

EXAM AND SOLUTIONS

M8 TRAFFIC FLOW DYNAMICS

2018-MODAN-2B

May 22, 2019

8:45-11:45

Room: NH124

The exam consists of 4 problems. Maximum scores are listed in the table below. End score is the total score divided by 7.

Good luck

Eric van Berkum

problem	max score
1	10
2	15
3	15
4	25
bonus	5
total	70

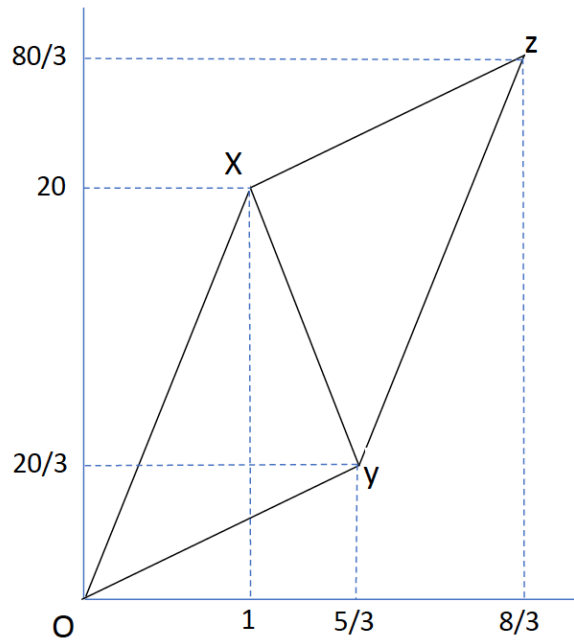


Figure 1: Trajectories

1. [10 points] Three friends take a long trip using a tandem bicycle for 2 persons. Because the bike riders travel at 20 km/h, independent of the number of riders, and all three persons walk at 4 km/h, they proceed as follows: to start the journey, friends A and B ride the bicycle and friend C walks; after a while, friend A drops of friend B who starts walking, and A rides the bicycle alone in the reverse direction. When A and C meet, they turn the bicycle around and ride forward until they catch up with B. At that moment, the three friends have complete a basic cycle of their strategy, which they then repeat a number of times until they reach their destination. What is their average travel speed? Tip: best way to solve this problem is to draw trajectories in the (t, x) -diagram.

Solution

Without loss of generality we assume that the tandem travels for 1 hour, from O to X. Obviously, since the speed is 20 km/h, the distance covered is 20 km. From here friend A returns and travel in the opposite direction to Y. The trajectory for friend A from X is

$x_A(x, t) = 20 - 20(t - 1)$. From origin O friend C has departed with speed 4 km/h. Trajectory from C is $x_C(t, x) = 4t$. To calculate the intersection Y it must hold that $x_A = x_C$, which yields $X = (5/3, 20/3)$. For reasons of symmetry it holds that $t_Z = t_X + t_Y = 8/3$, and $x_X + x_Y = 20 + 4 * 5/3 = 80/3$. As a result $\bar{v}$ the average speed to Z is $\bar{v} = \frac{80/3}{8/3} = 10$ km/h

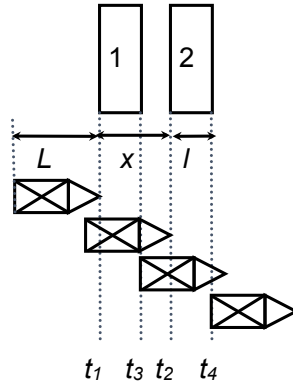


Figure 2: Loop configuration and time instants

2. [15 points] Rijkswaterstaat determines the locally measured individual vehicle speeds on motorways using two subsequent induction loops. Passing vehicles cause signals at different time instants, i.e.:

- t_1 , loop 1 is 'on'
- t_2 , loop 1 is 'off'
- t_3 , loop 2 is 'on'
- t_4 , loop 2 is 'off'

Length of a loop is l and the gross distance between two loops is x .

- (a) [4 points] Show how individual vehicle speed u_i of vehicle i can be determined from the different time instants.

Solution:

Speed v_i can be determined by:

$$v_i = \frac{\frac{x}{t_{i,2} - t_{i,1}} + \frac{x}{t_{i,4} - t_{i,3}}}{2}$$

- (b) [3 points] Derive how the mean speed for a period T can be computed. Is this time mean speed or space mean speed?

Solution:

Suppose during T there are n vehicles that passed, with speeds $v_1, \dots, v_n$.

$$\sum_{i=1}^n v_i$$

Then the mean speed is $\bar{v} = \frac{\sum_{i=1}^n v_i}{n}$

Since the measurements were recorded at one location over a period of time, it is time mean speed, or local mean speed.

- (c) [4 points] Show how an approximation of instantaneous mean speed can be determined for a period T

Solution:

When neglecting accelerations it can be shown that

$$u_M = \frac{1}{\frac{1}{n} \sum_{i=1}^n \frac{1}{v_i}}$$

So space mean speed equals the *harmonic average* of the speeds collected at a cross-section during a certain period T .

- (d) [4 points] Show how the length L_i of vehicle i can be determined

Solution:

Vehicle length L can be determined by:

$$L = v(t_4 - t_1) - x - l$$

3. [15 points] Consider a vehicle at an uncontrolled intersection waiting to merge into the main stream flow which is 2400 veh/h. The minor stream vehicle will merge only if the gap is at least 2 sec.
- (a) [5 points] What is the probability that a gap is sufficiently large for the vehicle to merge the main stream?

Solution:

For the main stream traffic flow we assume a Poisson distribution. When the arrival process is Poisson distributed, the headways are negative exponential distributed. Suppose flow is q , so $q = 2400$ veh/h = 0,6667 veh/sec, then for the number of arrivals k during time interval T it holds that the probability function is:

$$f(k) = \frac{\mu^k}{k!} e^{-\mu} = \frac{(qT)^k e^{-qT}}{k!}$$

The probability function for headways can be determined as

$$f(h) = qe^{-qh}$$

It can further easily be derived that mean headway is $\frac{1}{q} = 1,5$ sec. Then the probability that a gap is sufficiently large is

$$P(h \geq 2) = \int_2^{\infty} f(x) dx = e^{-4/3}$$

- (b) [5 points] Suppose we observe this intersection for one hour. What is the expected number of gaps that are sufficiently large?

Solution:

As was seen before the probability that a gap is sufficiently large is $e^{-4/3}$. In one hour 2400 vehicles will pass, thus the expected number of sufficiently large gaps is $2400 e^{-4/3}$.

- (c) [5 points] Analyses of the occurrence of different gap sizes when traffic volume is heavy have shown that the main discrepancies occur at gaps of short lengths (that is, less than 1 sec). The reason for this is that although theoretically there are definite probabilities for the occurrence of gaps between 0 and 1 sec, in reality

these gaps very rarely occur, since a driver will tend to keep a safe distance between his or her vehicle and the vehicle immediately in front. Write down the equation for the probability that a gap is sufficiently large, taking into account this restriction.

Solution:

Suppose the minimum gap is τ , e.g. $\tau = 1$ sec. Then

$$P(h \geq 2) = e^{-2/3(2-\tau)} = e^{-2/3}$$

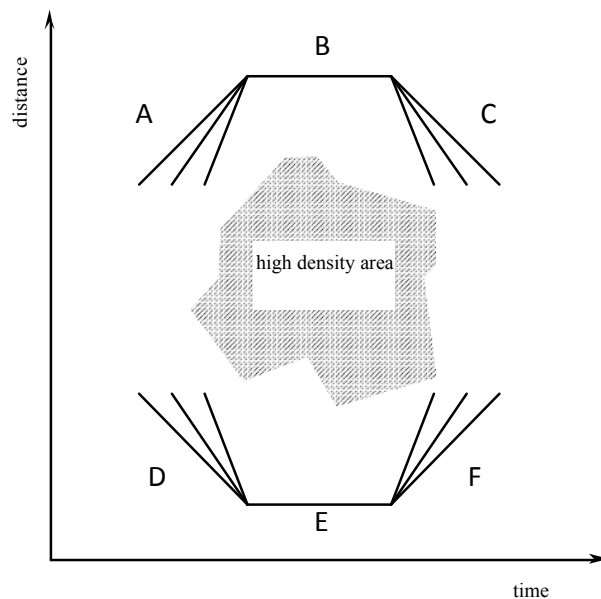


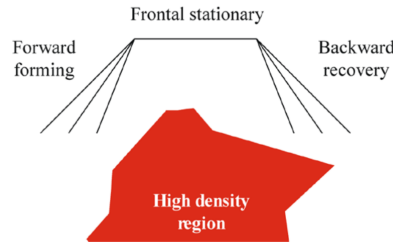
Figure 3: Classification of shockwaves

4. [25 points] In figure 3 in the (t, x) -plane a high density area is depicted, with six types of shockwaves.

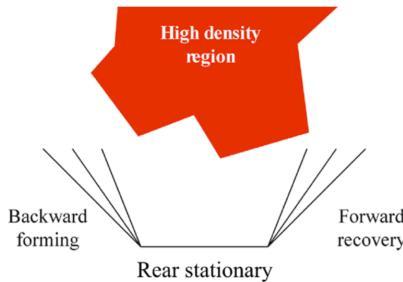
(a) [10 points] For all types, give the name and an example in which the type occurs.

Solution:

- *Frontal stationary*: head of a queue in case of stationary / temporary bottleneck
- *Forward forming*: moving bottleneck (slow vehicle moving in direction of the flow given limited passing opportunities)
- *Backward recovery*: dissolving queue in case of stationary or temporary bottleneck (demand < supply); forming or dissolving queue for moving bottleneck



- *Forward recovery*: removal of temporary bottleneck (e.g. clearance of incident, opening of bridge, signalized intersection)
- *Backward forming*: forming queue in case of stationary, temporary, or moving bottleneck (demand > supply);
- *Rear stationary*: tail of queue in case recurrent congestion when demand is approximately equal to the supply



- (b) [15 points] In 1975 Gerlough and Huber produced the Special Report 165 for the Transportation Research Board on traffic flow theory. This report contained the following example: "a steady flow state existed when a slow-moving truck entered the roadway and proceeded to travel at a constant lower speed for some distance before turning off the roadway. It is assumed that passing is not possible". Construct in the (t, x) -plane a diagram in which all shockwaves that occur are shown, as well as the trajectories of some selected vehicles. Also draw the density-flow fundamental diagram and depict in this diagram all traffic states that occur. Assume that the Greenshields relationships hold.

Solution:

We can identify 4 different traffic states, i.e.

- A, the normal traffic flow, somewhere in the free-flow regime.

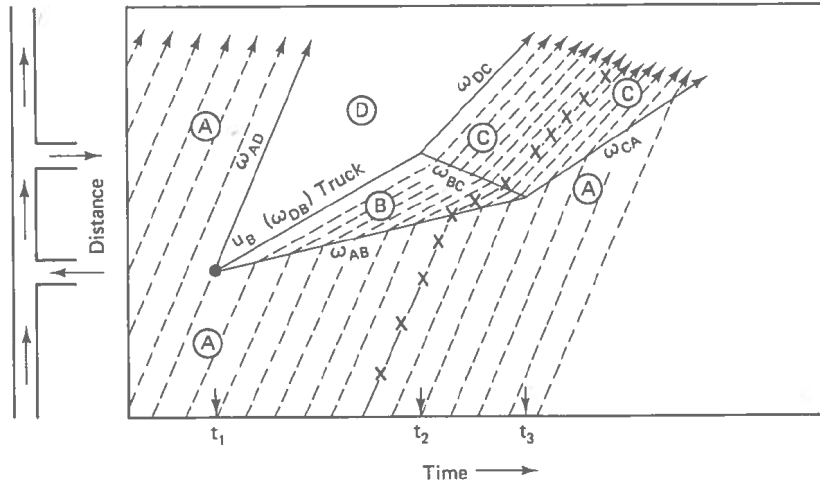


Figure 4: Shockwaves and trajectories

- B, the platoon of vehicles that follow the slow truck, so speed in the platoon equals the speed of the truck.
- C, capacity conditions that occur after the slow truck has disappeared
- D, the vacuum that occurs downstream the slow truck.

From these 4 traffic states we can derive 6 shockwaves, from $A \rightarrow D$, $D \rightarrow B$, $B \rightarrow A$, $D \rightarrow C$, $B \rightarrow C$ and $C \rightarrow A$. In the figures below, als shockwaves and some trajectories are depicted, and all states are depicted in the fundamental diagram (flow-density) which is a parabola since we assume Greenshields relations hold.

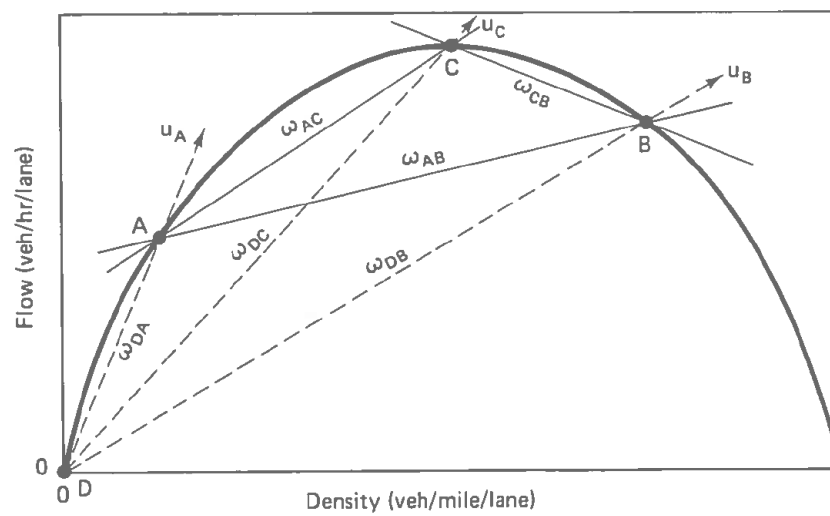


Figure 5: Traffic states in the fundamental diagram