

# VOOR Tentamenbank

## EXAM MATHEMATICAL OPTIMIZATION IN TRANSPORT

201100012-2A

April 14, 2014

13:45-17:15

Room: OH110

The exam consists of 5 problems. Maximum scores are listed in the table below. End score is the total score divided by 10:

**Good luck**

Georg Still and Eric van Berkum

| problem | max score |
|---------|-----------|
| 1       | 16        |
| 2       | 16        |
| 3       | 28        |
| 4       | 20        |
| 5       | 20        |
| total   | 100       |

1. [16 pts]

(a) [8 pts] Show that the function  $z(x_1, x_2) = x_1 \cdot x_2$  is not (strictly) convex.

(b) [8 pts] Now consider the function  $g(x_1, x_2) = x_1^4 + x_2^4 + x_1 x_2$ . Is this function convex? In which region of the  $(x_1, x_2)$ -plane is this function convex? (Make a rough sketch of the region).

2. [16 pts] Consider the unconstrained minimization problem

$$\min_{x \in \mathbb{R}^2} z(x_1, x_2) := \frac{1}{3}x_1^3 + \frac{1}{2}x_2^2 - x_1 x_2 - 2x_1$$

(a) [10 pts] Compute the stationary point(s) ( $\nabla z(x) = 0$ ) and check whether the point(s) are local minimizers.

(b) [6 pts] Is one of the points computed in (a) a global minimizer. Why? (or why not?)

3. [28 pts] Consider the minimization problem:

$$\min z(x_1, x_2) := (x_1 - 1)^2 + (x_2 - 2)^2$$

subject to

$$0 \leq x_1 \leq 2, \quad 0 \leq x_2 \leq 1.$$

- (a) [10 pts] Sketch the feasible set (and some level sets of  $z$ ). Guess the solution  $x^*$  of the problem from the sketch. Give the Kuhn-Tucker optimality conditions for this program.
  - (b) [10 pts] Use the Kuhn-Tucker theorem to find the minimizer (use the fact that at the minimizer only the constraint  $x_2 = 1$  will be active). Why is the point found really a (global) minimizer?
  - (c) [8 pts] From the starting point  $x^0 = (0, 0)$  perform one step of the Frank-Wolfe algorithm. (You may guess the vector  $y^0$  from the sketch in (a)).
4. [20 pts]
- (a) [8 pts] Consider an urban transport network, where at least one segment of an arterial is congested. In order to reduce congestion, the local transport authority considers the costs and benefits of widening this congested segment. Describe how, as a result of this road widening, given an existing travel demand, a new equilibrium between demand and supply will arise.
  - (b) [5 pts] Give the definition of the User Equilibrium and a System Optimum. Explain why in general the flow pattern that minimizes the SO (System Optimum) program is not an UE, user equilibrium, situation. Give an example in which SO and UE are equal.
  - (c) [7 pts] What is Braess's paradox? Give a numerical example.
5. [20 pts] Streets and roadways cannot carry more than a certain amount of vehicular flow known as "capacity."
- (a) [10 pts] Formulate the UE problem assuming that each network link is associated with a capacity  $c_a$
  - (b) [10 pts] Using the ideas outlined in the description of the feasible direction methods a modification to the convex combinations method was developed so that the algorithm would solve the capacity constrained UE problem. Devise a method for finding an initial feasible solution for such an algorithm.