

Exam: Vector Calculus for BMT, CE, CSE, ME

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8th November 2024, 13.45 – 15.45

2024/2025, Q-1A

Give a suitable explanation of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

Question 1. Use a line integral to calculate the area of one side of the "winding wall" standing orthogonally on the curve $C : x^2 + 4y^2 = 1$, and beneath the curve on the surface $z = f(x, y) = \sqrt{\frac{x^2}{4} + 4y^2}$.

Question 2. Consider the force field

$$\mathbf{F}(x, y, z) = \left(2(x-1)e^{y^2+z^2}\right) \mathbf{i} + \left(2(x-1)^2 y e^{y^2+z^2}\right) \mathbf{j} + \left(2(x-1)^2 z e^{y^2+z^2} + z\right) \mathbf{k}.$$

- (a) Use the component test to verify that $\mathbf{F}$ is conservative.
- (b) Determine a potential function for $\mathbf{F}$.
- (c) Let C be a curve connecting the points $A = (1, 0, 1)$ and $B = (2, 0, 0)$. Calculate the work done by $\mathbf{F}$ in moving an object along the curve C from A to B .

Question 3. Let S be the portion of the paraboloid $z = x^2 + y^2$ that lies inside the sphere $x^2 + y^2 + z^2 = 2$.

- (a) Make a clear sketch of S .
- (b) Calculate the surface area of S using a parametrization in cylindrical coordinates.

Question 4. Let D be the solid region in $\mathbb{R}^3$ between the spheres $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 1$. Consider the vector field

$$\mathbf{F}(x, y, z) = (x^3 + \sin(x) \cos(z)) \mathbf{i} + (2y^3 + 3x^2 y) \mathbf{j} + (2z^3 - \cos(x) \sin(z)) \mathbf{k}.$$

- a) Sketch D and describe the solid D using spherical coordinates.
- b) Calculate the outward flux of $\mathbf{F}$ across the boundary of D using an integration theorem (Gauß, Green, Stokes).

Points:

Q1.	4	Q2.	(a)	1	Q3.	(a)	1	Q4.	(a)	2
			(b)	1.5		(b)	4		(b)	3
			(c)	1.5						

Total: 18 + 2 = 20 points