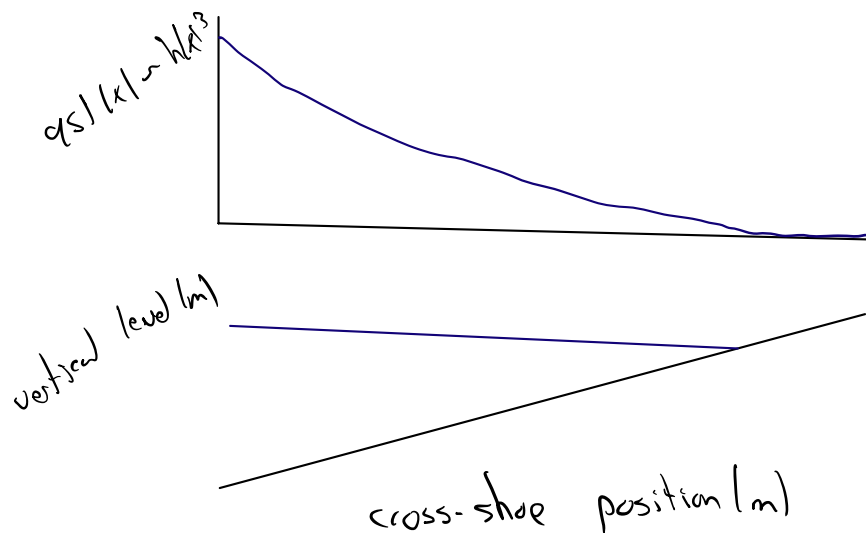
It is then related to water depth in the surf zone $q_{sl} \propto P_l \propto h^3$ ^{linear shore}

In reality,

There are a number of factors modifying this distribution such as wave breaking zone, instead of a breaking point.

On-shore and off-shore shifting breaking zone. due to varying wave conditions, non-linear beach slope.

Wave breaking not following simple linear model.



At the breaker line, the wave conditions are as follows: a wave height of 2.88 m and $\theta_w = 30^\circ$.
c. Compute the alongshore sediment transport rate and determine the direction of this transport. Explain the assumptions that you make.

CERC Formula:

Assuming $\gamma_w = 0.7$

$$Q_{sl} = 0.023 \frac{\sqrt{g}}{D} \cdot M_{s0}^{2.5} \cdot \sin(2\theta_{wb}) = 0.066 \text{ m/s}$$

Assuming sediment density 2650 kg/m^3
- shallow water

The deep-water wave climate is schematized in Figure D.6. This wave rose allows to determine wave heights and frequencies of occurrence per 30° direction bin.

This wave rose contains too much information for us, so we approximate this data as follows:

- For waves with $\gamma_w = 30^\circ$ (WNW) the following conditions are obtained: $H = 2 \text{ m}$ and $T = 7 \text{ s}$. This condition occurs during 23.0% of the time.
- For waves with $\gamma_w = 60^\circ$ (NNW): $H = 1.5 \text{ m}$ and $T = 7 \text{ s}$, during 13.2% of the time.
- For waves with $\gamma_w = 0^\circ$ (W): $H = 2.5 \text{ m}$ and $T = 7 \text{ s}$, during 21.6% of the time.
- For waves with $\gamma_w = 45^\circ$ (W): $H = 2 \text{ m}$ and $T = 7 \text{ s}$, during 10.5% of the time.
- During the rest of the time, there is no sediment transport at all.

d. Compute the annual net alongshore sediment transport and determine the direction of this transport by using the adapted CERC formula: $Q_s = 0.04 M_{s0}^2 \cos(\theta_0) \cdot \sin(\theta_b)$

in which H_0 is the deep-water wave height, c_{w0} the deep-water wave celerity, w_0 the deep-water wave angle and w_b the wave angle at the breaker line.

Considering $q_w = 0.7$, θ_{wb}

- 14
- 19
- 0
- -19

	w	Q_s	q_{op}	$Q_{s.p}$	Total
Thus	w_{c1}	0.36	23.90	0.083	
	w_{c2}	0.16	13.210	0.021	0.662
	w_{c3}	0	21.610	0	
	w_{c4}	-0.402	10.590	-0.042	

$\Delta_{total} = 195.5 \cdot 10^4 m^3$

Direction $\rightarrow$ onshore
Positive (North),

COASTAL MORPHOLOGY

Morphology is the study of the forms of things. Coastal morphology is about understanding why coasts look the way they do, and about predicting future changes due to natural processes and human interventions. This is very relevant as coastal zones hold high economic values and protect the hinterland against flooding. Anticipated sea-level rise related to climate change makes this task even more challenging, especially for low-lying delta areas such as The Netherlands.

The North Sea seabed displays 6 main bedform types according to my (subjective) opinion: ripples, megaripples, breaker bars, sand waves, sand banks and shoreface-connected ridges

Coastline dynamics

The position of the coastline is largely controlled by the alongshore sand transport or, more precisely, the alongshore sand transport gradients taking place in the active zone of the coastal profile.

The so-called one-line models further assume that active zone retains its coastal profile. In other words: redistribution of sand across the profile due to cross-shore transport is assumed to occur infinitely fast (i.e. much faster than the coastline changes).

his equation shows that a net (positive or negative) influx of alongshore sand transport during time step t results in a cross-shore shift (offshore or onshore) of the active profile of a coastal element with alongshore distance y

The longshore transport constant b can be determined numerically (by computer) with the help of a selected longshore transport formula and longshore transport computations for different coastal angles. It can also be determined analytically for coastal problems where the CERC formula is appropriate

Establishing the height h_p of the active zone is an uncertain element of the one-line model. The vertical distance between the level of the dune crest (e.g. 8 m above MSL = Mean Sea level) and the

level of the seabed at the offshore extension of the first (i.e. most offshore) breaker bar (e.g. 6 m below MSL) is a possibility. For the Dutch coast a typical coastal constant $a = 0.1-0.5 \cdot 10^6 \text{ m}^2/\text{year}$. Eq. (5.9) is a second-order ordinary differential equation. It requires 1 initial condition and 2 boundary conditions to be solved.

Infinitely long groyne

- Initial shoreline position: $x_k(y, 0) = 0$ (and so)
- No longshore transport at the groyne: $S_{sl}(0, t) = 0$, so $\nabla r = 0$, $\nabla k = \nabla w$, $\partial x_k(0, t) = \nabla w$. @y
- Longshore transport not affected infinitely far updrift from the groyne: $S_{sl}|y| \rightarrow \infty = \text{constant}$. Because of the constant wave conditions and the initial condition this means .

Downdrift an infinitely long groyne ($y > 0$) a solution can be found with the same initial and boundary conditions as for the updrift part, except for:

- Longshore transport not affected infinitely far downdrift ($y \rightarrow -\infty$ instead of $y \rightarrow \infty$) from the groyne: $S_{sl}|y| \rightarrow -\infty = \text{constant}$. Because of the constant wave conditions and the initial condition this means $\partial x_k(-\infty, t) = 0$. @y

The solution is a mirror image of the solution for the coastline updrift from the groyne. An (infinitely long) groyne thus results in updrift sedimentation and downdrift erosion. The shoreline change is strongest at the groyne ($y = 0$). The morphological changes slow down in time.

This is at the groyne ($y = 0$); morphological change will be slower at other alongshore locations. It shows that a large coastline shift takes more time, while a larger a (due to a smaller active height, or a larger change of alongshore transport with (relative) wave angle) and a larger (relative) wave angle result in a faster morphological development

Groyne with finite length

In case of a groyne with a finite length, sand can bypass leading to a stable equilibrium. Moreover, due to wave diffraction and variable wave conditions, a modified equilibrium coastline shape will develop

CROSS-SHORE MORPHODYNAMICS

The beach and upper shoreface (water depths smaller than $\hat{z} \approx 8 \text{ m}$) show a quick morphological response to changes in wave /current conditions (hours, days, weeks). All kinds of smaller-scale morphological features can be observed in this area

- swash bars
- rip channels
- beach cusps
- breaker bars
- bed ripples

The lower shoreface (water depths smaller between $\hat{z} \approx 8-20 \text{ m}$) and the shelf show a much slower morphodynamic behavior (months, years), due to the much lower magnitude of sediment transport rates (smaller wave influence).

BEACH STATES

- Reflective beaches for $\tan \beta < 2.5$. These relatively steep beaches allow for a strong reflection, standing waves and surging breakers. The reflection can generate (trapped) edge waves and the associated generation of beach cusps.

- Dissipative beaches for $\bar{b} > 20$. These relatively flat and wide beaches allow for dissipation of (short) wave energy accompanied by a shoreward growth of infragravity (long) wave energy, breaker bars and spilling breakers. (Dutch beaches)

As the beach bed slope is a function of the grain-size (in general coarser sands correspond to steeper beaches). Reflective beaches occur for $\bar{b} < 1$, and dissipative beaches for $\bar{b} > 6$ (Wright and Short, 1984). Suspended load dominates over bedload for high Dean numbers (and vice versa), such that suspended load is the dominant transport mode for dissipative beaches and bedload for reflective beaches.

DUNE EROSION

In extreme conditions, such as storms with wind-set-up in combination with an extreme high- water during spring tide, wave breaking may directly take place at the dune slope. Large suspended sediment loads, undertow and gravity effects lead to a dominant offshore-directed sediment transport, collapse of the dune front and sediment deposition on the beach and the shallow foreshore

The dune erosion process is a fast process with a morphological time scale of one storm (hours). It is of direct importance for the management of dune areas (danger of breaching, safety of the (low) lands behind the dunes).

- Swash/collision regime. Mass flux by waves returns as offshore return flow, keeps erosion process going by removing sand from slumping dune face (Figure 5.10).
- Overwash regime. Flow dominated by low frequency motion (wave groups), carrying water over dunes, landwards sand transport creates overwash fans.
- Inundation regime. Breaching occurs where a new channel is formed cutting through the dune.

BREAKER BARS

A bar is generated near the shoreline and grows in height and width while migrating offshore, before finally decaying at the seaward limit of the surf zone. It may take 10 to 15 years for a bar to exhibit this cycle. Four to five bars may occur simultaneously within a cross-shore bed profile. (Walstra, 2016) They can be detected remotely by the foam generated by depth-induced wave breaking. Breaker bar will dissipate wave energy. If the relative wave height, H_s/h , is large enough waves break on the bar, generating an offshore-directed undertow, turbulence and sand suspension. Low waves shoal on a breaker bar, due to the decreasing water depth, leading to an increased velocity skewness with higher onshore orbital velocities.

As a result, breaker bars typically migrate in the onshore direction during calm, low-energy wave conditions and in the offshore direction during stormy, high-energy wave conditions. On the longer term they move in the offshore direction. The migration direction is controlled by the sand transport direction. Under high waves the offshore-directed suspended load due to wave-breaking induced undertow dominates, whereas under low waves the wave skewness drives the sand transport in the onshore direction.

In the case of net onshore sand transport, an offshore (onshore) shift in the transport results in bar growth (decay) while migrating onshore. In the case of net offshore sand transport, an onshore (offshore) shift in the transport results in bar growth (decay) while migrating offshore.

NOURISHMENTS

The corresponding nourishment volume was about 6 million m^3 /year between 1990 and 2000. Besides beach nourishments, more economically attractive shoreface nourishments started becoming common practice. However, this policy did not consider the morphological development at a larger scale, induced for example by sand losses at larger water depth and by sea level rise.

In view of this new concept, the nourishment volumes for the whole Dutch coast were increased to 12 million m³ yearly.

Shoreface nourishments are placed just offshore from the outer breaker bar at water depths

between 5 and 8 m. It has two main effects (Figure 5.14):

- Lee effect. This is predominately a longshore effect. Large waves break at the shoreface nourishment causing a calmer wave climate behind the shoreface nourishment area (wave filtering) and a reduction of the longshore current and, hence, the transport capacity. This local decrease in longshore transport causes updrift accretion and downdrift erosion.
- Feeder effect. This is a cross-shore effect. The shoaling waves that remain after wave filtering generate onshore transport due to wave skewness. The smaller waves in the lee-side generate less stirring of the sediment and a reduced offshore-directed undertow. The increased onshore transport and decreased offshore transport result in a net onshore- directed transport bringing the nourished sediment to the shore.

Bruun rule: shoreline response to sea-level rise

The most well-known concept to quantify the shoreline response to sea-level rise (SLR) is the so-called Bruun rule (Bruun, 1962; Udo et al., 2020). The rule assumes that the active coastal profile, i.e. landward from the closure depth, moves upward and landward in response to SLR, keeping its equilibrium shape.

Bruun assumed that sand supply from offshore is negligible if the shoreface profile extends seaward up to the closure depth. The sand should then be supplied by the landward part of the active coastal profile, i.e. the dry beach and the foredune. This implies shoreline retreat (designated x_{slr}) and a corresponding landward shift of the equilibrium shoreface profile. The new equilibrium profile, $z_{b1}(x)$, is then equal to the old equilibrium profile with an upward shift equal to the sea-level rise h_{slr} and a landward shift x_{slr} (see Figure 5.15):

COASTAL AREA MORPHODYNAMICS

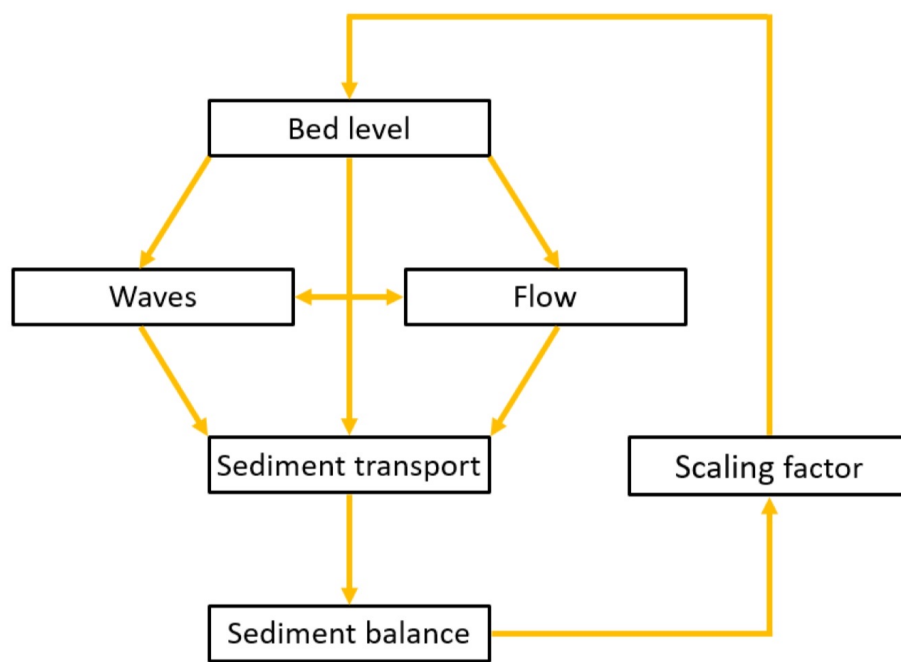
The morphological processes of the seabed have a more complex 2D/3D-character in case of, for example, non-uniform coastlines, around coastal structures (harbors, breakwaters, etc), around coastal headlands (e.g. rock/cut-offs), and in situations where tidal channels/shoals are present in the nearshore area.

MORPHOLOGICAL AREA MODELING

Morphological area models compute waves, water levels, currents, sediment transport and bed level changes on a grid with two horizontal dimensions (2DH), see Figure 5.16. It accounts for a two-way wave-current interaction: wave-generated currents and set-up/set-down, and wave propagation affected by time-varying tidal water levels and current-induced shoaling and refraction.

MorFac-values of 100, i.e. every flow time step the computed bed level is increased with a factor of 100, are not uncommon. This means that the simulation of 10 morphological years only requires the simulation of 1.2 hydrodynamic months. Of course, this should be a representative month, as well as it needs to be checked whether the morphological predictions are not seriously affected by the chosen MorFac-value.

The currents and sediment transport are usually solved in a depth-averaged, 2DH framework in order to avoid unpractically long computation times. This does mean that flow and sediment transport processes associated with a vertical structure are not (completely) resolved, such as the density-driven vertical circulation.



TIDAL INLET SYSTEM

In deep areas, such as the channels, the sediment transport is tide-dominant. In shallow areas, such as on the shoals and bars, and in the surfzone, the sediment transport is wave-dominated. Sand can bypass the tidal inlet through shoal migration of the edge of the outer delta, and via the tidal basin. Within two years after construction, there also occurred an unexpected, strong accretion on the downdrift/northern side (Figure 5.18). Possibly, the design study did not properly account for the sediment transport from the north, or the impact of the groyne on the Eijerlandse Gat. All in all it turned out to be one of the most successful Dutch coastal engineering works!

SAND MOTOR MEGA NOURISHMENT

Beach nourishments have a typical volume of $100\text{--}500 \times 10^3 \text{ m}^3$, and shoreface nourishments $1\text{--}5$ million m^3 . In line with expected sea-level rise, an innovative pilot project was developed to achieve a more efficient and sustainable nourishment approach with an upscaled volume: the Sand Motor³. This mega nourishment, built in 2011 along the Delfland coast, consists of a total sediment volume of 21 million m^3

The seaward edge of the peninsula erodes, and considerable accretion occurs on the adjacent coast to the northeast and southwest of the Sand Motor (see Figure 5.19). On the northern side of the peninsula, the morphological behavior is dominated by the dynamics of the lagoon entrance. Soon after construction of the peninsula, a spit developed, squeezing the lagoon entrance. Carefully set-up and calibrated depth-averaged (2DH) Delft3D numerical models were successful in reproducing the general morphological development of the Sand Motor

QUESTIONS

Q1 Consider the schematized cross-shore profile sketched in Figure D.8.

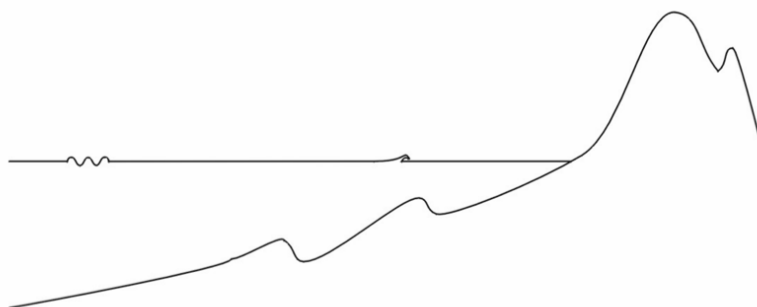


Figure 5.22: Schematized cross-shore profile.

a. Name at least 5 different zones of this cross-shore profile.

b. Describe briefly what the consequences are of a storm surge for this cross-shore profile (short-term and long-term). What happens during a storm? And what happens during more quiet periods? Make a sketch of the situations and the processes involved.

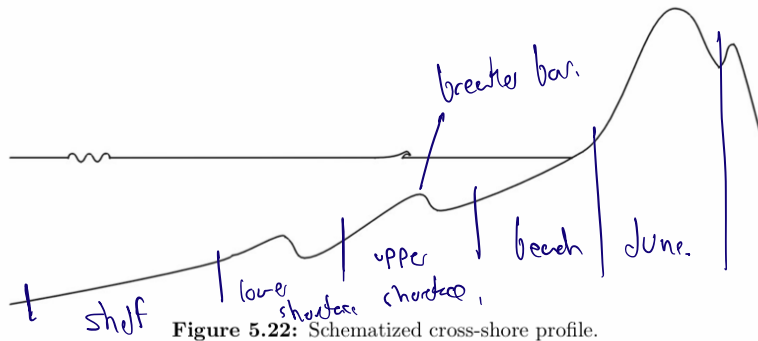


Figure 5.22: Schematized cross-shore profile.

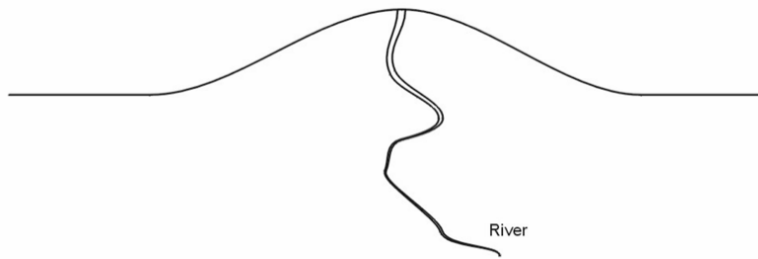
During a storm

During a storm the dunes will erode, given the higher undertow speeds the sediment will be transported offshore (making the bed flatter). Additionally the dune could erode too much and cause an overwash regime or inundation. $\rightarrow$ (more dissipative).
Elevated mean sea level.

After a storm

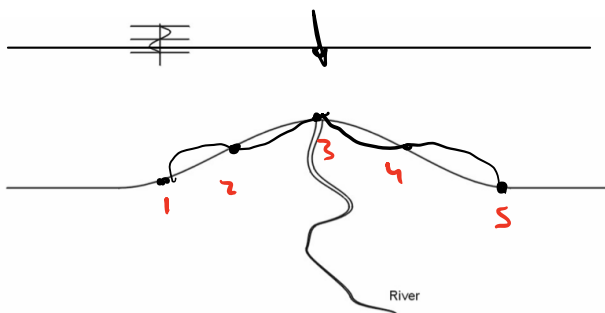
Waves will transport sediment onshore again, bringing the beach to a higher slope (more reflective).
Mean sea level re-establishes.

Q2 Consider the coastline sketched in Figure D.9.



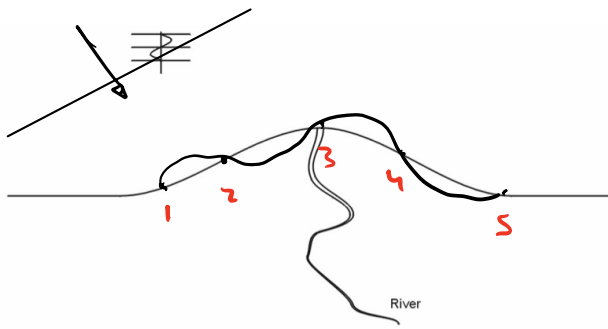
Describe the long-term development of the coast in the following cases:4

a. The dominant wave direction is perpendicular to the coast.



As the dominant wave direction is perpendicular
Max sediment at 45° (location 2 and 4)
From 1 to 4 and 5 to 4 there is accretion
From 2 to 3 and 3 to 4 there is erosion.
 $\propto \sin(2\theta)$

b. The dominant wave direction is 45° counter clockwise.



0. at 2 and 4.

Angle increase from 2 to 3

From 1-2 and 3-4 there is accretion

From 2-3 and 4-5 there is erosion

If angle increases $\rightarrow$ erosion - if angle decreases $\rightarrow$ accretion.

c. Discuss the role of the sediment supply from the river. What happens in case of plenty sediment supply? What if sediment supply is blocked by a barrage?

If there is plenty river sediment supply, the coastline will not erode and could even grow.

-If there is no sediment supply, the coastline will flat out.

Q3 Consider a tidal inlet system with sediment transport along the coast. The sediment transported by the alongshore current passes the tidal inlet via propagating shoals in the outer-delta of the tidal inlet. From time to time, these shoals reach the beach downstream of the tidal inlet, supplying this beach with sediment (Figure D.12).

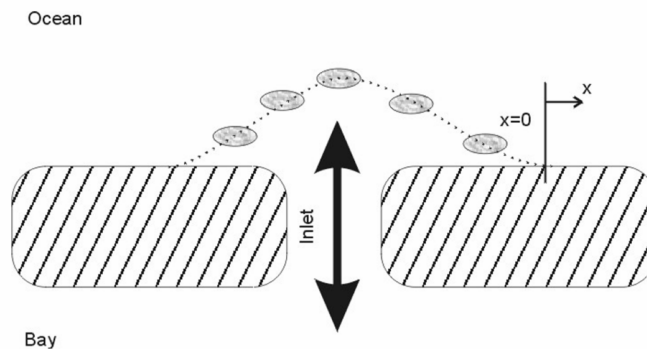


Figure 5.24: Schematic tidal inlet with shoals propagating across the outer-delta.

This process of landing of shoals can be seen as a periodic process with period T_r . Furthermore, we assume i) that the shoals land at one specific point ($x = 0$) at the downstream beach and ii) that at this point the coastline y_k follows a sinusoidal behavior in time (seaward progression followed by landward erosion).

a. Use a 1-line model to determine a solution for the behavior of the coastline $y_k(x, t)$ downstream of the tidal inlet at $x > 0$. Look for a solution of the form: $y_k = F(x)\cos(\omega_r t - kx)$ where $\omega_r = 2\pi/T_r$ and $F(x) = F_0 e^{-\lambda x}$, and $k = 2\pi/L_r$ are functions to be determined later.

Using

$$\frac{\partial y_k}{\partial t} - a \frac{\partial^2 y_k}{\partial x^2} = 0$$

with

$$y_k = F(x) \cdot \cos(\omega_r t - kx)$$

$$\frac{\partial F_0 e^{-\lambda x} \cos(\omega_r t - kx)}{\partial t} - a \frac{\partial F_0 e^{-\lambda x} \cos(\omega_r t - kx)}{\partial x^2} = 0$$

$$\frac{\partial}{\partial x} (-\omega F_0 e^{-\lambda x} \sin(\omega t - kx) - K(x)) = \frac{\partial}{\partial x} (-\lambda F_0 e^{-\lambda x} \cos(\omega t - kx) + K F_0 e^{-\lambda x} \sin(\omega t - kx))$$

$$-F_0 e^{-\lambda x} \sin(\omega t - kx) = \lambda^2 F_0 e^{-\lambda x} \cos(\omega t - kx) - \omega \lambda K F_0 e^{-\lambda x} \sin(\omega t - kx) - \omega K^2 F_0 e^{-\lambda x} \cos(\omega t - kx)$$

$$\sin(\omega t - kx) \cdot F_0 e^{-\lambda x} (+\lambda K - \omega \lambda K) = F_0 e^{-\lambda x} \cos(\omega t - kx) \cdot (\lambda^2 - \omega K^2)$$

$$(\lambda K - \omega \lambda K = 0) \rightarrow \lambda^2 = \frac{\omega^2}{2aTr}$$

$$(\lambda = K)$$

Solution

$$y_k = F_0 e^{-\sqrt{\frac{\omega^2}{2aTr}} \cdot x} \left(\cos\left(\frac{\omega^2}{Tr} \cdot t\right) - \sqrt{\frac{\omega^2}{2aTr}} \cdot x \right)$$

b. Describe briefly the solution you found. In order to do so, make a sketch of the coastal profile in time and space and explain your findings.

The 1-D model mainly depends on the tidal period, and the initial amplitude F_0 , that travels and decays exponentially in x -direction.

Exponentially decays in the alongshore direction.

c. How long does it take for a shoal to reach a location 10 km downstream of the landing point (i.e. at $x = 10$ km)? How strong is the effect of this shoal here?

Consider $Tr = 20$ years $a = 0.3 \cdot 10^6 \text{ m}^2/\text{year}$; $K = 7.2 \cdot 10^{-4} (1/\text{m})$

$$K = \frac{\omega^2}{Lr} \rightarrow Lr = 8.7 \text{ km}$$

with wave speed $c_w = \frac{Lr}{Tr} = 0.44 \text{ km/year}$

to reach 10 km $x = c_w t \rightarrow \frac{x}{c_w} = t \rightarrow t = \frac{10}{0.44} = 23 \text{ years}$