

Test: Structural Mechanics in Module 1
Unit of Study code: 2020-000-43

Date: 7-10-2021

Start of the exam: 13:45

End of the exam: 15:30

End of the exam for students with additional time: 15:55

Examiner: ing. G.H.Snellink

GRADING

- ✓ Weight of this test is 50% of the grade for Structural Mechanics.
- ✓ Within this test:
 - o Assignment 1: 30%
 - o Assignment 2: 40%
 - o Assignment 3: 30%

INSTRUCTIONS & PRACTICALITIES

- ✓ All Rules of Order for Testing as established by the examination board apply to this test. In case of inconsistencies with other rules set by the examiner, the Rules of Order for Testing prevail.
- ✓ Please read each question carefully before answering the question.
- ✓ All paper work has to be handed in at the end of the test, including scrap paper.
- ✓ For answering questions, use the attached working sheets. If one sheet is not enough, use the sheet with the text "Continuation of ..." in the top right corner. Enter the number of the question where this sheet is the follow-up. Do not make different tasks on the same working sheet.
- ✓ The elaboration of the question must be clear to all (i.e. the teacher). Show what you do, so use a lot of drawings, sketches, or schemas.

RESOURCES ALLOWED TO BE USED DURING THE EXAM:

You are allowed to use the following resources during the exam (i.e. any other resource not listed here is not allowed to be used during the exam):

- ✓ (graphical) calculator (not a communication device),
- ✓ pen or/and pencil,
- ✓ eraser,
- ✓ ruler.

Assignment 1. (30 %) Truss**Given:**

A truss with the characteristic load of $F=60\text{kN}$ has members with in genuine a cross section of 1000mm^2 (see Figure 1). The used material is steel $E=200\,000\text{N/mm}^2$

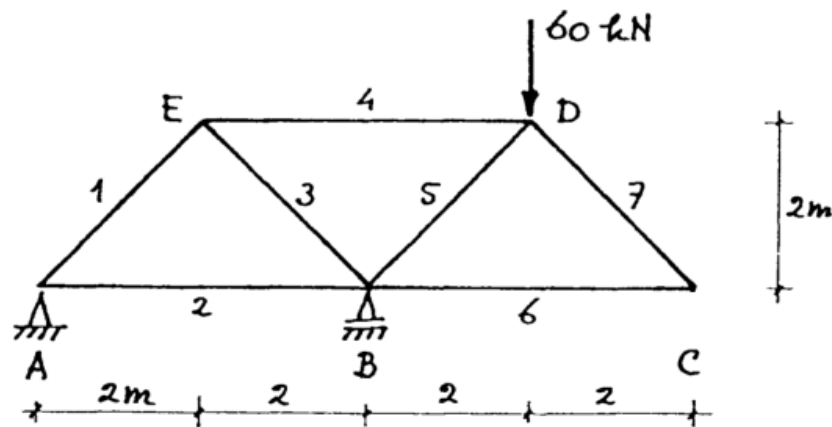


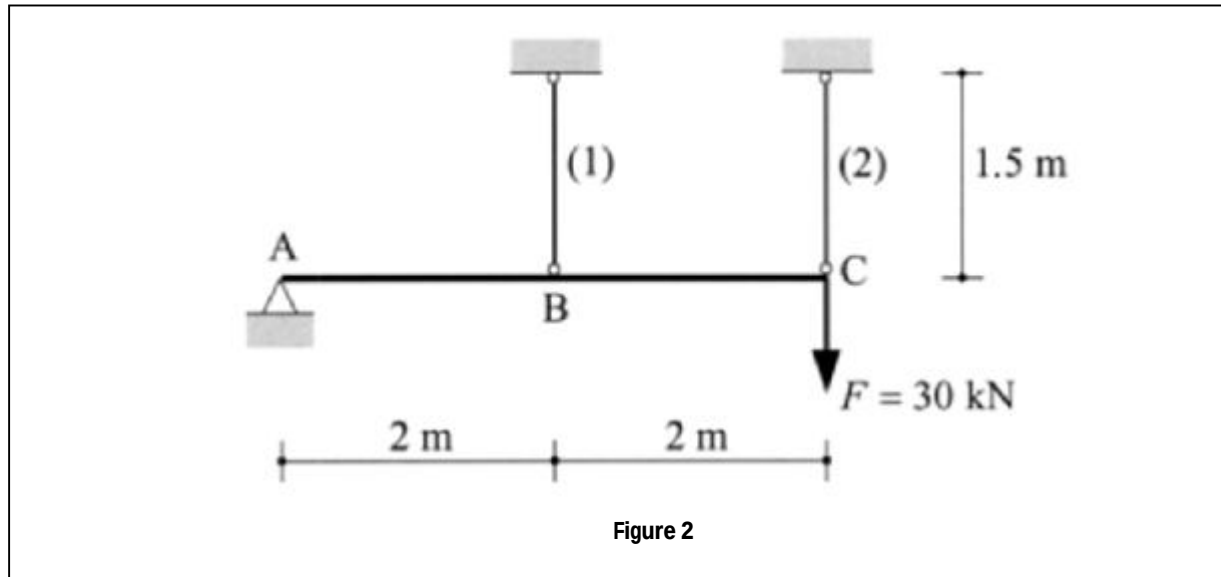
Figure 1

Question:

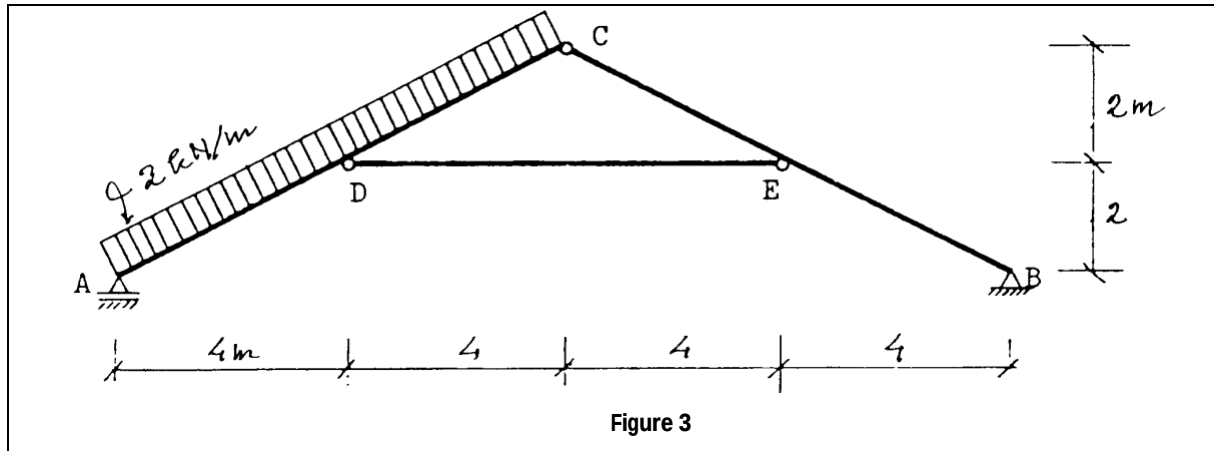
- a) Determine the internal forces in all members

Assignment 2. (40 %) Axial extension.**Given:**

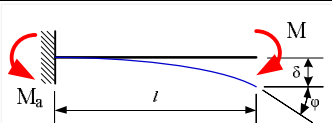
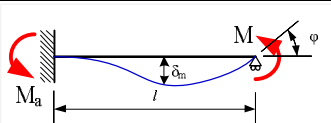
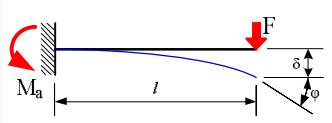
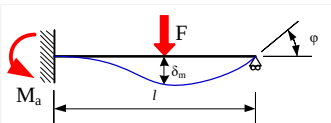
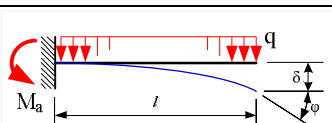
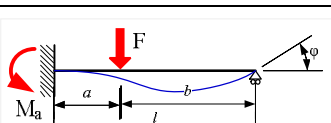
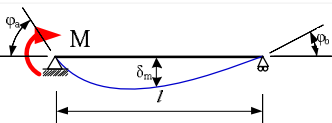
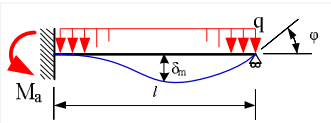
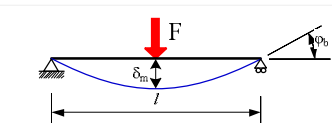
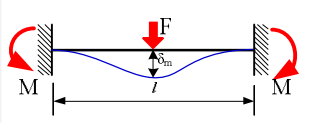
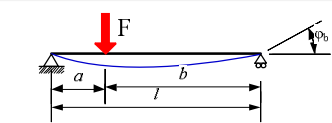
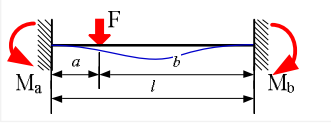
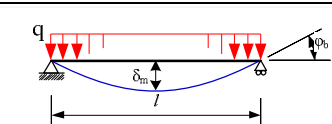
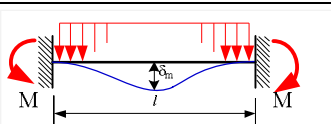
An entirely rigid and weightless beam ABC is supported by a hinge at A and suspended from two vertical bars at B and C. In the unloaded state the beam is horizontal. The structure is loaded at C by the force $F = 30 \text{ kN}$. For the axial stiffnesses of the bars use $EA_1 = 3000 \text{ kN}$ and $EA_2 = 1500 \text{ kN}$

**Questions:**

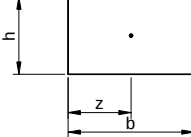
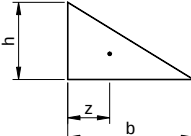
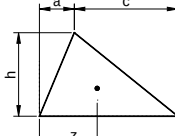
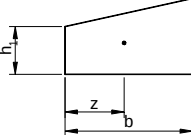
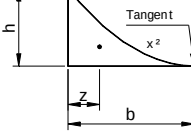
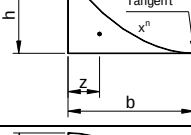
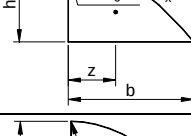
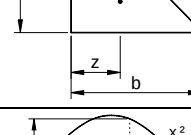
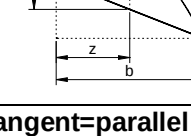
- How large are the strains and forces in the bars.
- Determine the magnitude and direction of the support reaction at A.

Assignment 3. (30 %) Support reactions**Given:****Questions:**

- a) Determine the internal forces of member DE.

Index M = midden	Moment	Rotation	Deflection:		Moment	Rotation	Deflection:
	$M_a = M$	$\varphi = \frac{Ml}{EI}$	$\delta = \frac{1}{2} \frac{Ml^2}{EI}$			$M_a = \frac{1}{2} M$	$\varphi = \frac{1}{4} \frac{Ml}{EI}$ $\delta_M = \frac{1}{32} \frac{Ml^2}{EI}$
	$M_a = Fl$	$\varphi = \frac{1}{2} \frac{Fl^2}{EI}$	$\delta = \frac{1}{3} \frac{Fl^3}{EI}$			$M_a = \frac{3}{16} Fl$ $M_M = \frac{5}{32} Fl$	$\varphi = \frac{1}{32} \frac{Fl^2}{EI}$ $\delta_M = \frac{7}{768} \frac{Fl^3}{EI}$
	$M_a = \frac{1}{2} ql^2$	$\varphi = \frac{1}{6} \frac{ql^3}{EI}$	$\delta = \frac{1}{8} \frac{ql^4}{EI}$			$M_a = \frac{Fb(l^2 - b^2)}{2l^2}$	$\varphi = \frac{1}{4} \frac{Fa^2b}{EI}$
		$\varphi_a = \frac{1}{3} \frac{Ml}{EI}$ $\varphi_b = \frac{1}{6} \frac{Ml}{EI}$	$\delta_M = \frac{1}{16} \frac{Ml^2}{EI}$			$M_a = \frac{1}{8} ql^2$	$\varphi = \frac{1}{48} \frac{ql^3}{EI}$ $\delta_M = \frac{1}{192} \frac{ql^4}{EI}$
	$M_M = \frac{1}{4} Fl$	$\varphi = \frac{1}{16} \frac{Fl^2}{EI}$	$\delta_M = \frac{1}{48} \frac{Fl^3}{EI}$			$M = \frac{1}{8} Fl$ $M_M = \frac{1}{8} Fl$	$\delta_M = \frac{1}{192} \frac{Fl^3}{EI}$
		$\varphi_a = \frac{1}{6} \frac{Fab(l+b)}{EI}$ $\varphi_b = \frac{1}{6} \frac{Fab(l+a)}{EI}$				$M_a = \frac{Fab^2}{l^2}$ $M_b = \frac{Fa^2b}{l^2}$	
	$M_M = \frac{1}{8} ql^2$	$\varphi = \frac{1}{24} \frac{ql^3}{EI}$	$\delta_M = \frac{5}{384} \frac{ql^4}{EI}$			$M = \frac{1}{12} ql^2$ $M_M = \frac{1}{24} ql^2$	$\delta_{mM} = \frac{1}{384} \frac{ql^4}{EI}$

Area+point of gravity

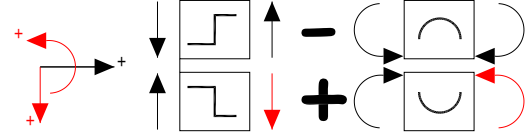
	$A = bh$ $z = \frac{1}{2}b$ $I = \frac{1}{12}bh^3$
	$A = \frac{1}{2}bh$ $z = \frac{1}{3}b$ $I = \frac{1}{36}bh^3$
	$A = \pi r^2$ $I = \frac{1}{4}\pi r^4$
	$A = \frac{1}{2}(a+c)h$ $z = \frac{1}{3}(2a+c)$
	$A = \frac{1}{2}b(h_1+h_2)$ $z = \frac{1}{3}b \frac{h_1+2h_2}{h_1+h_2}$
	$A = \frac{1}{3}bh$ $z = \frac{1}{4}b$
	$A = \frac{1}{n+1}bh$ $z = \frac{1}{n+2}b$
	$A = \frac{2}{3}bh$ $z = \frac{3}{8}b$
	$A = \frac{n}{n+1}bh$ $z = \frac{1}{2} \frac{(n+1)}{(n+2)}b$
	$A = \frac{2}{3}bh$ $z = \frac{1}{2}b$

Tangent=parallel to x-axis

Section properties

$A =$	$S_z =$	$I_{zz} =$
$\int_A dA$	$\int_A z dA$	$\int_A z^2 dA$

Sign conventions



Displacement rule of Steiner

$$I_{zz} = I_{zz(eigen)} + \bar{z}_c^2 A$$

Basic equation for bending

$$Q(x) = -EIw''''$$

$$V(x) = -EIw'''$$

$$M(x) = -EIw''$$

$$EI\varphi(x) = -EIw'$$

$$\Delta\varphi = \varphi_B - \varphi_A = \int_A^B \frac{M}{EI} dx$$

$$\Delta w = w_B - w_A = -\varphi_{A(x_B-x_A)} - \int_A^B \frac{M}{EI} (x_B - x) dx$$

Normal tension

$$\sigma(z) = \sigma_{N(z)} + \sigma_{M(z)} = \frac{N}{A} + \frac{M \cdot z}{I}$$

Extension by axial force

$$EA_x u'_x = N_x = \int -q_x dx$$

$$EA_x u_x = \int N_x dx = \iint -q_x d^2x$$

Prismatic bar & constant force

$$\Delta l = \frac{Nl}{EA}$$

Unit load method

$$\Delta = \sum \frac{F_u \cdot F_l \cdot l}{E \cdot A}$$

Shear stress

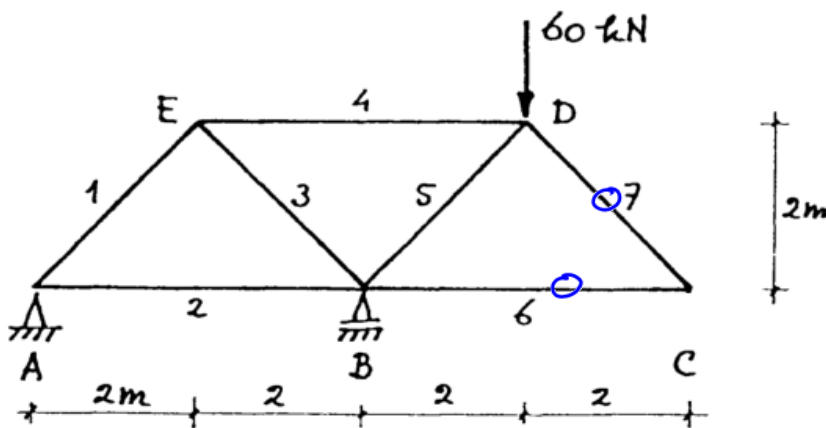
$$s_x^a = -\frac{V_z S_z^a}{I_{zz}} \quad \tau = -\frac{V_z S_z^a}{I_{zz} b^a}$$

Combinend stresses

$$\sigma_{x1} = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi - \tau_{zx} \sin 2\varphi$$

$$\sigma_{z1} = \frac{\sigma_x + \sigma_z}{2} - \frac{\sigma_x - \sigma_z}{2} \cos 2\varphi + \tau_{zx} \sin 2\varphi$$

$$\tau_{x1z1} = \tau_{xz} \cos 2\varphi + \frac{\sigma_x - \sigma_z}{2} \sin 2\varphi$$

Question:

- Determine internal forces in all members.

Key activities:

- per node $\rightarrow$ equilibrium.
- locate zero force members.
- Determine reaction forces (Equilibrium)

SOLUTION:Reaction forces:

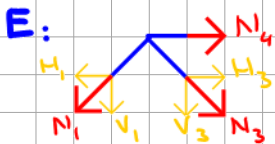
$$\begin{aligned}\sum H = 0 & \quad H_A = 0 \\ \sum M_A = 0 & \quad -4 \cdot V_B + 6 \cdot 60 = 0 \quad V_B = \frac{360}{4} = 90 \text{ kN} \\ \sum V = 0 & \quad -V_A - V_B + 60 = 0 \\ & \quad -V_A = V_B - 60 = 90 - 60 = 30 \\ & \quad V_A = -30 \text{ kN}\end{aligned}$$

Zero force members:

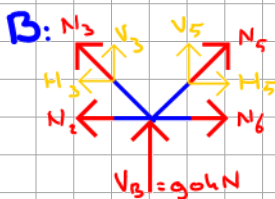
bar 6 & 7

Nodes:

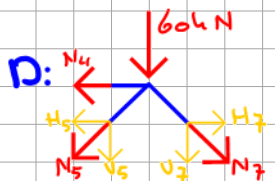
$$\begin{aligned}V_1 : H_1 &= 1:1 \Rightarrow H_1 = V_1 \\ \sum V = 0 & \quad V_A - V_1 = 0 \Rightarrow V_1 = V_A = 30 \text{ kN} \\ & \Rightarrow H_1 = 30 \text{ kN} \\ N_1 &= \sqrt{H_1^2 + V_1^2} = \sqrt{30^2 + 30^2} = 30\sqrt{2} \approx 42.5 \text{ kN} \\ \sum H = 0 & \quad H_1 + N_2 = 0 \Rightarrow N_2 = -H_1 = -30 \text{ kN}\end{aligned}$$



$$\begin{aligned}V_3 : H_3 &= 1:1 \Rightarrow H_3 = V_3 \\ \sum V = 0 & \quad V_1 + V_3 = 0 \Rightarrow V_3 = -V_1 = -30 \text{ kN} \\ & \Rightarrow H_3 = -30 \text{ kN} \\ N_3 &= \sqrt{H_3^2 + V_3^2} = \sqrt{30^2 + 30^2} = 30\sqrt{2} \approx 42.5 \text{ kN} \\ \sum H = 0 & \quad -H_1 + H_3 + N_4 = 0 \Rightarrow N_4 = H_1 - H_3 = 30 - (-30) = 60 \text{ kN}\end{aligned}$$



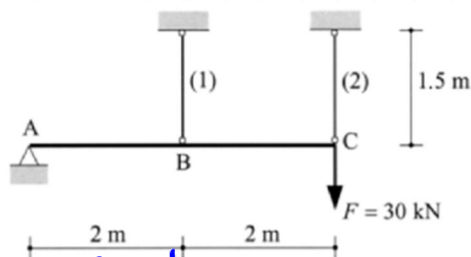
$$\begin{aligned}V_5 : H_5 &= 1:1 \Rightarrow H_5 = V_5 \\ \sum V = 0 & \quad -V_3 - V_5 - V_B = 0 \Rightarrow V_5 = -V_3 - V_B = -(-30) - 90 = -60 \text{ kN} \\ & \Rightarrow H_5 = -60 \text{ kN} \\ N_5 &= \sqrt{H_5^2 + V_5^2} = \sqrt{60^2 + 60^2} = 60\sqrt{2} \approx 84.9 \text{ kN} \\ \sum H = 0 & \quad -N_2 - H_3 + H_5 + N_6 = 0 \\ & \Rightarrow -(-30) - (-30) + (-60) + 0 = 0 \quad \text{correct.}\end{aligned}$$



$$\begin{aligned}\sum V = 0 & \quad V_5 + 60 + V_7 = 0 \Rightarrow -60 + 60 + V_7 = 0 \Rightarrow V_7 = 0 \\ \sum H = 0 & \quad -N_4 - H_5 + H_7 = 0 \Rightarrow -(60) - (-60) + H_7 = 0 \Rightarrow H_7 = 0 \\ & \quad N_7 = \text{zero force member}\end{aligned}$$

Final Answers

$$\begin{array}{lll} N_1 = +42,5 \text{ kN} & N_{11} = +60 \text{ kN} & N_7 = 0 \\ N_2 = -30 \text{ kN} & N_5 = -84,9 \text{ kN} & \\ N_3 = -42,5 \text{ kN} & N_6 = 0 & \end{array}$$

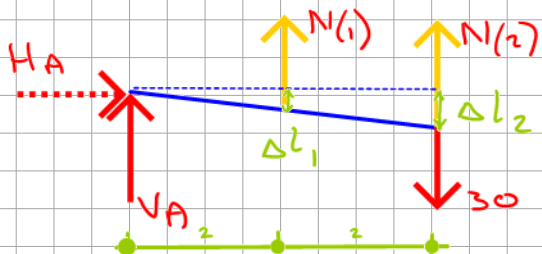
Questions:

- determine strain & forces in the bars
- determine reaction forces at A.

Given: $EA_1 = 3000 \text{ kN}$
 $EA_2 = 1500 \text{ kN}$

Key activities

- sketch displacement due load.
- determine forces in (1) & (2) $\Rightarrow \Delta L = \frac{NL}{EA}$ & $\sum H=0, \sum V=0, \sum M=0$
- determine reaction force in A using Equilibrium rules

ModelSolving

$$\Delta L_1 : \Delta L_2 = 2 : 4 = 1 : 2 \Rightarrow \Delta L_2 = 2 \Delta L_1$$

$$N(1) = \frac{\Delta L_1}{L} * EA_1$$

$$N(2) = \frac{\Delta L_2}{L} * EA_2 = \frac{2 \Delta L_1}{L} * EA_2$$

$$\sum M_A = 0 \quad 2 * N(1) + 4 * N(2) - 4 * 30 = 0 \Rightarrow 2 * \frac{\Delta L_1}{L} * EA_1 + 4 * \frac{2 \Delta L_1}{L} * EA_2 - 4 * 30 = 0$$

$$2 \Delta L_1 * \frac{3000}{1.5} + 4 * 2 \Delta L_1 * \frac{1500}{1.5} - 120 = 0$$

$$12000 \Delta L_1 - 120 = 0$$

$$\Delta L_1 = \frac{120}{12000} = 0.01 = 10 \text{ mm}$$

$$\Delta L_2 = 2 * \Delta L_1 = 2 * 10 = 20 \text{ mm}$$

$$\text{strain: } \epsilon_1 = \frac{\Delta L_1}{L_1} = \frac{10}{1500} = \frac{1}{150} \approx 0.00667$$

$$\epsilon_2 = \frac{\Delta L_2}{L_2} = \frac{20}{1500} = \frac{2}{150} \approx 0.01333$$

$$\text{force: } N(1) = \frac{\Delta L_1}{L} * EA_1 = \frac{1}{150} * 3000 = 20 \text{ kN}$$

$$N(2) = \frac{\Delta L_2}{L} * EA_2 = \frac{2}{150} * 1500 = 20 \text{ kN}$$

Reaction forces:

$$\sum V = 0 \quad -V_A - N(1) - N(2) + 30 = 0$$

$$-V_A - 20 - 20 + 30 = 0$$

$$-V_A - 10 = 0$$

$$V_A = -10 \text{ kN}$$

$$\text{check: } \sum M_A = 0 \quad +2 * 20 + 4 * 20 - 4 * 30 = 0$$

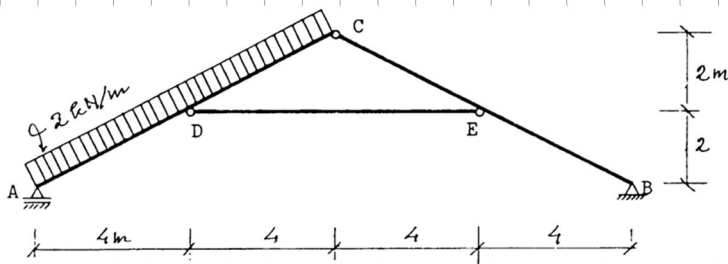
$$\sum H = 0 \quad H_A = 0$$

Answers

a: $\epsilon_1 = 0.00667$
 $\epsilon_2 = 0.01333$

$N(1) = +20 \text{ kN}$
 $N(2) = +20 \text{ kN}$

b: $V_A = -10 \text{ kN} (= 10 \text{ kN downwards}) \quad H_A = 0$

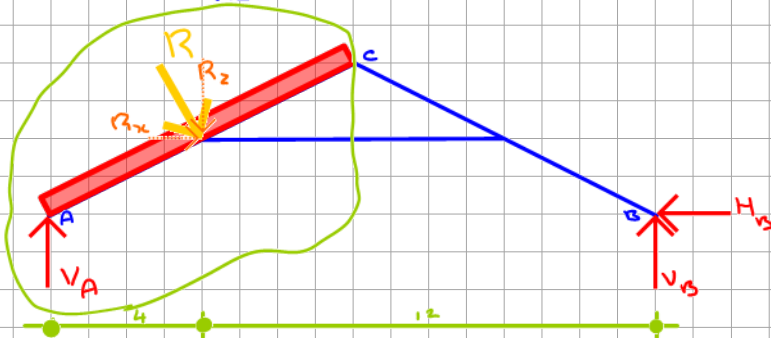
Question:

determine internal force in DE

Keyactivities- divide structure; use $\sum M; \sum H; \sum V = 0$

- bar DE: pandelnummer.

- determine support reactions:

 $\sum M; \sum H; \sum V = 0$ Free Body Diagram:

$$L_{AC} = \sqrt{8^2 + 4^2} = 4\sqrt{5} \approx 8.94\text{m}$$

$$R = 4\sqrt{5} \times 2 = 8\sqrt{5} \approx 17.89\text{kN}$$

$$R_x = \frac{4}{4\sqrt{5}} \times 8\sqrt{5} = 8\text{kN}$$

$$R_z = \frac{8}{4\sqrt{5}} \times 8\sqrt{5} = 16\text{kN}$$

Solution

$$\sum H = 0 \quad R_x - H_B = 0$$

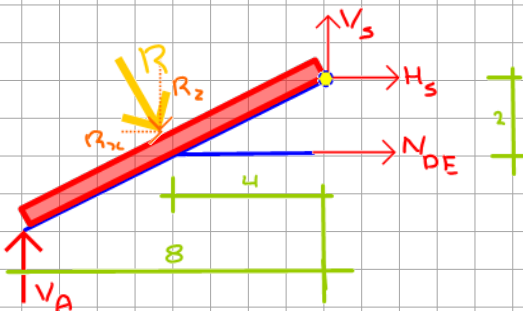
$$H_B = R_x = 8\text{kN}$$

$$\sum M_A = 0 \quad -4 \times R_z - 2 \times R_x + 16 \times V_B = 0$$

$$V_B = \frac{4 \times 16 + 2 \times 8}{16} = 5\text{kN}$$

$$\sum V = 0 \quad -V_A + R_z - V_B = 0$$

$$V_A = 16 - 5 = 11\text{kN}$$



$$\sum M_E = 0$$

$$+2 \times N_{DE} + 4 \times R_z + 2 \times R_x - 8 \times V_A = 0$$

$$N_{DE} = - \frac{(4 \times 16 + 2 \times 8 - 8 \times 11)}{2} = +4\text{kN}$$

Answer: $N_{DE} = +4\text{kN}$