

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.



Only for sorting

Answer form

Calculus 2 – Resit – Friday 3 July, 2026

Full name in BLOCK LETTERS CORRECTION MODEL

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. The direction vectors are:

$\langle 1, 0 \rangle$ and $\langle -\frac{4}{5}, -\frac{3}{5} \rangle$

Question 2(a). The point (x, y, z) at $(s, t) = (1, 0)$ is:

$(1, 2, 0)$

Question 2(b). $\frac{\partial f}{\partial s}$ at $(s, t) = (1, 0)$:

6π

Question 3. Write "TRUE" or "FALSE".

Points are awarded only if all three answers are correct.

(a) is FALSE (b) is TRUE (c) is FALSE

Question 4(a). Rewrite the integral using the order of integration $dx dz dy$:

$\int_0^1 \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} (7z + 1) dx dz dy$

Question 4(b). The volume of E is:

$\frac{4}{3}$

Question 5. The values of c are:

1 and -7

Question 6. $\bar{x} =$

$\frac{2}{21}$

1	2	3	4	5	6	7	8	9	10	Σ

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8π

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(a) is FALSE (b) is FALSE (c) is FALSE

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3 and -4

Question 6. $\bar{x} =$

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Question 1. The direction vectors are:

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5π

Question 3. Write "TRUE" or "FALSE".

Points are awarded only if all three answers are correct.

(a) is TRUE (b) is FALSE (c) is FALSE

Question 4(a). Rewrite the integral using the order of integration $dx dz dy$:

$\int_0^1 \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} (4z+1) dx dz dy$

Question 4(b). The volume of E is:

$4/3$

Question 5. The values of c are:

2 and -8

Question 6. $\bar{x} =$

$2/15$

Question 7. Write a full calculation/argumentation clearly in the box.

(a) $\nabla f = \left(y - \frac{5x}{x^2+y^2+1}, x - \frac{5y}{x^2+y^2+1} \right)$. Solve $\nabla f = \langle 0, 0 \rangle$:

$$\begin{cases} y(x^2+y^2+1) - 5x = 0 \\ x(x^2+y^2+1) - 5y = 0 \end{cases} \quad \text{i.e.} \quad \begin{cases} y(x^2+y^2+1) - 5x = 0 \\ x(x^2+y^2+1) - y(x^2+y^2+1) + 5x - 5y = 0 \end{cases}$$

From the 2nd equation: $(x-y)(x^2+y^2+1) + 5(x-y) = 0$

i.e. $(x-y)(x^2+y^2+6) = 0$

but $x^2+y^2+6 > 0$, so $x-y=0$, so $x=y$. So:

$$\begin{cases} x(2x^2+1) - 5x = 0 \\ x = y \end{cases} \quad \text{i.e.} \quad \begin{cases} x(2x^2-4) = 0 \\ x = y \end{cases} \quad \text{i.e.} \quad \begin{cases} x(x^2-2) = 0 \\ x = y \end{cases}$$

gives critical points $(0, 0)$, $(\sqrt{2}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{2})$.

(b) Second derivative test:

$$f_{xx}(x, y) = \frac{-5(x^2+y^2+1) + 10x^2}{(x^2+y^2+1)^2} \quad f_{xx}(0, 0) = -5$$

$$f_{yy}(x, y) = \frac{-5(x^2+y^2+1) + 10y^2}{(x^2+y^2+1)^2} \quad f_{yy}(0, 0) = -5$$

$$f_{xy}(x, y) = 1 + \frac{10xy}{(x^2+y^2+1)^2} \quad f_{xy}(0, 0) = 1$$

$$\Delta(0, 0) = (-5)(-5) - 1^2 = 24 > 0 \quad \text{and} \quad f_{xx}(0, 0) < 0,$$

so f attains a local maximum at $(0, 0)$.

Question 8. Write a full calculation/argumentation clearly in the box.

$$f(x,y) = x^2 - \ln(2y+3) \quad \text{Constraint: } g(x,y) = x^2 + y^2 - 1.$$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g=0 \end{cases} \quad \begin{cases} 2x = 2\lambda x & \leftarrow g' \text{ gives } x=0 \text{ or } \lambda=1 \\ -\frac{2}{2y+3} = 2\lambda y \\ x^2 + y^2 = 1 \end{cases}$$

$$\text{if } x=0: \quad \begin{cases} x=0 \\ \dots \\ y^2=1 \end{cases} \quad \text{yields crit. pt's } \underline{(0,1)}, \underline{(0,-1)}.$$

$$\text{if } \lambda=1: \quad \begin{cases} \lambda=1 \\ -1 = y(2y+3) \\ x^2 + y^2 = 1 \end{cases} \quad \begin{cases} \lambda=1 \\ 2y^2 + 3y + 1 = 0 \\ x^2 + y^2 = 1 \end{cases}$$

$$\text{Solve the 2nd equation for } y: \quad y = \frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4} = \begin{cases} -1 \\ -\frac{1}{2} \end{cases}$$

$$\text{If } y = -1, \text{ from the 3rd equation: } x^2 + 1 = 1 \text{ gives } x=0 \\ \text{critical point: } \underline{(0,-1)}. \text{ (Already found)}$$

$$\text{If } y = -\frac{1}{2}, \text{ from the 3rd equation: } x^2 + \frac{1}{4} = 1 \text{ gives } x = \pm \frac{\sqrt{3}}{2} \\ \text{critical points: } \underline{\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}, \underline{\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)}.$$

$$\text{Evaluation: } f(0,1) = -\ln 5 < 0.$$

$$f(0,-1) = 0.$$

$$f\left(\pm \frac{\sqrt{3}}{2}, -\frac{1}{2}\right) = \frac{3}{4} - \ln 2 > 0.$$

So the absolute maximum is $\frac{3}{4} - \ln 2$ attained at $\left(\pm \frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
and the absolute minimum is $-\ln 5$ attained at $(0,1)$.

Question 9. Write a full calculation/argumentation clearly in the box.

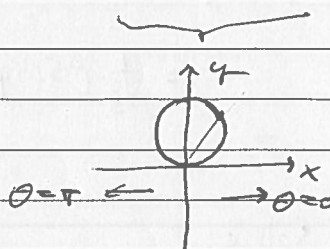
Cylindrical coordinates:

$$x^2 + (y-1)^2 = 1 \quad \text{iff} \quad r^2 \cos^2 \theta + r^2 \sin^2 \theta - 2r \sin \theta + 1 = 1$$

$$\text{iff} \quad r^2 = 2r \sin \theta \quad \text{iff} \quad (r=0 \text{ or}) \quad r = 2 \sin \theta$$

Floor: $z=0$

Ceiling: $z = 3 + xy$ i.e. $z = 3 + r^2 \cos \theta \sin \theta$.



$$\text{Volume} = \int_0^\pi \int_0^{2 \sin \theta} \int_0^{3 + r^2 \cos \theta \sin \theta} r \, dz \, dr \, d\theta =$$

$$\text{(or directly)} = \int_0^\pi \int_0^{2 \sin \theta} (3r + r^3 \cos \theta \sin \theta) \, dr \, d\theta =$$

$$= \int_0^\pi \left(\frac{3}{2} \cdot 4 \sin^2 \theta + \frac{1}{4} \cdot 16 \sin^4 \theta \cos \theta \sin \theta \right) d\theta =$$

$$= \int_0^\pi (6 \sin^2 \theta + 4 \sin^5 \theta \cos \theta) d\theta = \left[\text{use } \sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos(2\theta) \right]$$

$$= \left[6 \cdot \frac{\theta}{2} - 6 \cdot \frac{\sin(2\theta)}{4} + 4 \cdot \frac{\sin^6 \theta}{6} \right]_0^\pi = 3\pi - 0 + 0 - 0 + 0 - 0 = 3\pi.$$

ALTERNATIVELY

The volume of D is equal to the volume of a solid E bounded by $x^2 + y^2 = 1$, $z=0$ and $z = 3 + x(y+1) = 3 + x + xy$.

$$\text{Volume} = \int_0^{2\pi} \int_0^1 \int_0^{3 + r \cos \theta + r^2 \cos \theta \sin \theta} r \, dz \, dr \, d\theta =$$

$$= \int_0^{2\pi} \int_0^1 (3r + r^2 \cos \theta + r^3 \cos \theta \sin \theta) \, dr \, d\theta =$$

$$= \int_0^{2\pi} \left(\frac{3}{2} + \frac{1}{3} \cos \theta + \frac{1}{4} \cos \theta \sin \theta \right) d\theta = \frac{3}{2} \cdot 2\pi = 3\pi.$$

Question 10. Write a full calculation/argumentation clearly in the box.

(a) Cone:

$$\rho \cos \varphi = \frac{\sqrt{\rho^2 \cos^2 \theta \sin^2 \varphi + \rho^2 \sin^2 \theta \sin^2 \varphi}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \sqrt{\rho^2 \sin^2 \varphi} \quad \leftarrow \begin{array}{l} \rho \geq 0, \\ \sin \varphi \geq 0 \\ \text{for } \varphi \in [0, \pi] \end{array}$$

$$= \frac{1}{\sqrt{3}} \rho \sin \varphi \quad \text{gives } (\rho=0 \text{ or}) \quad \frac{\sin \varphi}{\cos \varphi} = \sqrt{3} \quad \text{i.e.} \quad \underline{\varphi = \pi/3}$$

Sphere:

$$\rho^2 \cos^2 \theta \sin^2 \varphi + \rho^2 \sin^2 \theta \sin^2 \varphi + \rho^2 \cos^2 \varphi - 2\rho \cos \varphi + 1 = 1$$

$$\text{i.e.} \quad \underbrace{\rho^2 \sin^2 \varphi + \rho^2 \cos^2 \varphi - 2\rho \cos \varphi}_{\rho^2 - 2\rho \cos \varphi} = 0$$

$$\rho^2 - 2\rho \cos \varphi = 0$$

$$\text{gives } (\rho=0 \text{ or}) \quad \underline{\rho = 2 \cos \varphi.}$$

$$(b) \text{ Compute } \int_0^{2\pi} \int_0^{\pi/3} \int_0^{2 \cos \varphi} \underbrace{\rho \cos \varphi}_{z} \cdot \rho^2 \sin \varphi \, d\rho \, d\varphi \, d\theta =$$

$$= \int_0^{2\pi} \int_0^{\pi/3} \frac{1}{4} \cdot 16 \cos^5 \varphi \sin \varphi \, d\varphi \, d\theta =$$

$$= 4 \cdot 2\pi \cdot \frac{1}{6} \cdot (-\cos^6(\frac{\pi}{3}) + \cos^6(0)) =$$

$$= \frac{4}{3} \pi \left(1 - \frac{1}{64}\right) = \frac{4}{3} \pi \cdot \frac{63}{64} = \frac{21}{16} \pi,$$

$$\text{So the average value is } \frac{21\pi}{16} / \text{vol}(D) = \frac{21\pi}{16} \cdot \frac{4}{5\pi} = \frac{21}{20}.$$