

RIVER FLOW PROCESSES

Trial Exam

October 22, 2025

Name: _____

Student Number: _____

- This is a closed book exam, hence readers, lecture notes and similar are not allowed during the exam. You are allowed to have only pens, ruler, calculator, dictionary and snacks/drinks on your desk. A formula sheet is given with the exam.
- Use a black or blue pen. Do not use a pencil or a red pen.
- You can write your answers below the questions in this document and at the back of each page.
- The numbers in the parentheses next to the questions indicate the number of points to be earned with each question. The total number of points is 120.
- Given data for the exam:

Gravitational acceleration:	$g = 9.81 \text{ m/s}^2$
Density of water:	$\rho = 1000 \text{ kg/m}^3$
Density of sediment:	$\rho_s = 2650 \text{ kg/m}^3$
Kinematic viscosity of water:	$\nu = 10^{-6} \text{ m}^2/\text{s}$
von Karman constant:	$\kappa = 0.41$

1. (5 points) Name the following set of equations and provide an explanation of the physical meaning of the groups of terms indicated by the numbers below the equations:

$$\begin{aligned}
 \underbrace{\frac{\partial u}{\partial t}}_1 + \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}}_2 &= \underbrace{g_x}_3 - \underbrace{\frac{1}{\rho} \frac{\partial P}{\partial x}}_4 + \underbrace{\nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)}_5 \\
 \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= g_y - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
 \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= g_z - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)
 \end{aligned}$$

Answer: These are the Navier-Stokes equations and describe any kind of fluid flow. The meaning of the terms is the following:

1. Temporal change of momentum in fluid element (local acceleration)
 2. Net flux of momentum through fluid element (convective acceleration)
 3. Gravity force acting on fluid element
 4. Net pressure force on fluid element
 5. Net viscous forces on fluid element (caused by molecular interactions)
2. (5 points) When solving river engineering problems the Reynolds' averaging is applied to the hydrodynamic variables. Explain what the Reynolds' averaging is and why it is needed to solve river engineering problems.

Answer: Reynolds' averaging means that the hydrodynamic variables are averaged over a certain period of time such that the fluctuations induced by turbulence are eliminated. This process introduces additional, turbulence induced, shear stresses to the equations named Reynolds' stresses. Reynolds' averaging makes it possible to solve the mean hydrodynamic variables without having to solve all turbulent scales, which is too demanding and often impossible for real world geometries.

3. (10 points) (a) The roughness of a river bed can be expressed in the Chezy coefficient C and the roughness height k_s ,
- Explain in physical terms the difference between the Chezy coefficient C and the roughness height k_s ;
 - Why is for a channel with a concrete bottom and a variable discharge the assumption of a constant C less realistic than the assumption of a constant k_s ?
 - Why can k_s also be variable in case of sandy river beds?

Answer:

- C is the hydraulic roughness (friction coefficient) and k_s is the physical roughness height of the wall (see also formula of White-Colebrook or Strickler).
- As C is depth dependent (see again formula of White-Colebrook), we cannot assume it is constant in the given situation. Opposite, k_s is not in a concrete channel.
- in sandy rivers, k_s can vary because of varying bedform dimensions (i.e. ripples, dunes, etc.)

- (b) In order to measure flow discharges in 2 rivers flow velocity measurements are carried out at several points in the river cross-section. The cross-section of both rivers can be schematized as a shallow rectangle (width $\gg$ depth). The river flow is steady and uniform.

River 1: depth $h_1 = 5m$, bed slope $i_b = 1.0 \cdot 10^{-4}$

River 2: depth $h_1 = 2.5m$, bed slope $i_b = 1.1 \cdot 10^{-4}$

The 2 rivers have the same bed roughness height k_s .

In which river will the largest flow velocity be measured at 50 cm above the bed? Explain your answer with an appropriate formula.

Answer:

We use the law of the wall equation to solve this question: $u(z) = \frac{u_*}{\kappa} \cdot \ln\left(\frac{33z}{k_s}\right)$ and divide out all constants:

$$\frac{u_1(50cm)}{u_2(50cm)} = \frac{u_{*,1}}{u_{*,2}} = \sqrt{\frac{h_1 i_{b,1}}{h_2 i_{b,2}}} \Rightarrow \sqrt{\frac{5 * 1.0 \cdot 10^{-4}}{2.5 * 1.1 \cdot 10^{-4}}} > 1$$

As the found answer > 1 , we find that largest flow velocity at 50 cm above the bed is measured in river 1.

- (c) In the transport formula of Meyer-Peter & Mueller the transport rate is a function of the flow-parameter Ψ . Also the Shields parameter τ^* plays an important role in the mechanism of sand transport.
- What is the physical meaning of the flow parameter Ψ and of the Shields parameter τ^* ? Explain their difference?
 - Is $\Psi > \tau^*$ or $\Psi < \tau^*$? Explain your answer.
 - Derive a formula for the minimum depth (depth-averaged) velocity u as required for the motion of sand particles, expressed in the grain diameter D , grain density ρ_{gs} and Chezy coefficient C . The critical Shields parameter is equal to 0.06.

Answer:

- Ψ (flow parameter) is the dimensionless effective bed shear stress (part of the bed shear stress responsible for bed-load), τ^* (Shields parameter) is the dimensionless total bed shear stress (including form drag on bed forms)
- Ψ is defined as $\Psi = \mu_r \cdot \tau^*$, where $\mu_r < 1$. So $\Psi < \tau^*$
- $u_{cr} = \sqrt{0.06 \cdot C^2 \cdot R \cdot D}$

4. (20 points) Consider a very wide river in equilibrium under steady and uniform flow conditions. During a flood event, the average flow depth is 5.5 m and the bed slope is $2 \cdot 10^{-4}$. The median sediment diameter is 0.2 mm and the Chezy coefficient is $50 \text{ m}^{1/2}/\text{s}$.
- (a) What is meant by the term ‘settling velocity of a sediment particle’? Calculate its value for the given conditions.
- (b) Generally, in the presence of bed forms in a river, the total bed shear stress is partitioned into two components. Which are they and give the physical explanation of both?
- (c) What type of bed forms do you observe for the given flow conditions? What does this tell you about the partitioning of the bed shear stress?
- (d) Assume that sediment transport can be estimated directly from the total bed shear stress that was calculated (i.e., without being partitioned). Is this a reasonable assumption for the type of bed forms that you estimated in (c)?

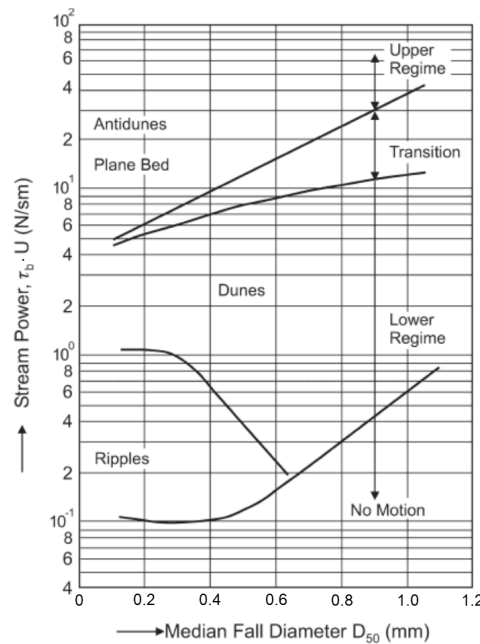


Figure 1: Bed form classification suggested by Simons and Richardson (1966) (figure adapted from Garcia, 2008). The median fall diameter is equal to the sediment median diameter D_{50} .

Answer:

(a) If a sediment grain is dropped into still water, it will initially accelerate under the action of gravity. While the particle accelerates, the drag on the particle (which is a function of the particle velocity) will increase. After a while, the exerted drag will become equal to the submerged weight of the particle and the particle will reach a terminal fall velocity, or settling velocity.

The submerged specific gravity of sediment, R , is:

$$R = \frac{\rho_s - \rho}{\rho} = \frac{2650 - 1000}{1000} = 1.65$$

For $D_{50} = 0.2$ mm, the settling velocity is:

$$w_s = \frac{10\nu}{D_{50}} \left(\sqrt{1 + \frac{0.01 Rg D_{50}^3}{\nu^2}} - 1 \right)$$

$$= \frac{10 \cdot 10^{-6}}{0.0002} \left(\sqrt{1 + \frac{0.01 \cdot 1.65 \cdot 9.81 \cdot 0.0002^3}{(10^{-6})^2}} - 1 \right) = 0.026 \text{ m/s}$$

(b) In the presence of bed forms, the bed shear stress is partitioned into the bed shear stress that is related to the grains (skin friction drag) and the bed shear stress that is related to the bed forms (pressure drag).

(c) The flow velocity is calculated with the Chezy equation for a very wide channel:

$$U = C\sqrt{hi_b} = 50\sqrt{5.5 \cdot 0.0002} = 1.66 \text{ m/s}$$

To be able to use Figure 1, we need to calculate the stream power, which is equal to the product of bed shear stress and flow velocity. The bed shear stress for steady and uniform flow conditions in a very wide channel is calculated from:

$$\tau_b = \rho g h i_b = 1000 \cdot 9.81 \cdot 5.5 \cdot 0.0002 = 10.79 \text{ Pa}$$

Thus, the stream power is $\tau_b U = 10.79 \cdot 1.66 = 17.9 \text{ N/(s}\cdot\text{m)}$.

We observe from Figure 1 that for $D_{50} = 0.2$ mm and $\tau_b U = 17.9 \text{ N/(s}\cdot\text{m)}$, we will have a plane bed at the upper flow regime, close to antidunes. For a plane bed, there is no pressure drag due to bed forms and the estimated bed shear stress is related only to skin friction.

(d) We consider the bed to be plane, so it is reasonable to expect that the entire bed shear stress will contribute to sediment transport through grain friction. Nevertheless, we are close to the antidune region, so there may also be some form drag.

5. (10 points) The figure below shows a modification of the Shields diagram together with data from bankfull flows in rivers. In such flows there is always sediment transport; however, a large number of data is below the Shields curve for $D_*^{3/2} > 100000$, implying that there is no sediment transport. This area of the diagram corresponds to gravel-bed rivers.

Why does the Shield diagram sometimes fail to predict the initiation of sediment motion in gravel-bed rivers?

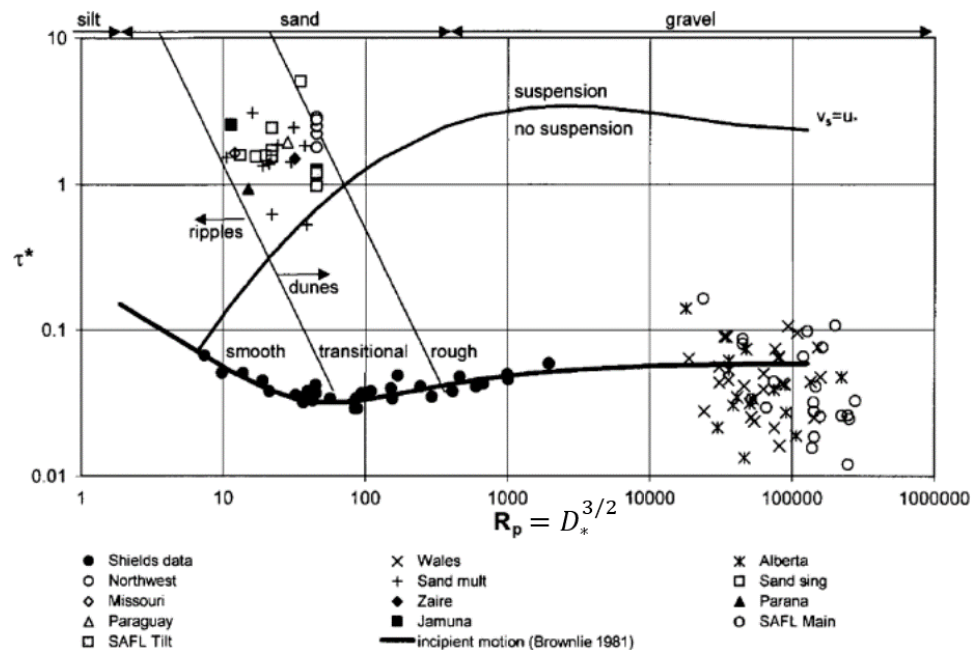


Figure 2: Shields diagram for the initiation of sediment motion with data from bankfull flows (figure adapted from Garcia, 2008)

Answer:

The Shields diagram uses time-averaged values and does not take into account instantaneous peaks of turbulent velocity near the bed. High-intensity turbulence bursts may be capable to dislodge a grain from its resting pocket and lead to particle entrainment from the flow. Another insufficiency of the Shields diagram is that it cannot capture the exposure of each particle to the flow. The bed consists of a large number of sediment particles, each of which has random orientation, different contact points with the surrounding particles, and protrudes to the flow to a different degree. Finally, the subjectivity of the observer on to what constitutes initiation of sediment transport (movement of just a few grains or the whole bed?) and experimental errors also contribute to occasional fails of the Shields diagram.

6. (25 points) A dam is constructed for electricity production. The dam blocks a river and creates a lake with a surface area of 50 km^2 and an average depth of 30 m. The river is in equilibrium and has an average discharge of $1500 \text{ m}^3/\text{s}$ and a width of 150 m. The bed sediment of the river is relatively uniform and the median grain size is 0.2 mm. The roughness is described with a Chezy coefficient of $50 \text{ m}^{1/2}/\text{s}$. The bed slope is 10^{-4} . Estimate how long it will take for the river to fill the reservoir with sediment reducing its water storage by 50%. You can assume that all sediment coming from the river is captured by the dam reservoir, the river is very wide, and the flow is steady and uniform. The porosity (ratio of volume of voids to total volume of sediment and voids) of the deposited sediment in the reservoir is $\epsilon = 0.4$.

Answer:

The river is very wide, so we can consider $R_h = h$. We can estimate the flow depth, h , with Chezy equation for a very wide channel.

$$U = C\sqrt{hi_b} \Rightarrow \frac{Q}{Wh} = C\sqrt{hi_b} \Rightarrow h = \left(\frac{Q}{WC\sqrt{i_b}} \right)^{2/3} = \left(\frac{1500}{150 \cdot 50 \cdot \sqrt{0.0001}} \right)^{2/3} = 7.37 \text{ m}$$

We observe that the median sediment diameter is 0.2 mm, hence we will estimate the sediment transport with the bed-material load formula of Engelund and Hansen. We also check if we have $w_s/u_* < 1$.

The submerged specific gravity of the sediment is:

$$R = \frac{\rho_s - \rho}{\rho} = \frac{2650 - 1000}{1000} = 1.65$$

For $D_{50} = 0.2 \text{ mm}$, the settling velocity, w_s , is:

$$w_s = \frac{10\nu}{D} \left(\sqrt{1 + \frac{0.01RgD^3}{\nu^2}} - 1 \right) = \frac{10 \cdot 10^{-6}}{0.0002} \left(\sqrt{1 + \frac{0.01 \cdot 1.65 \cdot 9.81 \cdot 0.0002^3}{(10^{-6})^2}} - 1 \right) = 0.026 \text{ m/s}$$

The shear velocity, u_* , for steady and uniform flow conditions in a very wide channel is:

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{\rho g h i_b}{\rho}} = \sqrt{g h i_b} = \sqrt{9.81 \cdot 7.37 \cdot 0.0001} = 0.085 \text{ m/s}$$

We observe that:

$$\frac{w_s}{u_*} = \frac{0.026}{0.085} = 0.3 < 1$$

which justifies the selection of the sediment transport formula of Engelund and Hansen.

The Shields parameter is:

$$\tau^* = \frac{\tau_b}{g(\rho_s - \rho) D_{50}} = \frac{\rho u_*^2}{g(\rho_s - \rho) D_{50}} = \frac{1000 \cdot 0.085^2}{9.81 (2650 - 1000) 0.0002} = 2.23$$

The parameter μ_r is:

$$\mu_r = \left(\frac{C^2}{g} \right)^{2/5} = \left(\frac{50^2}{9.81} \right)^{2/5} = 9.17$$

The flow parameter, ψ , for the Engelund and Hansen formula is:

$$\psi = \mu_r \tau^* = 9.17 \cdot 2.23 = 20.45$$

The sediment transport parameter for the Engelund and Hansen formula is:

$$\phi = 0.05\psi^{5/2} = 0.05 \cdot 20.45^{5/2} = 94.6$$

The volumetric sediment transport rate per unit width, q_s , is calculated from:

$$\phi = \frac{q_s}{\sqrt{gRD_{50}^3}} \Rightarrow q_s = \phi \sqrt{gRD_{50}^3} = 94.6 \sqrt{9.81 \cdot 1.65 \cdot 0.0002^3} = 0.0011 \text{ m}^2/\text{s}$$

We are looking for the time that is needed for the 50% of the reservoir to be filled with sediment. Thus, the requested volume, V_t , is:

$$V_t = \frac{1}{2} \cdot 30 \cdot 50000000 = 750000000 \text{ m}^3$$

Due to porosity, the volume V_t consists of the sum of the volume of the sediment, V_s , and the volume of the voids, V_v . We are interested only in the volume of sediment, which is the quantity that needs to be transported from upstream:

$$\epsilon = \frac{V_v}{V_s + V_v} \Rightarrow V_v = \epsilon V_t = 0.4 \cdot 750000000 = 300000000 \text{ m}^3$$

Thus, $V_s = V_t - V_v = 750000000 - 300000000 = 450000000 \text{ m}^3$.

Finally, we need to relate the sediment transport rate for the whole river width to the volume V_s , to get the requested time, t :

$$V_s = q_s W t \Rightarrow t = \frac{V_s}{q_s W} = \frac{450000000}{0.0011 \cdot 150} = 2727272727 \text{ sec} \quad \text{or} \quad 86.5 \text{ years}$$

7. (15 points) Consider a river with a bottom slope $i_b = 1 \cdot 10^{-4}$ and a depth of 10 m. The sediment has a median diameter of 0.25 mm. Use the following equation that describes the concentration of suspended sediment across the depth with constant sediment diffusivity

$$c(z) = c(z_a) e^{-6Z \frac{z - z_a}{h}}, \quad \text{where} \quad Z = \frac{w_s}{\kappa u_*}$$

to determine at what relative depth, z/h , the suspended sediment concentration, $c(z)$, has dropped by 90% with respect to the reference concentration $c(z_a)$ at a reference level z_a above the bed. You can assume the height of the reference level to be negligible compared to the depth. Assume the flow is steady and uniform under equilibrium conditions in a very wide river.

Answer: Since the river is very wide, we have $R_h = h$. The shear velocity for steady and uniform flow in a very wide channel is:

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{\rho g h i_b}{\rho}} = \sqrt{g h i_b} = \sqrt{9.81 \cdot 10 \cdot 0.0001} = 0.099 \text{ m/s}$$

The submerged specific gravity of sediment is:

$$R = \frac{\rho_s - \rho}{\rho} = \frac{2650 - 1000}{1000} = 1.65$$

For $D = 0.25$ mm, the settling velocity is calculated from:

$$w_s = \frac{10\nu}{D} \left(\sqrt{1 + \frac{0.01 R g D^3}{\nu^2}} - 1 \right) = \frac{10 \cdot 10^{-6}}{0.00025} \left(\sqrt{1 + \frac{0.01 \cdot 1.65 \cdot 9.81 \cdot 0.00025^3}{(10^{-6})^2}} - 1 \right) = 0.035 \text{ m/s}$$

The Rouse parameter is now computed from:

$$Z = \frac{w_s}{\kappa u_*} = \frac{0.035}{0.41 \cdot 0.099} = 0.86$$

Since the height of the reference level is negligible compared to depth, we get:

$$c(z) = c(z_a) e^{-6Z \frac{z - z_a}{h}} \Rightarrow c(z) = c(z_a) e^{-6Z \frac{z}{h}}$$

in which substituting $c(z) = 0.1c(z_a)$ and by solving for relative depth, z/h , leads to:

$$\frac{z}{h} = -\frac{1}{6Z} \ln \frac{c(z)}{c(z_a)} = -\frac{1}{6 \cdot 0.86} \ln(0.1) = 0.45$$