

River Morphodynamics - Formula sheet

Momentum equation for water flow:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial h}{\partial x} + g \frac{\partial z_b}{\partial x} = - \frac{g \bar{u}^2}{C^2 h}$$

Mass balance for water flow:

$$\frac{\partial h}{\partial t} + \frac{\partial (\bar{u} h)}{\partial x} = 0$$

Froude number: $Fr = \frac{\bar{u}}{\sqrt{gh}}$

Discharge per unit width for rectangular cross-section: $q = \frac{Q}{B} = \bar{u} h$

Sediment transport formula of Engelund and Hansen:

$$q_s = m \bar{u}^n = 0.05 \frac{\bar{u}^5}{\sqrt{g} C^3 \Delta^2 D_{50}}$$

Uniform flow

Flow velocity: $\bar{u} = C \sqrt{h i_b}$

Equilibrium depth: $h_e = \left(\frac{q^2}{C^2 i_b} \right)^{1/3}$

Critical depth: $h_c = \left(\frac{q^2}{g} \right)^{1/3}$

Steady non-uniform (gradually varied) flow

Method of Bresse:

$$x_1 - x_0 = \frac{h_e}{i_b} [(\zeta_1 - \zeta_0) - \gamma (\psi(\zeta_1) - \psi(\zeta_0))]$$

with $\gamma = 1 - \frac{C^2 i_b}{g}$, $\zeta = \frac{h}{h_e}$, and $\psi(\zeta)$ obtained from the table below.

ζ	$\psi(\zeta)$	ζ	$\psi(\zeta)$	ζ	$\psi(\zeta)$	ζ	$\psi(\zeta)$
0	0.	0.90	1.218	1.002	1.953	1.18	0.509
0.10	0.1	0.91	1.257	1.005	1.649	1.20	0.479
0.20	0.2	0.92	1.30	1.010	1.419	1.25	0.420
0.30	0.302	0.93	1.348	1.02	1.191	1.30	0.373
0.40	0.407	0.94	1.403	1.03	1.060	1.35	0.335
0.50	0.517	0.95	1.467	1.04	0.970	1.40	0.304
0.60	0.637	0.96	1.545	1.05	0.896	1.50	0.257
0.65	0.703	0.97	1.644	1.06	0.838	1.60	0.218
0.70	0.776	0.98	1.783	1.07	0.790	1.70	0.190
0.75	0.857	0.990	2.017	1.08	0.749	1.80	0.166
0.80	0.950	0.995	2.250	1.09	0.712	1.90	0.146
0.82	0.993	0.998	2.690	1.10	0.681	2.00	0.132
0.84	1.040	0.999	2.788	1.12	0.626	2.50	0.082
0.86	1.092	1.000	∞	1.14	0.580	3.00	0.055
0.88	1.151	1.001	2.184	1.16	0.541	large	$\frac{1}{2}\zeta^2$

Approximation according to Bresse:

$$h = h_e + (h_0 - h_e) \left(\frac{1}{2} \right)^{-\frac{x - x_0}{L_{1/2}}}$$

with
$$L_{1/2} = \frac{0.24 h_e}{i_b} \left(\frac{h_0}{h_e} \right)^{4/3}$$

Approximation for small deviations from the equilibrium depth:

$$h = h_e + (h_0 - h_e) e^{\frac{3(x - x_0)}{L}}$$

with
$$L = \frac{h_e^3 - h_c^3}{h_e^2 i_b} \quad \text{or} \quad \text{for} \quad Fr \ll 1 \quad L = \frac{h_e}{i_b}$$

Advection-diffusion equation for flood wave propagation

$$\frac{\partial h}{\partial t} + c_w \frac{\partial h}{\partial x} - K_w \frac{\partial^2 h}{\partial x^2} = 0$$

$$\text{with} \quad c_w = \frac{3}{2}\bar{u} \quad \text{and} \quad K_w = \frac{1}{2} \frac{C^2 h^2}{\bar{u}}$$

Exner equation

$$\frac{\partial q_s}{\partial x} + (1 - \varepsilon_p) \frac{\partial z_b}{\partial t} = 0$$

Morphodynamic change

$$\frac{\partial z_b}{\partial t} + c_m \frac{\partial z_b}{\partial x} = 0 \quad \text{with} \quad c_m = \frac{nq_s}{(1 - \varepsilon_p)h}$$

$$\frac{\partial z_b}{\partial t} - K_m \frac{\partial^2 z_b}{\partial x^2} = 0 \quad \text{with} \quad K_m = \frac{nq_s}{3(1 - \varepsilon_p)i_b}$$

Morphodynamic equilibrium

$$h_{eq} = \frac{Q}{\left(\frac{Q_s}{m}\right)^{\frac{1}{n}}} B^{\left(\frac{1}{n}-1\right)}$$
$$i_{b_{eq}} = \frac{1}{C^2} \frac{\left(\frac{Q_s}{m}\right)^{\frac{3}{n}}}{Q} B^{\left(1-\frac{3}{n}\right)}$$

List of symbols in the formula sheet

B	river width [m]
C	Chézy coefficient [$\text{m}^{1/2} \text{s}^{-1}$]
D	uniform sediment diameter [m]
Q	flow discharge [$\text{m}^3 \text{s}^{-1}$]
Q_s	volumetric sediment transport rate [$\text{m}^3 \text{s}^{-1}$]
Δ	submerged specific gravity of the sediment [-]
g	gravitational acceleration [m s^{-2}]
h	flow depth [m]
h_0	flow depth at x_0 [m]
i_b	bed slope [-]
m	coefficient in the sediment transport formula of Engelund and Hansen [$\text{m}^{-3} \text{s}^4$]
n	exponent in the sediment transport formula of Engelund and Hansen [-]
q	flow discharge per unit width [$\text{m}^2 \text{s}^{-1}$]
q_s	volumetric sediment transport rate per unit width [$\text{m}^2 \text{s}^{-1}$]
t	time [s]
$\bar{u}$	flow velocity [m s^{-1}]
x	longitudinal coordinate [m]
x_0	location of a reference point [m]
z_b	bed level [m]
z_w	water level [m]
ε_p	porosity of the river bed [-]

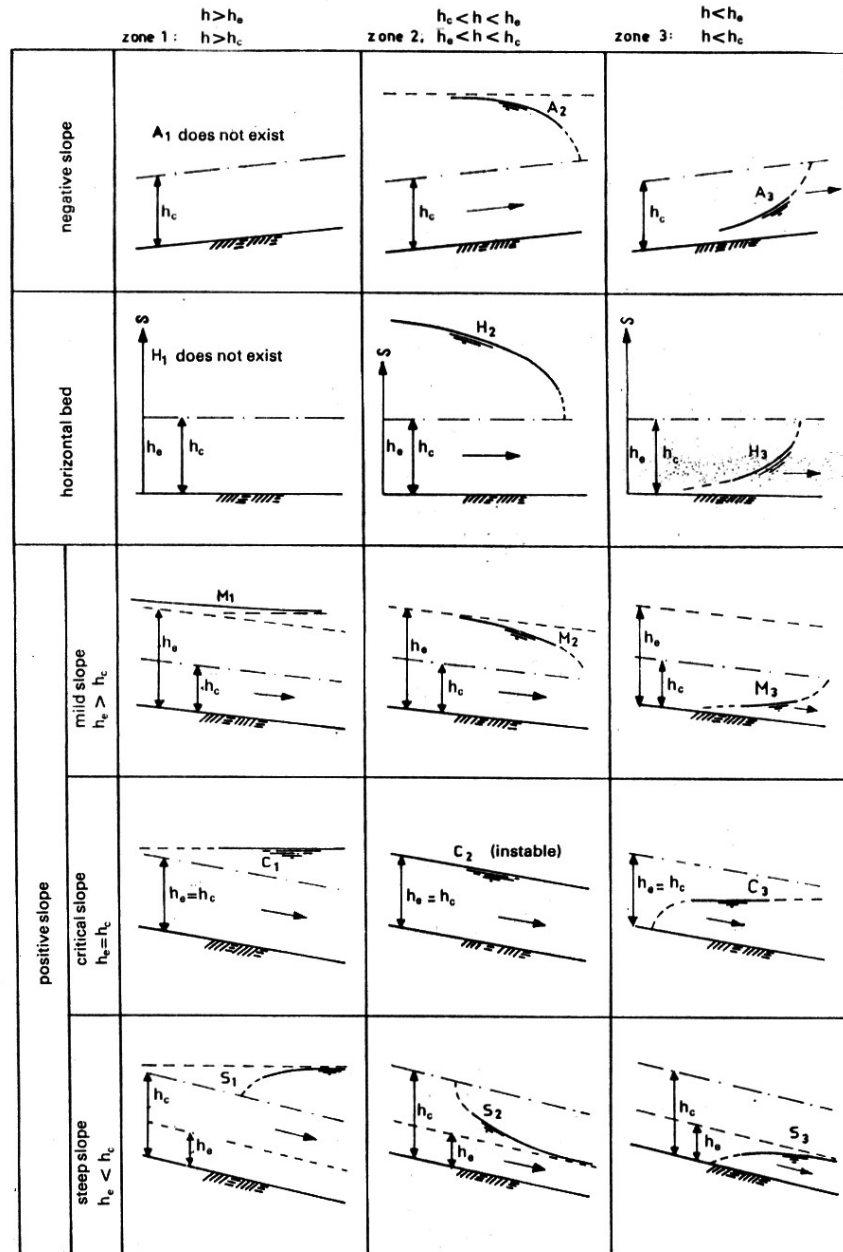


Figure 3.3: Classification of water surface curves (De Vries, 1985)