

Part III

Question 8

a) $TT = TT_{ff} * [1 + \alpha * (q/C)^\beta]$ and $D = TT - TT_{ff} \rightarrow D = TT_{ff} * \alpha * (q/C)^\beta$

$$err_x(D) = \partial D / \partial x * err(x).$$

$$\rightarrow err_{TT_{ff}}(D) = \alpha * (q/C)^\beta * err(TT_{ff}) = D * err(TT_{ff}) / TT_{ff}$$

$$\rightarrow err_q(D) = \beta * TT_{ff} * \alpha * q^{\beta-1} / C^\beta * err(q) = \beta * D * err(q) / q$$

$$\rightarrow err_C(D) = -\beta * TT_{ff} * \alpha * q^\beta / C^{\beta+1} * err(C) = -\beta * D * err(C) / C$$

$$\rightarrow err_\alpha(D) = TT_{ff} * (q/C)^\beta * err(\alpha) = D * err(\alpha) / \alpha$$

$$\rightarrow err_\beta(D) = TT_{ff} * \alpha * (q/C)^\beta * \ln(q/C) * err(\beta) = \beta * \ln(q/C) * D * err(\beta) / \beta$$

b) The relative errors in C, α and β are equal and they are 10%. When you compare magnitudes of errors, you are always comparing positive numbers:

$$\rightarrow err_C(D)/D = |-\beta| * err(C) / C = 2 * err(C) / C = 20\% \rightarrow \text{the relative error increased}$$

$$\rightarrow err_\alpha(D)/D = err(\alpha) / \alpha = 10\% \rightarrow \text{the relative error remains the same}$$

$$\rightarrow err_\beta(D)/D = |\beta * \ln(q/C)| * err(\beta) / \beta: \text{the magnitude depends on } q/C. \text{ For small } q, \text{ the error increases relatively much. For large } q, \text{ the relative increase is smaller.}$$

c) The 5 minute periods were the same $\rightarrow$ the time-series are paired:

$$z = TT_1 - TT_2 \rightarrow \text{is } mean(z) \text{ significantly different from 0?}$$

$$mean(z) = \sum_{i:1..5} z_i / 5 = 3.74$$

$$stdev(z) = \sqrt{\sum_{i:1..5} (z_i - mean(z))^2 / 4} = 3.6$$

$$stdev(mean(z)) = stdev(z) / \sqrt{5} = 1.60$$

$$mean(z) - 2 * stdev(mean(z)) = 0.54, mean(z) + 2 * stdev(mean(z)) = 6.94$$

$$\rightarrow 0 \text{ is not in the 2-sigma interval} \rightarrow mean(z) \text{ is significantly different from } 0.$$

d) Observed:

$$D_1 = [4.26, 11.86, 8.22, 14.61, 2.18], D_2 = [4.46, 7.09, 4.43, 5.56, 0.91]$$

$$\Delta TT^{obs} = D_1 - D_2 = [-0.20, 4.77, 3.79, 9.05, 1.27]$$

Theoretical delays:

$$D_1 = 30 * 0.5 * (q_1/4000)^2 = [3.38, 9.13, 5.96, 10.90, 2.87]$$

$$D_2 = 30 * 0.5 * (q_2/3500)^2 = [3.54, 5.45, 3.21, 4.15, 1.20]$$

$$\Delta TT^{BPR} = D_1 - D_2 = [-0.15, 3.67, 2.74, 6.76, 1.67]$$

e) $err_C(D) = -2 * D * err(C) / C$, $err(C_1) / C_1 = err(C_2) / C_2 = 0.1$, and errors in C_1 and C_2 are positively correlated with correlation coefficient = 1

$$\rightarrow err(\Delta TT^{BPR}) = err(D_1 - D_2) = err_C(D_1 - D_2) = -2 * (D_1 - D_2) * 0.1$$

$$err(\Delta TT^{BPR}) = [0.03, -0.73, -0.55, -1.35, -0.33]$$

If you only want to indicate the magnitude of the error (as is also done with $err(C)$), it is more convenient to take the absolute values (see also question b)

$$\rightarrow err(\Delta TT^{BPR}) = [0.03, 0.73, 0.55, 1.35, 0.33]$$

$$f) \Delta TT^{obs} - \Delta TT^{BPR} = [-0.04, 1.10, 1.04, 2.30, -0.40]$$

→ $\chi^2 = 11.88$, degrees of freedom = 5

For 99% confidence level the BPR function is not rejected.

g) $\Delta TT^{BPR} = TT_1 - TT_2 = [6.76, 1.67]$ for the last two periods

model 1: $p_1 = [0, 0.333]$, $p_2 = [1, 0.677]$

ln likelihood model 1: $\ln L = 0 \cdot \ln(0) + 100 \cdot \ln(1) + 40 \cdot \ln(0.333) + 60 \cdot \ln(0.677) = -67.38$

model 2: $p_1 = [0.1, 0.333]$, $p_2 = [0.9, 0.677]$

ln likelihood model 1: $\ln L = 0 \cdot \ln(0.1) + 100 \cdot \ln(0.9) + 40 \cdot \ln(0.333) + 60 \cdot \ln(0.677) = -77.93$

→ likelihood is larger for model 1 → model 1 is better

h) ln likelihood model 1: $\ln L = 1 \cdot \ln(0) + 100 \cdot \ln(1) + 40 \cdot \ln(0.333) + 60 \cdot \ln(0.677) = -\infty$,

because $1 \cdot \ln(0) = -\infty$

In this case model 2 is better. Model 1 is very bad, because it predicted with certainty that a certain event could not happen. But it did.