

RIVER FLOW PROCESSES 2025-26

Formula sheet

Submerged specific gravity of the sediment: $R = \frac{\rho_s - \rho}{\rho}$

$$\text{Particle settling velocity: } w_s = \begin{cases} \frac{Rg}{18\nu}D^2, & \text{for } 0.001 < D \leq 0.1 \text{ mm} \\ \frac{10\nu}{D} \left(\sqrt{1 + \frac{0.01RgD^3}{\nu^2}} - 1 \right), & \text{for } 0.1 < D \leq 1 \text{ mm} \\ 1.1\sqrt{RgD}, & \text{for } D > 1 \text{ mm} \end{cases}$$

Hydraulic radius: $R_h = \frac{A}{O}$

Chezy equation for steady and uniform flow: $U = C\sqrt{R_h i_b}$

White-Colebrook equation for the Chezy coefficient: $C = 18 \log \left(\frac{12R_h}{k_s} \right)$

Manning's equation for steady and uniform flow: $U = \frac{1}{n} R_h^{2/3} i_b^{1/2}$

Manning's coefficient for hydraulically rough bed: $n = 0.04 k_s^{1/6}$

Darcy-Weisbach equation for steady and uniform flow: $U = \sqrt{\frac{8u_*^2}{f_{DW}}}$

Roughness height for flat bed: $k_s \approx D_{50}$ (if not given at a different form)

Logarithmic velocity profile: $\bar{u}(z) = \frac{u_*}{\kappa} \ln \left(\frac{z}{z_0} \right)$

Shear Reynolds number: $\text{Re}_* = \frac{u_* k_s}{\nu}$

$$\text{Level above the bed where the flow velocity is zero: } z_0 = \begin{cases} 0.11 \frac{\nu}{u_*}, & \text{for } \text{Re}_* < 5 \\ 0.11 \frac{\nu}{u_*} + \frac{k_s}{33}, & \text{for } 5 \leq \text{Re}_* \leq 70 \\ \frac{k_s}{33}, & \text{for } \text{Re}_* > 70 \end{cases}$$

Bed shear stress for steady and uniform flow conditions: $\tau_b = \rho g R_h i_b$

$$\text{Shear velocity: } u_* = \sqrt{\frac{\tau_b}{\rho}}$$

$$\text{Shields stress: } \tau^* = \frac{\tau_b}{g(\rho_s - \rho)D}$$

$$\text{Particle parameter: } D_* = D \left[\frac{(\rho_s - \rho) g}{\rho \nu^2} \right]^{1/3}$$

$$\text{Critical Shields parameter: } \tau_{cr}^* = \begin{cases} \frac{0.24}{D_*} & \text{if } 1 < D_* \leq 4 \\ \frac{0.14}{D_*^{0.64}} & \text{if } 4 < D_* \leq 10 \\ \frac{0.04}{D_*^{0.1}} & \text{if } 10 < D_* \leq 20 \\ 0.013 D_*^{0.29} & \text{if } 20 < D_* \leq 150 \\ 0.055 & \text{if } D_* > 150 \end{cases}$$

$$\text{Sediment transport parameter: } \phi = \frac{q_s}{\sqrt{g R D_{50}^3}}$$

$$\text{Flow parameter: } \psi = \mu_r \tau^*$$

Sediment transport formula of Meyer-Peter and Muller:

$$\phi = 8(\psi - 0.047)^{3/2}, \quad \text{with} \quad \mu_r = \left(\frac{C}{C_{90}} \right)^{3/2} \quad \text{and} \quad C_{90} = 18 \log \left(\frac{12R_h}{D_{90}} \right)$$

valid for

- $\mu_r \tau^* < 0.2$
- $D > 0.4 \text{ mm}$
- $w_s/u_* > 1$

Sediment transport formula of Engelund and Hansen:

$$\phi = 0.05\psi^{5/2}, \quad \text{with} \quad \mu_r = \left(\frac{C^2}{g} \right)^{2/5}$$

valid for

- $w_s/u_* < 1$
- $0.07 < \tau^* < 6$
- $0.19 < D_{50} < 0.93 \text{ mm}$

Methodology of van Rijn for the estimation of the total roughness height with dunes:

Roughness height due to grain roughness according to van Rijn:

$$k'_s = 3D_{90}$$

Roughness height due to dunes according to van Rijn:

$$k''_s = 1.1\gamma_d\eta_d \left[1 - e^{-25\eta_d/\lambda_d} \right]$$

where:

$$\gamma_d = 0.7$$

$$\eta_d = 0.11h \left(\frac{D_{50}}{h} \right)^{0.3} \left[1 - \exp \left(-0.5 \frac{\tau'_b - \tau_{bcr}}{\tau_{bcr}} \right) \right] \left(25 - \frac{\tau'_b - \tau_{bcr}}{\tau_{bcr}} \right)$$

$$\lambda_d = 7.3h$$

Methodology of Engelund for estimating depth and velocity from given discharge:

According to this methodology, the bed shear stress decomposition $\tau_b = \tau'_b + \tau''_b$ relates to the corresponding decomposition of hydraulic radiuses $R_h = R'_h + R''_h$ (single-prime denotes the effect of grain roughness and double-prime denotes the effect of form drag due to bed forms). The flow velocity is not affected by this decomposition and the velocity for the total bed roughness is used. For this method, the roughness height due to grain roughness is $k'_s = 2.5D_{50}$.

Bed form classification:

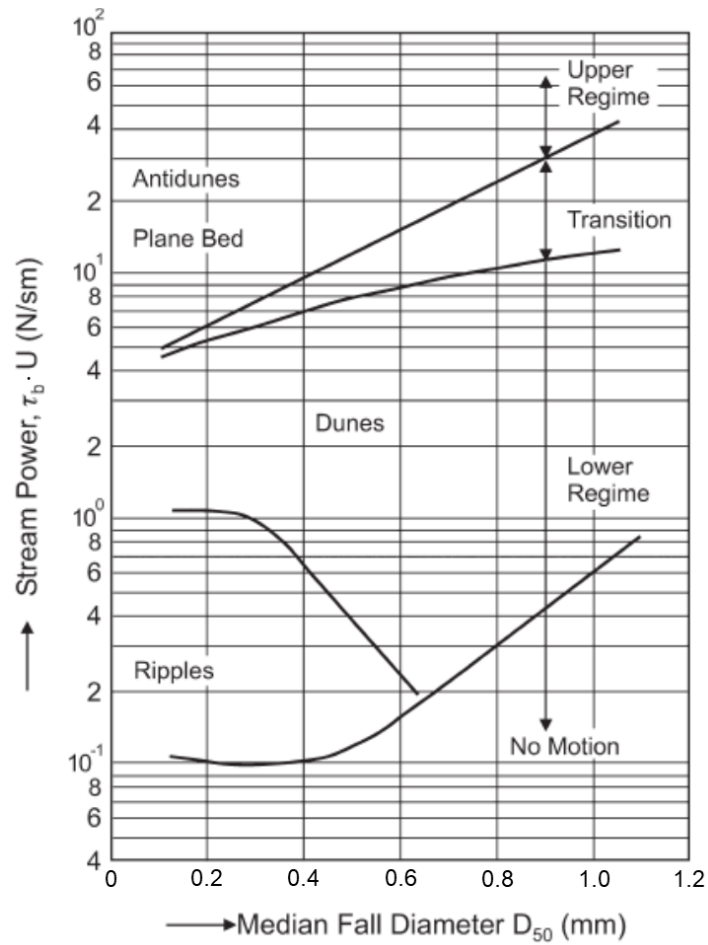


Figure 1: Bed form classification suggested by Simons and Richardson (1966) (figure from Garcia, 2008)

Suspended sediment concentration across the depth with constant sediment diffusivity:

$$c(z) = c(z_a) e^{-6Z \frac{z - z_a}{h}}$$

Suspended sediment concentration across the depth with parabolic sediment diffusivity:

$$c(z) = c(z_a) \left(\frac{h - z}{z} \frac{z_a}{h - z_a} \right)^Z$$

Rouse parameter: $Z = \frac{w_s}{\kappa u_*}$

Suspended sediment concentration at the reference level, z_a :

$$c(z_a) = 0.015 \frac{D_{50}}{z_a} \frac{\left(\frac{\tau'_b - \tau_{bcr}}{\tau_{bcr}} \right)^{1.5}}{D_*^{0.3}}, \text{ where } \tau'_b = \left(\frac{C}{C_{90}} \right)^2 \tau_b \text{ and } C_{90} = 18 \log \left(\frac{12R_h}{D_{90}} \right)$$

List of symbols in the formula sheet

A	cross-sectional area [L^2]
C	Chézy coefficient [$L^{1/2} T^{-1}$]
C_{90}	Chézy coefficient related to D_{90} [$L^{1/2} T^{-1}$]
D	sediment diameter [L]
D_{50}	median sediment diameter [L]
D_x	sediment diameter from which $x\%$ of the sediment mixture by weight is finer [L]
D_*	particle parameter [-]
O	wet perimeter of a cross-section [L]
R	submerged specific gravity of the sediment [-]
R_h	hydraulic radius [L]
U	time- and depth-averaged flow velocity for the longitudinal direction [$L T^{-1}$]
W	river width [L]
Z	Rouse parameter [-]
c	sediment concentration [-]
f_{DW}	Darcy-Weisbach coefficient [-]
g	gravitational acceleration [$L T^{-2}$]
h	flow depth [L]
i_b	slope [-]
k_s	roughness height [L]
k'_s	roughness height due to grain roughness [L]
k''_s	roughness height due to dunes [L]
n	Manning coefficient [$T L^{-1/3}$]
q	flow discharge per unit width [$L^2 T^{-1}$]
q_s	volumetric sediment transport rate per unit width [$L^2 T^{-1}$]
$\bar{u}$	time-averaged flow velocity for the longitudinal direction [$L T^{-1}$]
u_*	shear velocity [$L T^{-1}$]
w_s	settling or fall velocity of a sediment particle [$L T^{-1}$]
z	vertical coordinate [L]
z_0	level above the bed where the flow velocity is zero [L]
z_a	reference level [L]
κ	von Karman constant [-]
μ_r	ripple factor [-]
ν	kinematic viscosity of water [$L^2 T^{-1}$]
ρ	density of water [$M L^{-3}$]
ρ_s	density of sediment [$M L^{-3}$]
τ_b	bed shear stress [$M L^{-1} T^{-2}$]

τ^*	dimensionless shear stress or Shields parameter [-]
τ_{cr}^*	critical dimensionless shear stress or critical Shields parameter [-]
γ_d	form factor for field conditions [-]
η_d	dune height [L]
λ_d	dune length [L]
ϕ	sediment transport parameter [-]
ψ	flow parameter [-]