

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.



Answer Form

Calculus 2 - Exam - Friday 14 June, 2024

Full name in BLOCK LETTERS CORRECTION MODEL

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

Question 1. The value of $\frac{dy}{dx}$ at the point $(0, 4)$ is:

3

Question 2(a). The coefficient of xy in $P_2(x, y)$ is:

-6π

Question 2(b). $P_2(1, \pi) =$

$-1 - 3\pi^2$

Question 3. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
Points are not awarded for fewer than two correct answers.

At $(0, -\sqrt{2})$, f has: ☐ a local maximum ☐ a local minimum ☒ a saddle point

At $(0, 0)$, f has: ☒ a local maximum ☐ a local minimum ☐ a saddle point

At $(\sqrt{2}, -\sqrt{2})$, f has: ☐ a local maximum ☒ a local minimum ☐ a saddle point

Question 4. The integral is

$\int_0^{\pi/4} \int_1^{\sqrt{2}(\cos\theta + \sin\theta)} r^3 \cos^2\theta \, dr \, d\theta$

Question 5. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
Points are not awarded for fewer than two correct answers.

For the coordinates (x, y, z) of P , it is true that:

☒ $x > 0$ ☐ $x < 0$ ☐ $x = 0$

☒ $y > 0$ ☐ $y < 0$ ☐ $y = 0$

☐ $z > 0$ ☒ $z < 0$ ☐ $z = 0$

Question 6. The area of R is:

6

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.



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Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

Question 1. The value of $\frac{dy}{dx}$ at the point $(0, -4)$ is:

1

Question 2(a). The coefficient of xy in $P_2(x, y)$ is:

-8π

Question 2(b). $P_2(1, \pi) =$

$-1 - 4\pi^2$

Question 3. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
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At $(-\sqrt{2}, \sqrt{2})$, f has: ☐ a local maximum ☒ a local minimum ☐ a saddle point

At $(-\sqrt{2}, 0)$, f has: ☐ a local maximum ☐ a local minimum ☒ a saddle point

At $(0, 0)$, f has: ☒ a local maximum ☐ a local minimum ☐ a saddle point

Question 4. The integral is

$\int_{\boxed{0}}^{\boxed{\pi/4}} \int_{\boxed{1}}^{\boxed{\sqrt{2}/(\cos\theta + \sin\theta)}} \boxed{r^3 \sin^2\theta} \, dr \, d\theta$

Question 5. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
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☐ $y > 0$ ☒ $y < 0$ ☐ $y = 0$

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Question 6. The area of R is:

8

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.



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Full name in BLOCK LETTERS _____

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

Question 1. The value of $\frac{dy}{dx}$ at the point $(0,3)$ is:

4

Question 2(a). The coefficient of xy in $P_2(x, y)$ is:

6π

Question 2(b). $P_2(1, \pi) =$

$1 + 3\pi^2$

Question 3. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
Points are not awarded for fewer than two correct answers.

At $(0,0)$, f has: ☒ a local maximum ☐ a local minimum ☐ a saddle point

At $(-\sqrt{2}, -\sqrt{2})$, f has: ☐ a local maximum ☒ a local minimum ☐ a saddle point

At $(0, \sqrt{2})$, f has: ☐ a local maximum ☐ a local minimum ☒ a saddle point

Question 4. The integral is

$\int_0^{\pi/4} \int_1^{\sqrt{2}/(\cos\theta + \sin\theta)} r^3 \sin^2\theta \, dr \, d\theta$

Question 5. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
Points are not awarded for fewer than two correct answers.

For the coordinates (x, y, z) of P , it is true that:

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☐ $y > 0$ ☒ $y < 0$ ☐ $y = 0$

☒ $z > 0$ ☐ $z < 0$ ☐ $z = 0$

Question 6. The area of R is:

4

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.



Answer Form

Calculus 2 – Exam – Friday 14 June, 2024

Full name in BLOCK LETTERS _____

Student number _____

Programme

☐ BMT

☐ CE

☐ CSE

☐ IEM

☐ Other: _____

Question 1. The value of $\frac{dy}{dx}$ at the point $(0, -3)$ is:

2

Question 2(a). The coefficient of xy in $P_2(x, y)$ is:

8π

Question 2(b). $P_2(1, \pi) =$

$1 + 4\pi^2$

Question 3. Mark the correct options with an X.

Ambiguous or empty choices are marked as wrong. Please issue any correction using the space on the right.
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At $(\sqrt{2}, 0)$, f has: ☐ a local maximum ☐ a local minimum ☒ a saddle point

At $(\sqrt{2}, -\sqrt{2})$, f has: ☐ a local maximum ☒ a local minimum ☐ a saddle point

At $(0, 0)$, f has: ☒ a local maximum ☐ a local minimum ☐ a saddle point

Question 4. The integral is

$$\int_0^{\pi/4} \int_1^{\sqrt{2}/(\cos\theta + \sin\theta)} r^3 \cos^2\theta \, dr \, d\theta$$

Question 5. Mark the correct options with an X.

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For the coordinates (x, y, z) of P , it is true that:

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☒ $z > 0$ ☐ $z < 0$ ☐ $z = 0$

Question 6. The area of R is:

10

Question 7(a). Write a full calculation/argumentation clearly in the box.

Constraint: $g(x,y) = x^2 + 2y^2 - 1$. Set $\nabla f = \lambda \nabla g$, $g=0$:

$$\begin{cases} 3x^2 = 2\lambda x \\ -8y = 4\lambda y \\ x^2 + 2y^2 = 1 \end{cases} \Leftrightarrow \begin{cases} x(3x - 2\lambda) = 0 \\ y(\lambda + 2) = 0 \\ x^2 + 2y^2 = 1 \end{cases} \leadsto y=0 \text{ or } \lambda = -2$$

⊗ ⊗⊗

⊗ $\begin{cases} \dots \\ y=0 \\ x^2=1 \end{cases}$ yields critical points $(1,0)$, $(-1,0)$

⊗⊗ $\begin{cases} x(3x+4)=0 \\ \lambda = -2 \\ x^2 + 2y^2 = 1 \end{cases} \leadsto x=0 \text{ or } x = -\frac{4}{3}$

⊙ ⊙⊙

• $\begin{cases} x=0 \\ \lambda = -2 \\ 2y^2 = 1 \end{cases}$ yields critical points $(0, \frac{1}{\sqrt{2}})$, $(0, -\frac{1}{\sqrt{2}})$

•• $\begin{cases} x = -\frac{4}{3} \\ \lambda = -2 \\ \frac{16}{9} + 2y^2 = 1 \end{cases}$ yields no real solution ($y^2 < 0$).

$f(1,0) = 0$, $f(-1,0) = -2$, $f(0, \pm \frac{1}{\sqrt{2}}) = -3$,

So the minimum value of f on the ellipse is -3 ,
attained at $(0, \frac{1}{\sqrt{2}})$ and $(0, -\frac{1}{\sqrt{2}})$.

Question 7(b). Write a full calculation/argumentation clearly in the box.

No: if it did, then there would be points on the ellipse where the function attains a value greater than its maximum.

Question 8. Write a full calculation/argumentation clearly in the box.

$$\text{Average value} = \frac{\iiint_E f \, dV}{\text{Vol}(E)}$$

We use cylindrical coordinates: the surfaces intersect at $\begin{cases} z = r^2 + 4 \\ z = 2r^2 \end{cases} \quad \text{i.e.} \quad \begin{cases} r = 2 \\ z = 8 \end{cases}$

$$f(r \cos \theta, r \sin \theta, z) = r^2, \quad \text{so}$$

$$\begin{aligned} \iiint_E f(x, y, z) \, dV &= \int_0^{2\pi} \int_0^2 \int_{2r^2}^{r^2+4} r^3 \, dz \, dr \, d\theta = \\ &= 2\pi \int_0^2 r^3 (4 - r^2) \, dr = 2\pi \left[r^4 - \frac{r^6}{6} \right]_0^2 = \\ &= 2\pi \left(16 - \frac{32}{3} \right) = 2\pi \cdot 32 \cdot \frac{1}{6} = \frac{32}{3} \pi. \end{aligned}$$

$$\begin{aligned} \text{Vol}(E) &= \iiint_E dV = \int_0^{2\pi} \int_0^2 \int_{2r^2}^{r^2+4} r \, dz \, dr \, d\theta = \\ &= 2\pi \int_0^2 r (4 - r^2) \, dr = 2\pi \left[2r^2 - \frac{r^4}{4} \right]_0^2 = \\ &= 2\pi (8 - 4) = 8\pi. \end{aligned}$$

So the average value of f over E is $\frac{32}{3} \pi \cdot \frac{1}{8\pi} = \frac{4}{3}$.

Question 9. Write a full calculation/argumentation clearly in the box.

- Critical points of f in the interior of R : $\nabla f = \underline{0}$.

$$\begin{cases} 2x(1 - e^{1-x^2-y^2}) = 0 \\ -2y \underbrace{(1 + e^{1-x^2-y^2})}_{>0} = 0 \end{cases} \Leftrightarrow \begin{cases} x(1 - e^{1-x^2}) = 0 \\ y = 0 \end{cases} \quad \text{gives}$$

$(0,0)$, $(1,0)$ and $(-1,0)$ as critical points.

- Inspection of the boundary: this can be done in several ways. One of them is via a parametrisation

$$\begin{aligned} g(t) &= f(2\cos t, 2\sin t) = 4\cos^2 t - 4\sin^2 t + e^{1-4(\cos^2 t + \sin^2 t)} = \\ &= 4\cos^2 t - 4\sin^2 t + e^{-3}, \quad \text{for } t \in [0, 2\pi]. \end{aligned}$$

$$g'(t) = -8\cos t \sin t - 8\sin t \cos t = -16\cos t \sin t$$

$g'(t) = 0$ (in $[0, 2\pi]$) for $t=0$, $t=\frac{\pi}{2}$, $t=\pi$ and $t=\frac{3\pi}{2}$, corresponding to the points $(2,0)$, $(0,2)$, $(-2,0)$, $(0,-2)$.

- Evaluation of the function:

$$\begin{aligned} f(0,0) &= e, & f(\pm 1, 0) &= 2, & f(\pm 2, 0) &= 4 + e^{-3}, \\ & & & & f(0, \pm 2) &= -4 + e^{-3}. \end{aligned}$$

So the maximum is $4 + e^{-3}$ attained at $(2,0)$ and $(-2,0)$ and the minimum is $-4 + e^{-3}$ attained at $(0,2)$ and $(0,-2)$.

Question 10. Write a full calculation/argumentation clearly in the box.

$\bar{x} = \frac{M_y}{M}$ where $M = \frac{16}{3}$ is given, and

$$\begin{aligned} M_y &= \iint_T x \cdot \rho(x,y) dA = \iint_R x \cdot \rho(x,y) dA + \iint_S x \cdot \rho(x,y) dA \\ &= \iint_R x dA + \iint_S (x + xy) dA \\ &= \iint_R x dA + \iint_S x dA + \iint_S xy dA \\ &= \iint_T x dA + \iint_S xy dA \quad * \\ &= \int_0^2 \int_0^{2-x} xy dy dx = \int_0^2 \frac{1}{2} x (2-x)^2 dx \\ &= \frac{1}{2} \int_0^2 (x^3 - 4x^2 + 4x) dx = \\ &= \frac{1}{2} \left[\frac{x^4}{4} - \frac{4}{3} x^3 + 2x^2 \right]_0^2 = \frac{1}{2} \left(4 - \frac{32}{3} + 8 \right) \\ &= 6 - \frac{16}{3} = \frac{2}{3}. \end{aligned}$$

* Note that $\iint_T x dA = 0$ by symmetry.

$$\text{So } \bar{x} = \frac{2}{3} \cdot \frac{3}{16} = \frac{1}{8}.$$