

Tag : Toetsen/21-22/Calc1A.21-22[01].CorrectionModel
 Course : **Calculus 1A**
 Date : Friday October 29th, 2021
 Time : 13:45 – 15:45

Solutions

1. *This is a final-answer question.*

(a) [1 pt]

Versions	$\mathbf{u} \times \mathbf{v}$
V1	$\langle 2, 1, 0 \rangle$
V2	$\langle 2, 0, -2 \rangle$
V3	$\langle -2, 1, 0 \rangle$
V4	$\langle 6, -8, -5 \rangle$

(b) [2 pt]

Versions	$\text{proj}_{\mathbf{v}} \mathbf{u}$
V1	$\langle \frac{2}{9}, -\frac{4}{9}, \frac{4}{9} \rangle$
V2	$\langle 0, 1, 0 \rangle$
V3	$\langle 0, 0, -2 \rangle$
V4	$\langle \frac{8}{5}, \frac{6}{5}, 0 \rangle$

2. *This is a final-answer question.*

(a) [2 pt] Note that the below parametric equations are calculated for the point P and the direction vector $\overrightarrow{PQ}$. Other solutions are also possible.

Versions	V1	V2	V3	V4
Par. eqns. for l_1	$\begin{cases} x = 1 + 2t, \\ y = -1 + 3t, \\ z = 4 - t \end{cases}$	$\begin{cases} x = 1 - 2t, \\ y = 1 - t, \\ z = 2 - 4t \end{cases}$	$\begin{cases} x = 2 + t, \\ y = -1 + t, \\ z = 1 - 2t \end{cases}$	$\begin{cases} x = 4 - t, \\ y = 2 - 2t, \\ z = -1 + 3t \end{cases}$

(b) [2 pt]

Versions	A simplified component equation for the plane
V1	$x + 2y + z = 3$
V2	$x + 2y + 2z = 7$
V3	$2x - 3y + z = 8$
V4	$x - 2y - z = 1$

3. Define the function $f: \mathbb{R} \rightarrow \mathbb{R}$ as follows:

$$f(x) = \sqrt{x^2 \cos x + 3x^2}$$

(a) [3 pt] Note that $f(x) = |x|\sqrt{\cos x + 3}$.

The piecewise $f(x)$ function is also accepted.

$$\lim_{x \rightarrow 0^+} \frac{f(x)}{x} = \lim_{x \rightarrow 0^+} \frac{|x|}{x} \sqrt{\cos x + 3}$$

and

$$\lim_{x \rightarrow 0^-} \frac{f(x)}{x} = \lim_{x \rightarrow 0^-} \frac{|x|}{x} \sqrt{\cos x + 3}$$

Since $\frac{|x|}{x} = +1$ for $x > 0$, and $\frac{|x|}{x} = -1$ for $x < 0$ we have

$$\lim_{x \rightarrow 0^+} \frac{|x|}{x} \sqrt{\cos x + 3} = \boxed{+2}$$

and

$$\lim_{x \rightarrow 0^-} \frac{|x|}{x} \sqrt{\cos x + 3} = \boxed{-2}$$

Left- and right limit are not the same, so $\lim_{x \rightarrow 0} \frac{f(x)}{x}$ does not exist

(b) [2 pt] The derivative of f at 0 is

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{f(h)}{h},$$

which does not exist according to assignment (a).

4. Define the function $f: \mathbb{R} \rightarrow \mathbb{R}$ as follows:

$$f(x) = x^4 - 2x^2$$

- (a) [2 pt] Critical points:

- The function f is polynomial, so the only critical points are values x for which $f'(x) = 0$:

$$f'(x) = 4x^3 - 4x$$

So $f'(x) = 0$ whenever $4x(x^2 - 1) = 0$, which is the case if $x = -1$, $x = 0$ and $x = 1$.

- (b) [2 pt] Check the function values of the critical values on $[-1, 2]$ and the boundary points:

x	$f(x)$
-1	-1
0	0
1	-1
2	8

The minimum value is -1 (attained at -1 as well as at 1), and the maximum value is 8 (attained at 2).

5. [3 pt] If polar coordinates are used, the calculation should look like this:

$$\begin{aligned} \lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + 2xy^2}{\sqrt{x^2 + y^2}} &= \lim_{r \rightarrow 0^+} \frac{r^3 \cos^3 \theta + 2r^3 \cos \theta \sin^2 \theta}{r} \\ &= \lim_{r \rightarrow 0^+} r^2 (\cos^3 \theta + 2 \cos \theta \sin^2 \theta), \end{aligned}$$

which is in the form of $\lim_{r \rightarrow 0} f(r, \theta)$. Here $f(r, \theta) = r^2 g(\theta)$, where $g(\theta) = \cos^3 \theta + 2 \cos \theta \sin^2 \theta$. It can be shown that $g(\theta)$ is bounded, that is $-3 \leq \cos^3 \theta + 2 \cos \theta \sin^2 \theta \leq 3$. Then,

$$\lim_{r \rightarrow 0^+} r^2 (\cos^3 \theta + 2 \cos \theta \sin^2 \theta) = 0$$

We can also acknowledge the boundedness of $g(\theta)$ by expressing the limit above using the Sandwich Theorem. Finally, the limit exists and it is zero.

6. *This is a final-answer question.*

- (a) [2 pt]

$$(a, b, c) = (-4, 3, 5)$$

- (b) [1 pt] Do not penalize if the answer is not simplified.

$$\text{Linearization of } f : 1 - \frac{x}{2} + \frac{2y}{3}$$

Total: 22 points