

Part II

Please write your derivations and answers for Part II on separate paper sheets.

Question 5 (20 points)

Consider a series of 10 observed route choices for two different OD pairs. For both OD pairs, 2 routes occur in the observations. These are denoted by r_1 & r_2 .

- The first OD-pair yields: $[r_1, r_2, r_2, r_1, r_1, r_1, r_2, r_1, r_2, r_2]$
- The second OD-pair yields: $[r_1, r_1, r_2, r_1, r_1, r_1, r_1, r_1, r_2, r_1]$

We now consider a route choice model, called the RC-model, in which the model probabilities are the same for all travellers per OD pair.

- For the first OD pair, the model predicts: $p_1 = 0.40, p_2 = 0.60$
- For the second OD pair, the model predicts: $p_1 = 0.90, p_2 = 0.10$

(a) Give the log (with base 10) likelihood of obtaining the observed choices given the RC-model. Do this for both OD pairs.

(b) Give the maximum likelihood, i.e. the likelihood of obtaining these observations given the best model for these observations (yielding the maximum likelihood). Do this for both OD pairs.

(c) The reliability of the RC-model is evaluated for both sets of observed route choices. Explain which model result (OD 1 or OD 2) is most reliable?

The study has been extended and in the end the total number of observed route choices is equal to 100. For OD 1, 15 travellers chose route 1 and 25 travellers chose route 2. For OD 2, 50 travellers chose route 1 and 10 travellers chose route 2

(d) Use the Chi-square goodness of fit to test whether the RC-model should be rejected or not. Use a 95% confidence level.

Question 6 (30 points)

Car travel demand in urban areas depends on weather. The following model was estimated for an urban corridor without public transport alternative:

$$q_{model} = q_0 [1 + a \sqrt{t_p}], \quad \text{with}$$

- q_{model} : the modelled volume
- $q_0 > 0$: the volume under dry conditions
- $t_p \geq 0$: the duration of rain (in hours)
- $a > 0$: proportionality coefficient

(a) Use the first order approximation to express the error in q_{model} due to the error in q_0 as function of the error in q_0 , and the error in q_{model} due to the error in a as function of the error in a .

- (b) If the relative errors in q_0 ($\text{err}(q_0)/q_0$) and a ($\text{err}(a)/a$) are the same, which error has the largest influence on the error in q_{model} ?
- (c) Suppose that (the individual errors in) q_0 and a are uncorrelated. Express the total error in q_{model} as combination of the errors determined in (a)
- (d) Suppose that (the individual errors in) q_0 and a are negatively correlated with a correlation coefficient of -1. Express the total error in q_{model} as combination of the errors determined in (a).
- (e) Is the total error (in q_{model}) in (d) smaller or larger than in (c) or does it depend on the values of q_0 , $\text{err}(q_0)$, a , $\text{err}(a)$ or t_p ? Explain your answer
- (g) Suppose $\text{err}(q_0)/q_0 = 0.05$ (5%), $\text{err}(a)/a = 0.1$ (10%) and $a = 1/3$. For the correlated case (d), calculate the errors in q_{model} (as function of q_0) for different durations of rainfall, i.e., $t_p = 0, 1$ and 9 hours. Explain the differences in the obtained errors.
- (f) Suppose $\text{err}(q_0)/q_0 = \text{err}(a)/a = x$ ($100 \cdot x\%$). For the correlated case (d), calculate (or express) the error in q_{model} (if necessary as function of q_0 , a , x and / or t_p). Do the errors in q_0 and a strengthen each other, weaken each other or do they not influence each other? Explain your answer.

Chi Square Distribution Table							
d.f.	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$	$\chi^2_{.001}$
1	1.32	2.71	3.84	5.02	6.63	7.88	10.8
2	2.77	4.61	5.99	7.38	9.21	10.6	13.8
3	4.11	6.25	7.81	9.35	11.3	12.8	16.3
4	5.39	7.78	9.49	11.1	13.3	14.9	18.5
5	6.63	9.24	11.1	12.8	15.1	16.7	20.5
6	7.84	10.6	12.6	14.4	16.8	18.5	22.5
7	9.04	12.0	14.1	16.0	18.5	20.3	24.3
8	10.2	13.4	15.5	17.5	20.1	22.0	26.1
9	11.4	14.7	16.9	19.0	21.7	23.6	27.9
10	12.5	16.0	18.3	20.5	23.2	25.2	29.6
11	13.7	17.3	19.7	21.9	24.7	26.8	31.3
12	14.8	18.5	21.0	23.3	26.2	28.3	32.9
13	16.0	19.8	22.4	24.7	27.7	29.8	34.5
14	17.1	21.1	23.7	26.1	29.1	31.3	36.1
15	18.2	22.3	25.0	27.5	30.6	32.8	37.7
16	19.4	23.5	26.3	28.8	32.0	34.3	39.3
17	20.5	24.8	27.6	30.2	33.4	35.7	40.8
18	21.6	26.0	28.9	31.5	34.8	37.2	42.3
19	22.7	27.2	30.1	32.9	36.2	38.6	43.8
20	23.8	28.4	31.4	34.2	37.6	40.0	45.3
21	24.9	29.6	32.7	35.5	38.9	41.4	46.8
22	26.0	30.8	33.9	36.8	40.3	42.8	48.3
23	27.1	32.0	35.2	38.1	41.6	44.2	49.7
24	28.2	33.2	36.4	39.4	42.8	45.6	51.2
25	29.3	34.4	37.7	40.6	44.3	46.9	52.6
26	30.4	35.6	38.9	41.9	45.6	48.3	54.1
27	31.5	36.7	40.1	43.2	47.0	49.6	55.5
28	32.6	37.9	41.3	44.5	48.3	51.0	56.9
29	33.7	39.1	42.6	45.7	49.6	52.3	58.3
30	34.8	40.3	43.8	47.0	50.9	53.7	59.7
40	45.6	51.8	55.8	59.3	63.7	66.8	73.4
50	56.3	63.2	67.5	71.4	76.2	79.5	86.7
60	67.0	74.4	79.1	83.3	88.4	92.0	99.6
70	77.6	85.5	90.5	95.0	100	104	112
80	88.1	96.6	102	107	112	116	125
90	98.6	108	113	118	124	128	137
100	109	118	124	130	136	140	149
Table from Ronald J. Wonnacott and Thomas H. Wonnacott, <i>Statistics: Discovering Its Power</i> , New York: John Wiley and Sons, 1982, p.352.							