

LINEAR ALGEBRA_A Date : April 03, 2023
Time : 08.45 – 10.45 hrs

First read these instructions carefully:

This test contains 10 exercises. The complete solutions of Exercises 3, 4 (only c), 7, 8, 9, and 10 must be accurately written down including calculations and argumentation on the answer sheet at the end of this test. For the other exercises you are only required to fill in the final answers on the answer sheet at the end of this test. You must hand in the answer sheet. The use of dictionaries or electronic devices is not allowed. The grade is determined by dividing the total number of points by 4 and adding 1.

1. (2pts.) *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

The linear system of equations is given below:

$$x_1 + 2x_2 + 4x_3 = -3, \quad 2x_1 + 4x_2 + 8x_3 = -6, \quad -x_1 - x_2 + 5x_3 = 5.$$

Which of the below is an **augmented matrix** corresponding to the linear systems above?

(A) $\left(\begin{array}{ccc|c} 1 & 0 & -14 & 7 \\ 0 & 1 & 9 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$

(C) $\left(\begin{array}{ccc|c} 1 & 2 & 4 & \\ 2 & 4 & 8 & \\ -1 & -2 & 5 & \end{array} \right)$

(B) $\left(\begin{array}{ccc|c} 1 & 2 & 4 & -3 \\ -1 & -1 & 5 & 5 \\ 2 & 4 & 8 & -6 \end{array} \right)$

(D) $\left(\begin{array}{ccc|c} 1 & 2 & 4 & \\ 0 & 1 & 9 & \\ 0 & 0 & 0 & \end{array} \right)$

2. (2pts.) *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

Which of the following matrices are in reduced echelon form? (There might be more than one correct answer.)

(A) $\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right)$

(C) $\left(\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$

(E) $\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right)$

(G) $\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{array} \right)$

(B) $\left(\begin{array}{ccc} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{array} \right)$

(D) $\left(\begin{array}{ccc} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right)$

(F) $\left(\begin{array}{ccc} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$

(H) $\left(\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} \right)$

3. (3pts.) *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

Find all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ for which $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \mathbf{y}$.

4. Consider the coefficient matrix $A \in \mathbb{R}^{3 \times 4}$ and the reduced echelon form of the augmented matrix

$$R \text{ to the system } A\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}.$$

$$A = \begin{pmatrix} 1 & -1 & 1 & -3 \\ 2 & -1 & 3 & -6 \\ 1 & 2 & 5 & -4 \end{pmatrix}, \quad R = \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & -4 \\ 0 & 0 & 1 & -1 & 3 \end{array} \right).$$

- a) (2pts.) *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

Give the solution set of the system $A\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$ in parametric vector form.

- b) (1pt.) *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

Find the null space of A (Null A).

- c) (3pts.) *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

Determine a basis that includes the vector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ for the column space Col A .

5. *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

- a) (3pts.) For which $k \in \mathbb{R}$ is the matrix below invertible?

$$\begin{pmatrix} 1 & 0 & -2 \\ 3 & k & -6 \\ 1 & -3 & k-4 \end{pmatrix}$$

- b) (2pts.) Let A and B be 3×3 matrices where $\det A = -2$ and $\det B = 3$. Compute the following:

i) $\det B^T A$

ii) $\det B^{-1}$

iii) $\det A^3$

iv) $\det 5A$

6. (2pts.) *This is a final answer question. Provide your final answer (and only your final answer) on the supplied space on the answer sheet.*

Find the volume of the parallelepiped in $\mathbb{R}^3$ which has corners: $\mathbf{0}, \mathbf{u}, \mathbf{v}, \mathbf{w}$ given as:

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} -3 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 2 \\ -7 \\ 3 \end{pmatrix}.$$

7. (3pts.) *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

The matrix A is given by:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Show that if $ad - bc \neq 0$ then

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

8. (3pts.) *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

Let a matrix A has eigenvalues: $-1, 0$, and 1 . Determine all of the eigenvalues of $5I + 3A$.

9. (4pts.) *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

The 3×3 matrix N is given:

$$N = \begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{pmatrix}$$

Determine the matrices P and D , such that $N = PDP^{-1}$.

10. *Use the supplied space on the answer sheet and include clear argumentation and calculation. All your work will be graded.*

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that:

$$T \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}, \quad T \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}.$$

- a) (3pts.) Find a general formula for $T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.
- b) (3pts.) Determine whether T is surjective (onto) and/or injective (one-to-one).

Do not remove the staple!

Linear Algebra_A April 03, 2023

Answer sheet Answer the questions legibly in the spaces provided. If you run out of room for an answer, continue on the back of the answer sheet by clearly referring to the question.

Name and Student Number: _____

Programme: _____

Question	1	2	3	4	5	6	7	8	9	10	Total
Points Possible	2	2	3	2+1+3	3+2	2	3	3	4	3+3	36
Points Earned											

1. Which of the options is the augmented matrix of the given system?

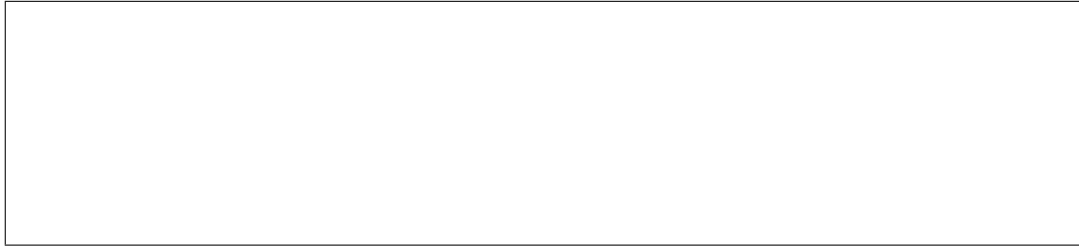
☐ (A) ☐ (C)
☐ (B) ☐ (D)

2. Add a check mark (✓) for each option in the reduced echelon form.

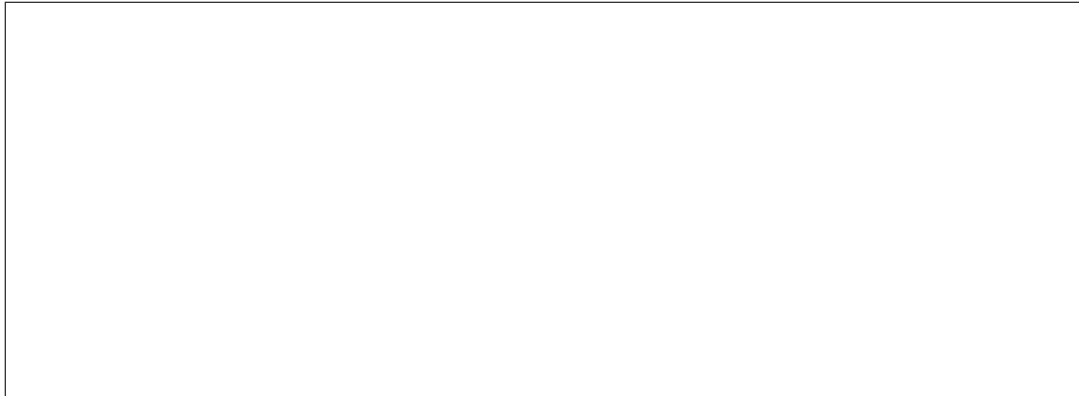
☐ (A) ☐ (C) ☐ (E) ☐ (G)
☐ (B) ☐ (D) ☐ (F) ☐ (H)

3. Show all your work.

4. a) Only provide your final answer.

A large, empty rectangular box with a thin black border, intended for the student to write their final answer to question 4a.

- b) Only provide your final answer.

A large, empty rectangular box with a thin black border, intended for the student to write their final answer to question 4b.

- c) Show all your work.

A large, empty rectangular box with a thin black border, intended for the student to show all their work for question 4c.

A

5. a) Only provide your final answer.

- b) Only provide your final answer.

i) $\det B^T A$

ii) $\det B^{-1}$

iii) $\det A^3$

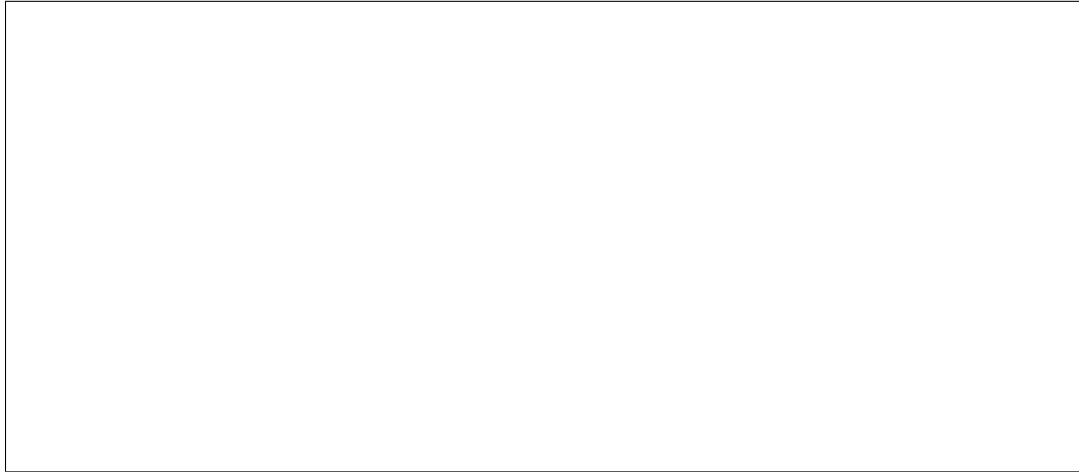
iv) $\det 5A$

6. Only provide your final answer.

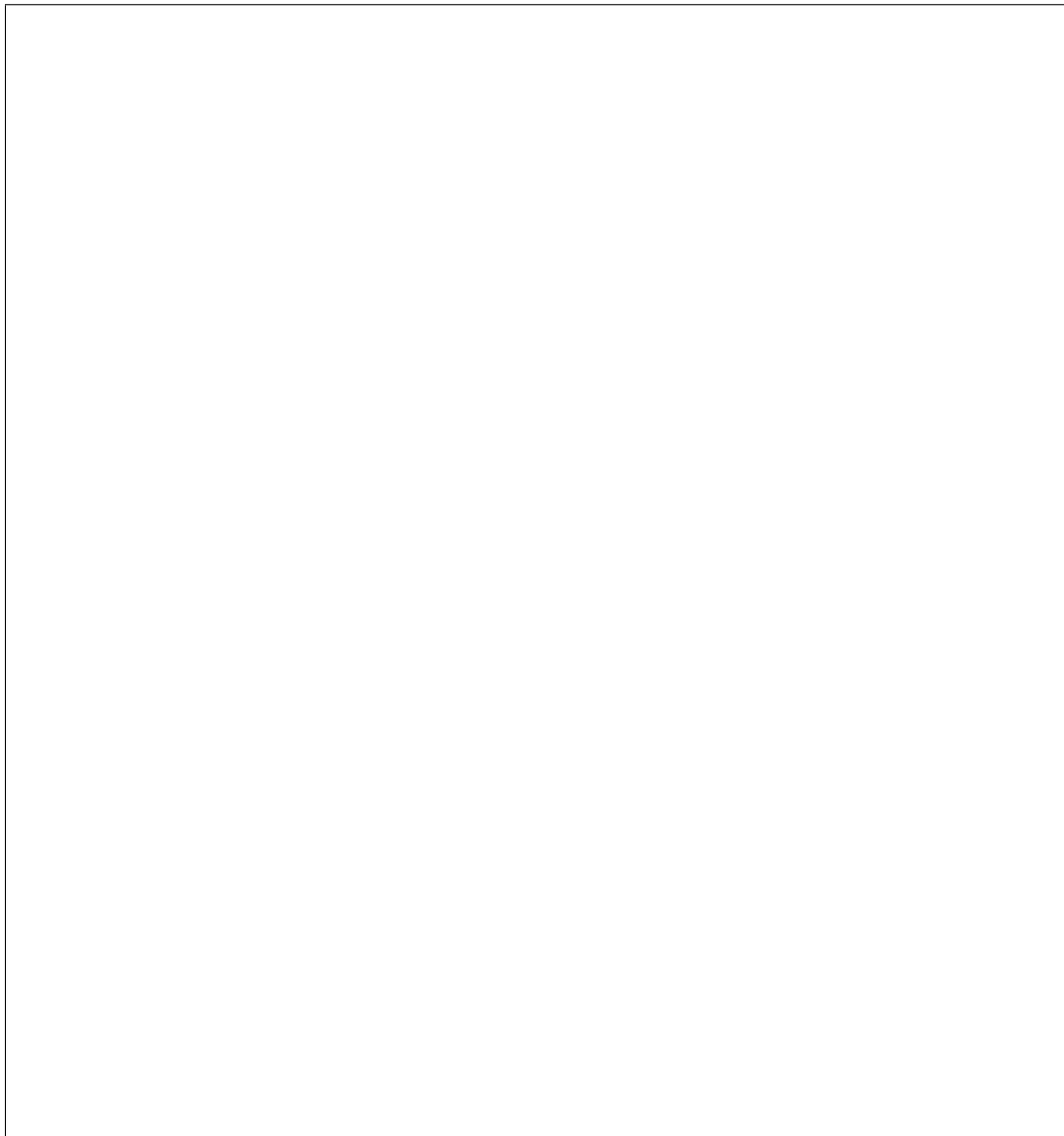
7. Show all your work.

A

8. Show all your work.

A large, empty rectangular box with a thin black border, intended for the student to show their work for problem 8.

9. Show all your work.

A large, empty rectangular box with a thin black border, intended for the student to show their work for problem 9.

10. a) Show all your work.

- b) Show all your work.

Extra writing space (Include the question number for each answer!)