

# Exam: Vector Calculus for BMT, CE, CSE, ME

October 7th, 2022 , 13.45 - 15.45 hrs

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Give a suitable explanation of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

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1. An object of mass  $m$  slides down a wire in the shape of the curve

$$x = \sin(t), \quad y = \cos(t), \quad z = -t, \quad 0 \leq t \leq 6\pi.$$

Compute the work done by the gravitational force  $\mathbf{F} = -mg\mathbf{k}$  on the object during its descent.

*Remark:* As usual,  $g$  denotes the gravitational constant.

2. In the following you should compute  $\int_C \mathbf{G} \bullet d\mathbf{r}$  for

$$\mathbf{G} = (1+x)e^{x+y}\mathbf{i} + (xe^{x+y} + 2z)\mathbf{j} - 2y\mathbf{k},$$

and the curve  $C$  with parametrization

$$\mathbf{r}(t) = (1-t)e^t\mathbf{i} + t\mathbf{j} + 2t\mathbf{k}, \quad 0 \leq t \leq 1.$$

A *direct* computation is, however, *too difficult*. Instead, proceed along the following steps:

- (a) Find a potential for  $\mathbf{F} = (1+x)e^{x+y}\mathbf{i} + (xe^{x+y} + 2y)\mathbf{j} - 2z\mathbf{k}$ .
  - (b) Compute  $\mathbf{H} = \mathbf{G} - \mathbf{F}$  explicitly, and compute  $\int_C \mathbf{H} \bullet d\mathbf{r}$  using the given parametrization of  $C$ .
  - (c) Use (a) and (b) to compute  $\int_C \mathbf{G} \bullet d\mathbf{r}$ .
3. Let the curve  $C$  be the intersection of the cylinder  $x^2 + y^2 = 1$  and the plane  $2x + 2y + z = 3$ , oriented counter-clockwise when viewed from above, that is from the positive  $z$ -axis.
- (a) Make a clear sketch of the curve  $C$  and its orientation.
  - (b) Calculate the circulation of  $\mathbf{F}(x, y, z) = -y^3\mathbf{i} + x^3\mathbf{j} - z^3\mathbf{k}$  around  $C$  using Stokes' theorem.

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**Points:**

|           |   |              |     |              |   |
|-----------|---|--------------|-----|--------------|---|
| <b>1.</b> | 3 | <b>2.(a)</b> | 3   | <b>3.(a)</b> | 1 |
|           |   | (b)          | 1.5 | (b)          | 7 |
|           |   | (c)          | 2.5 |              |   |

**Total:**  $18 + 2 = 20$  points