

EXAM- M8 TRAFFIC FLOW DYNAMICS

code: 202000083

May 19, 2021
NH207 and NH209
13:45-16:45

Dear student,

The exam consists of 4 problems. Maximum scores are listed in the table below. End score is the total score divided by 10.

Good luck

Eric van Berkum

problem	max score
1	10
2	20
3	30
4	40
total	100

1. [10 points] In the peak hour the intensity of left turning vehicles at an intersection equals 360 veh/h and the period they are confronted with red light is 60 s. Suppose the goal is to guarantee that in 96 % of the cycles the length of the lane is sufficient. What is the minimum length in meters of a lane for left turning vehicles. Assume that the arrival process is Poisson distributed and assume further that jam density is 160 veh/km. (in case you did not bring a calculator, please describe in an equation how you would solve this problem)

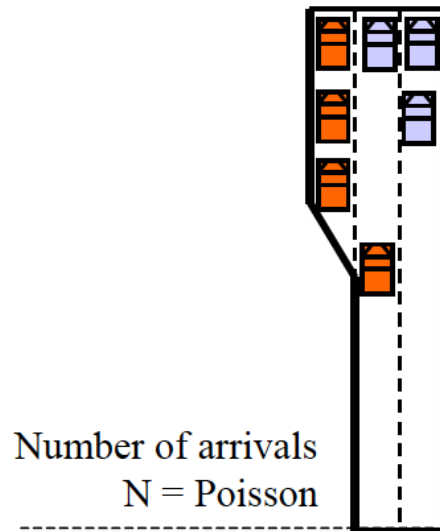


Figure 1: Example of a left lane with capacity for 3 vehicles

Solution: In 60 s will arrive on average $(60/3600) \times 360 = 6$ veh. We assume arrivals to be Poisson distributed. Then we must find the maximum value K for which it holds that:

$$\sum_{k=0}^K \frac{6^k}{k!} e^{-6} < 0.96$$

which yields $K = 10$. Jam density is 160 veh/km so 10 vehicles require a length of 62,5 m.

2. [20 points] A study of freeway flow at a particular site has resulted in a calibrated speed-density relationship, as follows:

$$u = 57.5(1 - 0.008k)$$

From this relationship:

- (a) [5 points] Find the free-flow speed and jam density
Solution: $u = 57.5 - 0.46k$. Notice that this equation is linear with respect to speed and density and is of the form of Greenshield's equation: $u = u_f - (u_f/k_j)k$; Free flow speed $u_f = 57.5$ km/h. To calculate jam density: $u_f/k_j = 0.46$ gives $k_j = 125$ veh/km
- (b) [10 points] Derive the equations describing flow versus speed and flow versus density.
 $q = 57.5k - 0.46k^2$ veh/h
gives flow as a function of density (note: it is a quadratic in k)
To derive flow as a function of speed:
 $0.46k = 57.5 - u$
 $k = (57.5 - u)/0.46 = 125 - (u/0.46)$
 $q = u(125 - (u/0.46)) = 125u - u^2/0.46$ veh/h (note that it is a quadratic in u)
- (c) [5 points] Determine the capacity of the site mathematically
Solution: To determine the capacity of the site we need to determine the maximum flow:
 $dq/dk = 57.5 - 0.46(2)k = 0$
 $57.5 = 0.46(2)k$
 $k = 57.5/(0.46(2)) = 62.5$ veh/km = k_m = density at maximum flow
 $q = 57.5k - 0.46k^2$
 $q = 57.5(62.5) - 0.46(62.5)^2$
 $q = 3593.75 - 1796.875$
 $q = 1796.875$ veh/h = q_m
speed at maximum flow = $u_m = 57.5 - 0.46(62.5) = 28.75$ km/h

3. [30 points] Consider a 3-lane freeway, with a triangular fundamental diagram. The free flow speed is 80 km/h, the capacity is 2000 veh/h/lane and the jam density is 150 veh/km/lane. The demand is constant at 2500 veh/h. From $t = 1$ h to $t = 2$ h, an incident occurs at $x = 10$, limiting the capacity to 1000 veh/h.

- (a) [10 points] Determine and calculate the different states of the traffic flow that occur before, during and after the incident.

Solution: For referring to certain states, we will first show the resulting states, and then explain how these states are constructed. Figure shows the fundamental diagram and the occurring states, and how the states move in space and time. The details of the states can be found in table, and the details of the shock waves can be found in table.

At the start, there are free flow conditions (state A) at an inflow of 2500 veh/h. For the assumed triangular fundamental diagram, the speed for uncongested conditions is equal to the free flow speed, so 80 km/h. The matching density can be found by applying

$$k_A = \frac{q_A}{u_A} = \frac{2500}{80} = 31.25 \text{ veh/km}$$

From $t = 1$ to $t=2$, a flow limiting condition is introduced. We draw this in the space time diagram. The flow is too high to pass the bottleneck, so the moment the bottleneck occurs, a congested state (B) will form upstream. Downstream of the bottleneck we find uncongested conditions (once the vehicles have passed the bottleneck, there is no further restriction in their progress): state C. For state C, the flow equals the flow that can pass the bottleneck, which is given at 1000 veh/h. The speed is the free flow speed of 80 km/h, so the matching density can be found by applying $k_C = \frac{q}{u} = \frac{1000}{80} = 12.5 \text{ veh/km}$. The speed of the shock wave between state A and C can be calculated using the shock wave equation

$$\omega_{AC} = \frac{q_A - q_C}{k_A - k_C} = \frac{2500 - 1000}{31.25 - 12.5} = 80 \text{ km/h}$$

This equals the free flow speed. Graphically, we understand this because both states can be found at the free flow branch of the fundamental diagram and the shock wave speed is the slope of the

line segment connecting these states. Because the fundamental diagram is triangular, this slope is equal to the slope at the origin (i.e., the free flow speed). Upstream of the bottleneck a congested state forms (B). The flow in this area must be the same as the flow which can pass the bottleneck. This is because at the bottleneck no new vehicles can be formed. That means state B is a congested state with a flow of 1000 veh/h. From the fundamental diagram we find the matching density in the congested branch, 387.5 veh/km. The speed at which the shock between states A and B now moves, can be calculated using

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 1000}{31.25 - 387.5} = -4.2 \text{ km/h}$$

The minus sign means that the shock wave moves in the opposite direction of the traffic, upstream. We could also graphically derive the speed of the shock wave by the slope of the line between point A and B in the fundamental diagram. Once the temporal bottleneck has been removed, the vehicles can drive out of the queue: state D. Because state C is congested, the outflow will be capacity (or the queue discharge rate in case there is a capacity drop). That is for this case a flow of $3 \times 2000 \text{ veh/h} = 6000 \text{ veh/h}$. Realising the vehicle speed equals the free flow speed of 80 km/h, the density can be found using

$$k_D = \frac{q_D}{v_D} = \frac{6000}{80} = 75 \text{ veh/km}$$

The shock wave between state B and D moves backward. The speed thereof can be found by applying

$$\omega_{BD} = \frac{q_B - q_D}{k_B - k_D} = \frac{1000 - 6000}{387.5 - 75} = -16 \text{ km/h}$$

The negative shock wave speed means the wave moves upstream. Intuitively, this is right, since the vehicles at the head of the queue can accelerate out of the queue, and thus the head moves backwards.

- (b) [10 points] Sketch the fundamental diagram for the freeway in the flow-density plane, and position the states in the diagram.

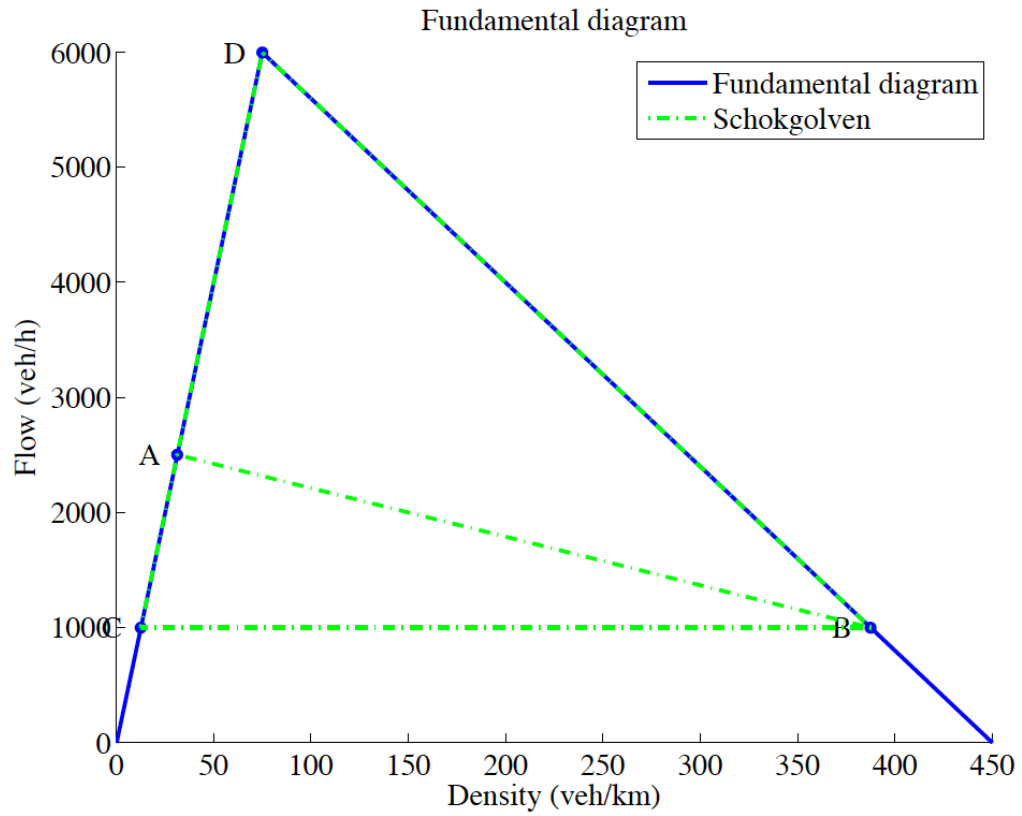


Figure 2: Position of traffic states at the fundamental diagram

- (c) [10 points] Sketch the shockwaves and states in the (x, t) -plane.
Determine the speed of the shockwaves using shockwave theory.

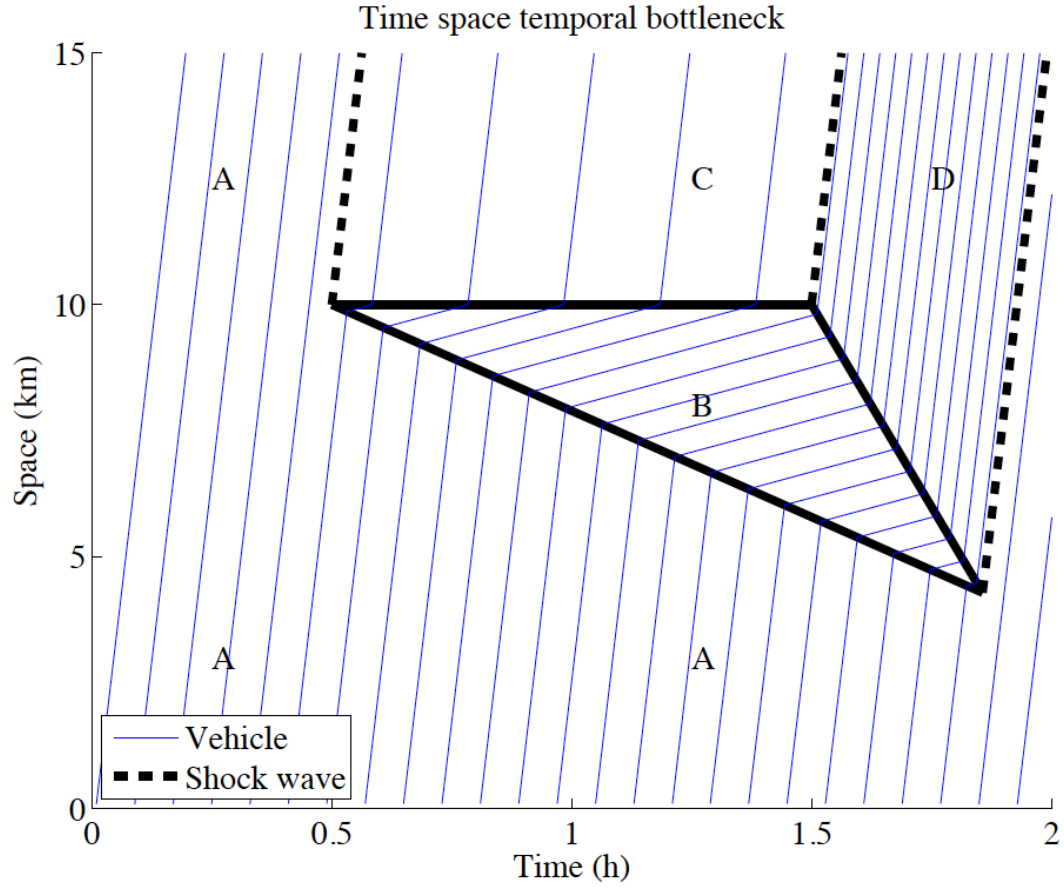


Figure 3: Shock waves in space-time

4. [40 points] Carfollowing behaviour.

- (a) [10 points] The General Motors suite of carfollowing models are so-called stimulus-response models. Consider the GM-5 model. What specifically is considered to be the stimulus and what the response? How is sensitivity (of the response) being modelled?

Solution: the equation for GM5 is:

$$a_{n+1}(t + \Delta t) = \frac{\alpha_{l,m}[u_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l} [u_n(t) - u_{n+1}(t)]$$

Stimulus is the difference in speed, i.e $[u_n(t) - u_{n+1}(t)]$, and re-

sponse is the acceleration $a_{n+1}(t + \Delta t)$ with response time Δt . Further, sensitivity depends on the speed and the distance headway, the higher the speed the more sensitivity, and the higher the distance headway, the lower the sensitivity. It is modeled as
$$\frac{\alpha_{l,m}[u_{n+1}(t + \Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l}$$

- (b) [10 points] Following this type of rather mechanistic models so-called psycho-spacing models were developed. Which issues did these models try to overcome? Describe the action-point model as developed by Wiedemann.

Solution: The only human element in the GM-models is the presence of a finite response time, but for the rest a more or less perfect driver is assumed. In reality a driver is not able to:

- observe a stimulus lower than a given values (perception threshold);
- evaluate a situation and determine the required response precisely, for instance due to observation errors resulting from radial motion observation;
- manipulate the gas and brake pedal precisely;

and does not want to be permanently occupied with the car-following task. The Wiedemann model was further explained in the lecture about carfollowing.

- (c) [10 points] The Dutch government recommends drivers in fog: "Half the speed, double the space headway to the predecessor". Suppose everyone takes this advice, how would the capacity of a road change?

Solution: Consider the FD in speed-density. Its form does not change, only the distance changes hence the values at the density axis are halved. Moreover, the speed reduces to half, hence the values on the speed axis are also halved. The effect on capacity is that the product of density and speed is half times half = 1 quarter of the original capacity.

- (d) [10 points] Argue how automated driving would impact traffic flow stability, from the theory of car-following.

Solution: in CF there is no longer a response time $\Delta t > 0$, and the automated vehicle is perfectly capable of determining speed

differences, so ideally the automated vehicle would be in constant CF-mode, keeping a safe distance at all times. However, the resulting acceleration behaviour may be far from comfortable. As a result a kind of Wiedemann approach is considered to be more realistic, except for the thresholds which in turn may yield uncomfortable behaviour, so these may be smoothed. Further, an automated vehicle is considered to make no errors in determining the stimuli, e.g. speed difference, and no error in response, so the right amount of deceleration or acceleration is determined, as a result, the flow becomes way more stable.