

Tag : Toetsen/21-22/I2M.22-23[v3].Test
Course : **Intro to Math + Calculus 1A**
Date : Friday September 23rd, 2022
Time : 13:45 – 14:45

Motivate all your answers.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Final answer

On the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

1. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Let $A_i = [\frac{1}{i}, i]$ for $i \neq 0$.

(a) [1 pt] Determine

$$\bigcup_{i=6}^{60} A_i.$$

(b) [1 pt] Determine

$$\bigcap_{i=6}^{60} A_i.$$

2. [2 pt] *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet.*

For each of the following statements, state whether it is true or false.

- (i) For all sets A : $\emptyset \in A$
- (ii) For all sets A : $\emptyset \subseteq A$
- (iii) For all sets A, B : $\overline{A} \cap \overline{B} \subset \overline{A \cup B}$
- (iv) For all sets A, B : $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

3. [3 pt] Prove the following statement:

For any numbers p, q : if $p + q \notin \mathbb{Q}$, then $p \notin \mathbb{Q}$ or $q \notin \mathbb{Q}$.

4. [4 pt] Use mathematical induction on n to prove that for all $n \in \mathbb{N}$,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

5. (a) [1.5 pt] How many 8-digit numbers consist of exactly 5 ones and 3 twos? For example, 12211121 is such a number. Provide both the computation and the final answer (For example, if the answer were $3! + 4!$, then you should write $3! + 4! = 30$.)
- (b) [1.5 pt] How many 8-digit numbers consist of exactly 5 ones and 3 other digits? For this exercise, numbers could start with a 0. For example, 05011111 is such a number. You don't need to evaluate your answer. (For example, if the answer were $3! + 4!$, then $3! + 4!$ is a correct answer.)

Total: 14 points