

Course : Introduction to Mathematics & Calculus 1A
 Module : 1A
 Course code : 2020011[87-93]

Date : October 24, 2025
 Time : 13:45 – 16:45
 Reference : Test 1

Introduction to Mathematics & Calculus 1A

Instructions

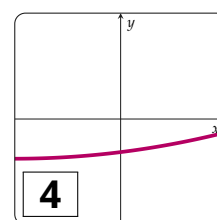
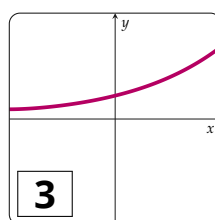
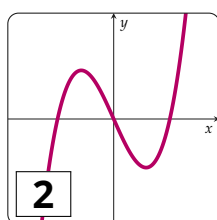
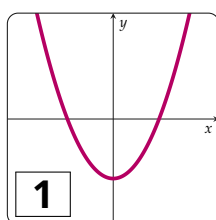
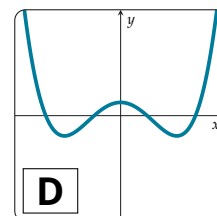
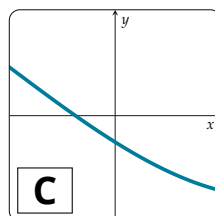
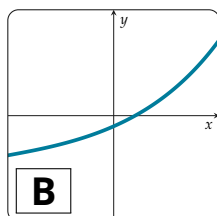
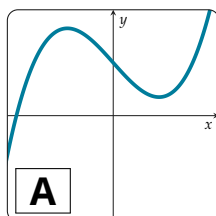
- For questions 1–5, you are only required to fill in the final answer on the answer form.
 - For questions 6–13, you are required to write down a full calculation and argumentation.
- Please hand in your answer form only. If you run out of space, you can use the extra space at the end of the answer form. Refer clearly to that space in the original answer.
- Do not write with red pen or pencil. Do not use correction fluid or tape.

The use of electronic devices is not allowed!

Final answer questions

Report all your answers on the answer form.

- Let $A = \{2 - \frac{4}{n} \mid n \in \mathbb{N}\}$. [2 pt]
 For each of the following: if it exists, determine its value, otherwise write 'does not exist'.
 (a) $\inf(A)$ (b) $\min(A)$
 (c) $\max(A)$ (d) $\sup(A)$
- For each of the following statements, determine whether or not it is true for all sets A, B . [2 pt]
 (a) $(A = B) \rightarrow (\forall x \in A(x \in B))$
 (b) $(\forall x \in A(x \in B)) \rightarrow (A = B)$
 (c) $(A = B) \rightarrow (\exists x(x \in A \wedge x \in B))$
 (d) $(\exists x(x \in A \wedge x \in B)) \rightarrow (A = B)$
- Identify all horizontal and vertical asymptotes of the function $f(x) = \frac{7x^2 - 3}{3x^2 - 4x + 1}$. [2 pt]
- Consider the points $P(1, 1, 1)$, $Q(2, 3, 1)$, and $R(1, 2, -2)$.
 (a) Find the equation of the plane passing through P , Q and R . [2 pt]
 (b) Compute the area of the triangle whose vertices are P , Q and R . [1 pt]
 (c) Compute the cosine of the (smallest) angle between the vectors $\vec{PQ}$ and $\vec{PR}$. [1 pt]
- Match each of the functions A—D in the first row to its derivative 1—4 in the second row. [2 pt]



Open questions

The full solutions to questions 6–13 must be clearly written down on the answer form, including calculations and argumentations. Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

6. Prove the following statement:

[3 pt]

For all $a, b \in \mathbb{Z}$: If $ab + a + 1$ is odd, then: (a is even or b is odd).

7. For each integer $n \geq 0$, the *Fibonacci number* F_n is recursively defined by:

[4 pt]

1. $F_0 = 0, F_1 = 1,$

2. for each $n \geq 2$: $F_n = F_{n-1} + F_{n-2}.$

Prove the following statement with mathematical induction:

$$\text{For all non-negative integers } n \geq 0 : \sum_{i=0}^n F_{2i} = F_{2n+1} - 1.$$

8. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many subsets S of A satisfy:

[3 pt]

(a) $|S| = 4$ and the smallest element in S is 3. Express your answer as a number.

(b) $|S| = 4$ and the smallest element in S is even. Express your answer as a number.

9. Consider the function $f(x) = \frac{1}{x^2 + x + 1}.$

[3 pt]

Sketch the graph of f .

Clearly label local extreme value(s) and the point(s) at which they are attained, as well as point(s) of inflection; indicate where the function is increasing/decreasing and where it is concave up/concave down; find all points at which the graph of the function intersects the x -axis or the y -axis; find the horizontal and vertical asymptotes of the function (if they exist).

10. Compute the following limit:

[2 pt]

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x + 3} - 2}.$$

11. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \sin(x) e^{-\cos^2(x)}.$

(a) Show that $x = \frac{\pi}{2}$ is the only critical point of f in the interval $\left[\frac{\pi}{4}, \pi\right].$

[2 pt]

(b) Find the absolute maximum and minimum values attained by f in the interval $\left[\frac{\pi}{4}, \pi\right].$

[1 pt]

12. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = -x^3 + 2x$. It is known that the line $y = \frac{5}{4}x - \frac{1}{4}$ is tangent to the graph of f at a certain point $(a, f(a)).$

[3 pt]

Find a .

13. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \ln(x^2 - 4x + 5).$

[3 pt]

Compute the Taylor polynomial of order 2 of f at the point $x = 2$.

Total: 36 pt

1	2	3	4	5	6	7	8	9	10	11	12	13	Σ

Do not fill in this table.

A

Only for sorting.

Answer form

Introduction to Mathematics & Calculus 1A – Test – October 24, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. No points are awarded for fewer than 3 correct answers. Fill in the correct answers:

(a) $\inf(A) =$		(b) $\min(A) =$	
(c) $\max(A) =$		(d) $\sup(A) =$	

Question 2. No points are awarded for fewer than 3 correct answers. Fill in the correct answers:

(a) ☐ True	☐ False
(b) ☐ True	☐ False
(c) ☐ True	☐ False
(d) ☐ True	☐ False

Question 3. Write only your final answer in each box.

horizontal asymptote(s):	
vertical asymptote(s):	

Question 4. Write only your final answer in each box.

(a)	
(b)	
(c)	

Question 5. Write your answers 1–4 in the corresponding spaces.

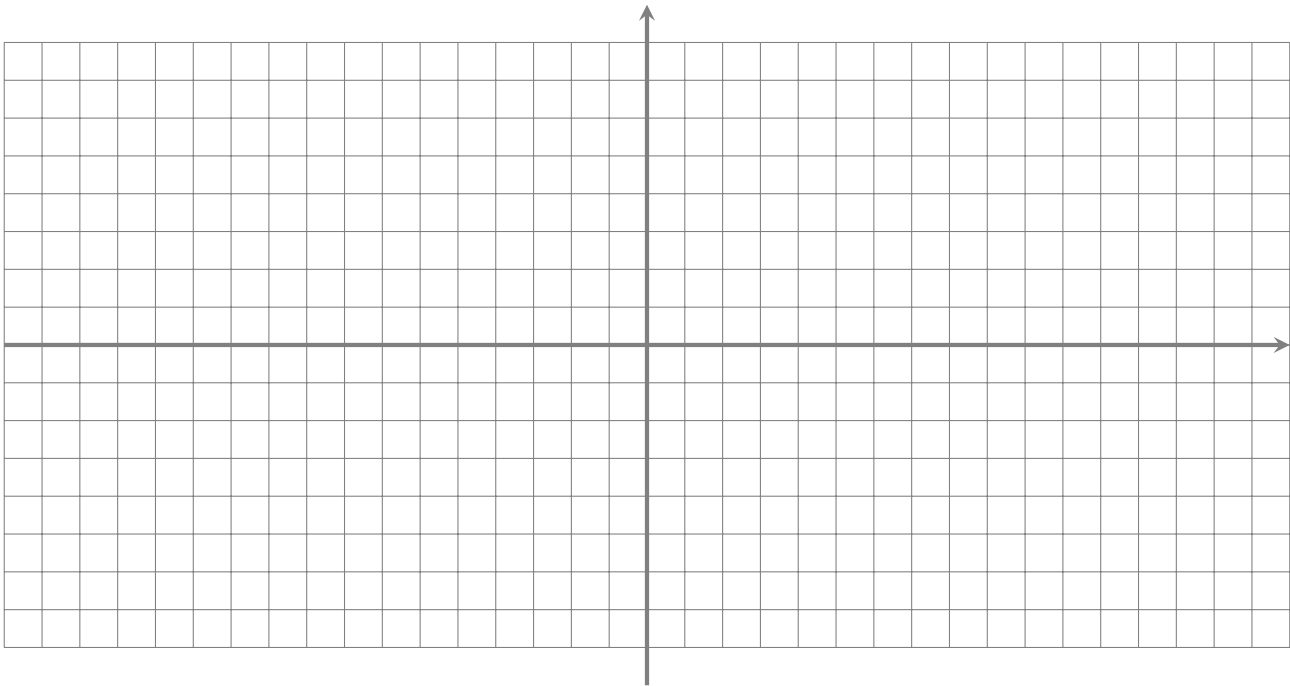
☐ A		☐ B		☐ C		☐ D	
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[illegible]

[illegible]

[illegible][illegible]

Question 9. Sketch the graph of f and include all the information that is asked for.



Provide a full calculation/argumentation for your sketch.

Question 10. Write a full calculation/argumentation clearly in the box.

Question 11a. Write a full calculation/argumentation clearly in the box.

Question 11b. Write a full calculation/argumentation clearly in the box.

[illegible]

[illegible]

[illegible]

[illegible]