

Question

Five points per question

Consider the BPR function to estimate travel time TT , given an intensity-capacity ratio q/C with q the intensity (per hour) and C the capacity (per hour):

$$TT = TT_{ff} * [1 + \alpha * (q/C)^\beta]$$

With TT_{ff} the free-flow travel time, and α and β two coefficients of the BPR function.

The delay is given by: $D = TT - TT_{ff}$

a) Give an approximation of the error in D , $err(D)$, due to the error in TT_{ff} , $err(TT_{ff})$. Do the same for the error in D due to the error in q , $err(q)$, due to the error in C , $err(C)$, due to the error in α , $err(\alpha)$, and due to the error in β , $err(\beta)$.

For the following questions, $\alpha = 0.5$, $\beta = 2$ and $TT_{ff} = 30$ (min), and $err(TT_{ff}) = 0$ and $err(q) = 0$.

b) The **relative** errors in C , α , and β , i.e., $err(C) / C$, $err(\alpha) / \alpha$, and $err(\beta) / \beta$ respectively, are 0.1 (10%). Use the answers from a) to derive the relative error in D , i.e., $err(D)/D$, due to the (relative) error in C , due to the (relative) error in α , and due to the (relative) error in β . Is the relative error in D larger than that in the input variables? Which (relative) error has the largest effect on D ?

Consider two routes from A to B. For both routes, the travel times were measured during 5 periods, which were the same for both routes. For route 1 and 2, the travel times are given by TT_1 and TT_2 respectively:

- $TT_1 = [34.26, 41.86, 38.22, 44.61, 32.18]$
- $TT_2 = [34.46, 37.09, 34.43, 35.56, 30.91]$

c) Are the mean travel times $mean(TT_1)$ and $mean(TT_2)$ significantly different? Use a 2-sigma confidence level. Explain all steps.

The free flow Travel Time is 30 minutes for both routes $TT_{ff} = 30$ (min), $\alpha = 0.5$ and $\beta = 2$. The capacities are: $C_1 = 4000$ for the first route and $C_2 = 3500$ for the second route. The respective intensities in the 5 periods are:

- $q_1 = [1900, 3120, 2520, 3410, 1750]$
- $q_2 = [1700, 2110, 1620, 1840, 990]$

The travel time difference is given by: $\Delta TT = TT_1 - TT_2 = D_1 - D_2$. Note that the free-flow travel times are equal.

d) For all 5 periods, derive the observed delays for both routes and the corresponding travel time differences. Also derive the theoretical delays and corresponding travel time differences according to the BPR function.

Suppose now that $\text{err}(\alpha) = 0$ and $\text{err}(\beta) = 0$. The relative errors in the capacity are 10%, i.e., $\text{err}(C_1)/C_1 = \text{err}(C_2)/C_2 = 0.1$. In addition, both errors are positively correlated with correlation coefficient equal to 1.

e) Give the expression for the error in the travel time difference. Derive the error in the travel time difference for the 5 periods.

f) Perform a Chi-square goodness of fit test for the BPR function. Should the BPR function be rejected? Take a 99% confidence interval.

We have two route choice models: model 1 and 2.

Model 1 is described by:

$$\begin{aligned} P_1 &= 0.5 - 0.1 (TT_1 - TT_2) && \text{for } -5 < TT_1 - TT_2 < 5 \\ P_1 &= 0 && \text{for } TT_1 - TT_2 \geq 5 \\ P_1 &= 1 && \text{for } TT_1 - TT_2 \leq -5 \\ P_2 &= 1 - P_1 \end{aligned}$$

Model 2 is described by:

$$\begin{aligned} P_1 &= 0.5 - 0.1 (TT_1 - TT_2) && \text{for } -4 < TT_1 - TT_2 < 4 \\ P_1 &= 0.1 && \text{for } TT_1 - TT_2 \geq 4 \\ P_1 &= 0.9 && \text{for } TT_1 - TT_2 \leq -4 \\ P_2 &= 1 - P_1 \end{aligned}$$

For the last 2 periods, we observe the route choice. For each period, we have 100 observations. We observe the following fractions that took route 1:
Fraction over route 1 = [0, 0.40]

g) Give the ln likelihoods for both models. Which model is better?

h) Suppose that one person took route 1 during the fourth period (instead of zero persons). What is the likelihood for model 1 in this case? Which model is better in that case?