

Exam: Vector Calculus for BMT, CE, CSE, ME

October 6th, 2023 , 13.45 - 15.45 hrs

Give a suitable explanation of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

1. Let C be the curve given by the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = x$ in the first octant. Compute the mass of a wire with mass density $\delta(x, y, z) = xy$ occupied by C .
2. Let the surface S be the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $C: x^2 + y^2 = 2y$.

(a) Sketch S .

(b) Compute the area of S .

Hint: You may construct a parametrization of S by first parametrizing the points inside the cylinder C that lie in the plane $z = 0$. Observe that $x^2 + y^2 = 2y$ is equivalent to $x^2 + (y - 1)^2 = 1$.

3. Let a, b, c be real numbers and

$$\mathbf{F} = (axy + 3yz)\mathbf{i} + (x^2 + 3xz + by^2z)\mathbf{j} + (bxy + cy^3)\mathbf{k}.$$

(a) Determine all values of a, b, c for which you can evaluate the integral $\int_C \mathbf{F} \cdot \mathbf{T} ds$ along a curve C from $A = (0, 1, -1)$ to $B = (2, 1, 1)$ without knowing exactly which curve.

(b) Determine the value of $\int_C \mathbf{F} \cdot \mathbf{T} ds$ for $a = 2, b = 3$ and $c = 1$.

4. Let D be the volume inside the sphere $x^2 + y^2 + z^2 = 4$ and outside the cylinder $x^2 + y^2 \geq 1$. Let S_1 denote the cylindrical part of the boundary of D and let S_2 denote the spherical part. Let

$$\mathbf{F} = (x + yz)\mathbf{i} + (y - xz)\mathbf{j} + (z - e^x \sin(y))\mathbf{k}.$$

(a) Sketch the boundary of D .

(b) Evaluate the flux of $\mathbf{F}$ out of D through the boundary of D using an integration theorem (Green, Gauss, Stokes).

(c) Evaluate the flux of $\mathbf{F}$ out of D through the boundary of S_1 using an integration theorem (Green, Gauss, Stokes).

(d) Evaluate the flux of $\mathbf{F}$ out of D through the boundary of S_2 .

Hint: You can use (without verification) that the volume of the sphere $x^2 + y^2 + z^2 \leq 4$ is $32\pi/3$, and that the volume of D is $4\sqrt{3}\pi$.

Points:

1.	3	2.(a)	0.5	3.(a)	1.5	4.(a)	0.5
		(b)	4.5	(b)	3	(b)	2.5
						(c)	1.5
						(d)	1

Total: $18 + 2 = 20$ points