

EXAM AND SOLUTIONS

M8 TRAFFIC FLOW DYNAMICS

202000414

May 18, 2020

13:45-15:45

Room:

The exam consists of 4 problems. Maximum scores are listed in the table below. End score is the total score divided by 5.

Good luck

Eric van Berkum

problem	max score
1	10
2	30
3	30
4	15
bonus	5
total	90

1. [10 points] On a particular road the Greenshields fundamental diagram holds, with a free flow speed of 80 km/h and a jam density of 100 veh/km. Does this determine the capacity with no further assumptions? If so, calculate. If not, why not.

Solution Yes, it does. The critical density for the Greenshields fundamental diagram is halfway the density range, hence at 50 veh/km. The speed at that density is half the free flow speed (linear relationship), being 40 km/h. The capacity hence is $q = ku = 40 * 50 = 2000$ veh/h

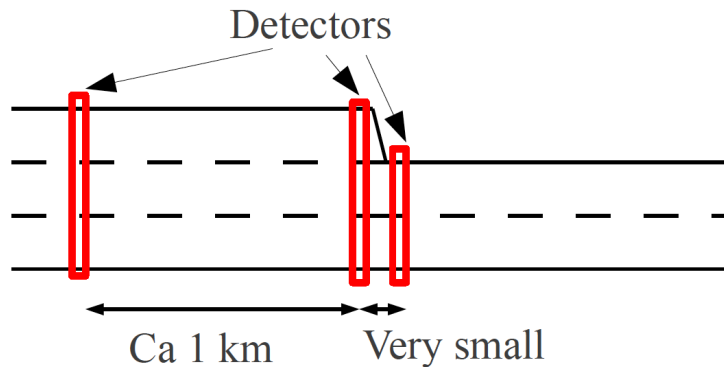


Figure 1: Problem 2: Road layout

2. [30 points] Consider the following road layout as depicted in figure 1, with a lane drop from 3 to 2 lanes; traffic is flowing left to right.

The demand, as well as the flow at the downstream detector, is given in the figure 2. The speed at the detector, located just upstream (you may assume no spacing between the bottleneck and the detector) of the bottleneck, is depicted in figure 3. Assume the fundamental diagram per lane is the same for all lanes, at all locations in this setting.

To answer the questions, you might need values from the graph. Slightly misreading the graph is not a problem, but mention the values you directly read from the graph and where you find these.

- (a) [4 points] In figure 2 with the flow and the demand, which line is the demand and which line the flow. Argue why.

Solution: The red line is the flow. In fact, flow can never be higher than capacity, while demand can be as high as one can imagine. so flow is truncated by capacity, whereas demand is not. When demand exceeds capacity, excess demand must be stored, which is in a queue. In the figure this means that the area where demand is higher than flow equals the area where flow is higher than demand.

- (b) [6 points] Give the free flow capacity (and the reasoning how you find it).

Solution Free flow capacity can be found by the flow at the up-

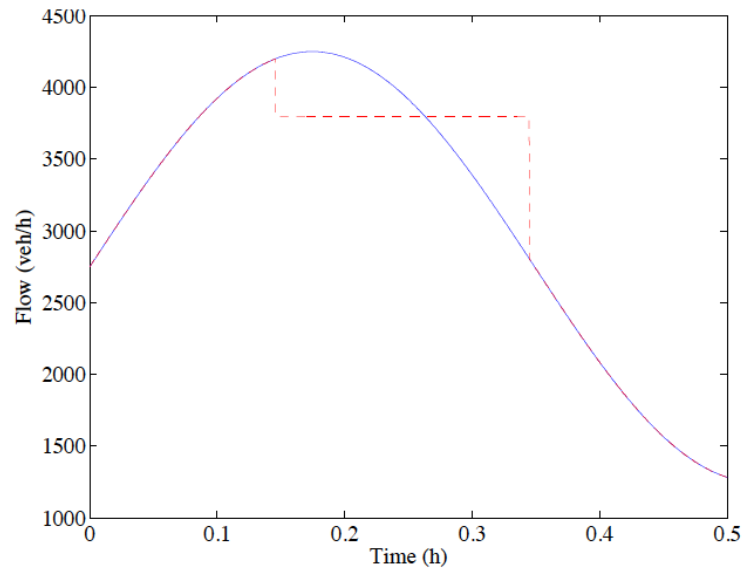


Figure 2: Problem 2: Flow and demand

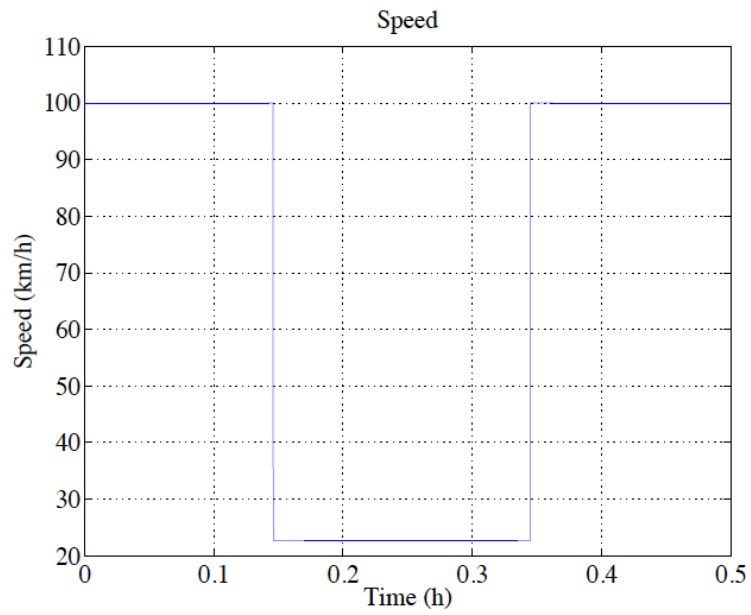
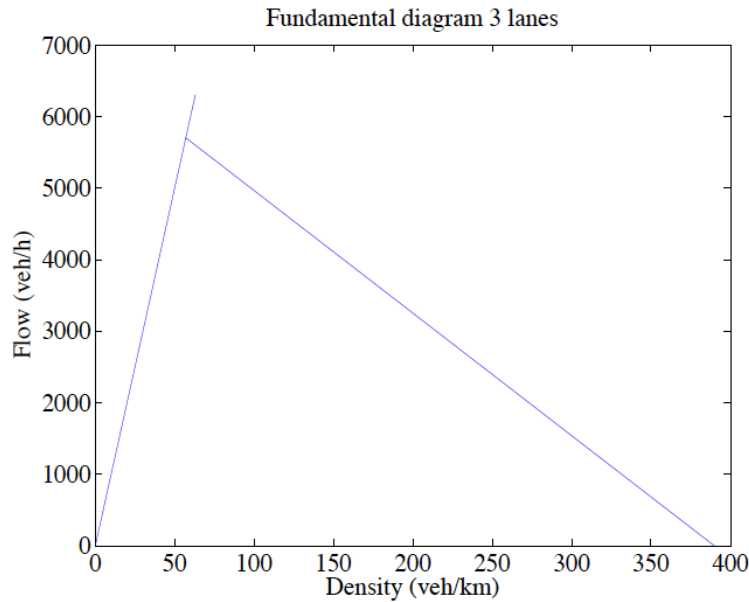


Figure 3: Problem 2: Speed



stream detector before congestion sets in, so 4200 veh/h (read from the graph at the time congestion sets in)

- (c) [6 points] Give the queue discharge rate. (and the reasoning how you find it).

Solution: The queue discharge rate can be found by the flow after congestion has set in, so 3800 veh/h

- (d) [4 points] Explain why can this situation not be described with a triangular fundamental diagram (density-flow).

Solution Free flow capacity is higher than the queue discharge rate, i.e., there is a capacity drop

- (e) [10 points] Draw the simplest fundamental diagram (density-flow) possible for the three lane section (aggregated over all lanes). Explain how you find the values for the relevant points (also provide calculations)

Solution The FD should have a capacity drop (see previous questions), so the simplest is to assume an inverse lambda shaped FD. The free flow capacity is found in question a: 3×2100 veh/h. Also the free flow speed is known (from the speed figure): 100 km/h. This gives the free flow branch. The density matching the queue

outflow rate can be found on the free flow branch, by looking up the density for the queue outflow rate. $k_c = q_c/v_{free} = 1900/100 = 19$ veh/km. (multiply by 3 for the 3-lane section). Furthermore, it is known that with a flow of 2 lanes x 1900 veh/h/lane (the flow through the bottleneck), the speed is 22,6 km/h (read from graph), which gives a density of $3800/22.6=168$ veh/km . Now, the congested branch can be constructed. We find the wave speed $\omega = \Delta q/\Delta k = ((3*1900) - (2*1900))/(3*19 - 168) = -17$ km/h. The jam density is found by $k_j = k_c - q_c/\omega = 3 * (19 - 1900/ - 17.1) = 390$ veh/km. The FD is depicted above.



Figure 4: Problem 3: Road layout

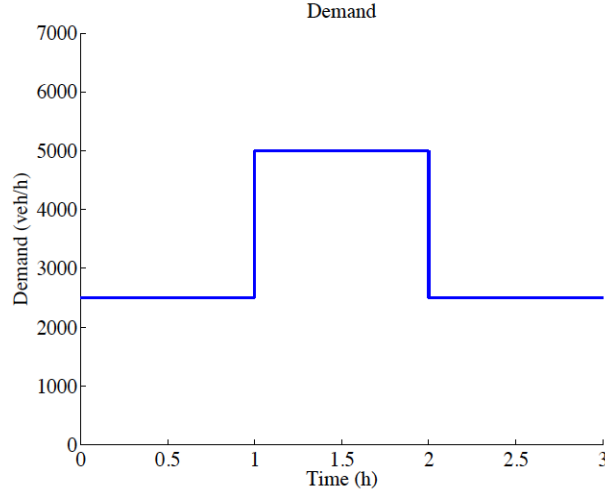
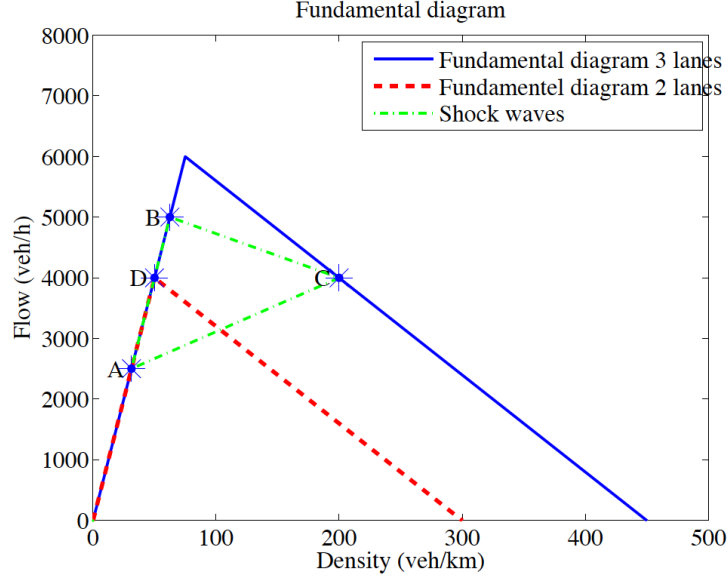


Figure 5: Problem 3: Demand

3. [30 points] Consider a 3 lane road with a reduction to 2 lanes over a 1 km section between $x = 10$ and $x = 12.5$ (figure 4). For the road, we assume lanes with equal characteristics, described by a triangular fundamental diagram with a free speed of 80 km/h, a capacity of 2000 veh/h/lane and a jam density of 150 veh/km/lane. At the start of the road, there is a demand of 2500 veh/h which increases to 5000 veh/h between $t = 1$ h and $t = 2$ h (figure 5). What are the resulting traffic conditions, i.e.

- (a) [15 points] Define all different traffic states, draw the fundamental diagrams (density-flow, so x-axis density, y-axis flow) for both road sections (2 and 3 lanes) into one figure, and depict the states in the diagram.

Solution: The final answer to the question are the traffic states which are shown in table 1, and shown on the fundamental di-



agram in the figure below. We will now explain how this solution can be found. The inflow to the system is given, being 2500 veh/h (state A) and 5000 veh/h (state B) on a three lane road. The matching densities can be computed from the fundamental diagram for a three lane road. This gives densities of $k_A = \frac{q_A}{u_A} = \frac{2500}{80} = 31.25$ veh/km and $k_B = \frac{q_B}{u_B} = \frac{5000}{80} = 62.5$ veh/km. The separation with the empty road moves forward with a speed of 80 km/h, i.e. the speed of the vehicles. This can also be found by the shock wave equation. The difference in flow is 2500 veh/h and the difference in density is 31.25 veh/km. The shock wave then moves with:

$$\omega_{0A} = \frac{q_0 - q_A}{k_0 - k_A} = \frac{-2500}{-31.25} = 80 \text{ km/h} \quad (1)$$

Also the wave between state A and B moves forward with 80 km/h, since

$$\omega_{AB} = \frac{q_A - q_B}{k_A - k_B} = \frac{2500 - 5000}{31.25 - 62.25} = 80 \text{ km/h} \quad (2)$$

Note that this equals the free flow speed. This is because we use a triangular fundamental diagram. The speed ω_{AB} is the direction

of the line segment AB. This has the same slope as the slope of the free flow speed (i.e. the slope of the fundamental diagram at the origin) because its shape is triangular. Now this wave hits the two lane section. The flow is higher than the capacity of the two lane section. That means that downstream, the road will operate at capacity, and upstream a queue will form (i.e. we have a congested state). The capacity of the downstream part is, according to the fundamental diagram, 4000 veh/h (state D). The speed follows from the (triangular) fundamental diagram, and is 80 km/h. The density hence is:

$$k_D = \frac{q_D}{u_D} = \frac{4000}{80} = 50 \text{ veh/h} \quad (3)$$

If 4000 veh/h drive onto the two lane segment, 4000 veh/h have to drive off the three lane segment (no vehicles can be lost or created at the transition from three to two lanes). That means that upstream of the transition, we have a congested state with a flow of 4000 veh/h. The density is derived from the fundamental diagram, specifically the congested branch:

$$q_{cong} = q_{cap} - q_{cap} \frac{k - k_c}{k_j - k_c} = 4000 \text{ veh/h} \quad (4)$$

Substituting the parameters for the fundamental diagram, and realising that the capacity is found by $q_{cap} = l u_{cap} k_c$ (l is the number of lanes) we calculate the density at point C, 200 veh/km. Graphically, we can find point C on the fundamental diagram by the intersection of the congested branch and a line at a constant flow value of 4000 veh/h.

The speed at which the tail of the queue moves backwards, i.e., the speed of the boundary between B and C, is calculated by the shock wave equation

$$\omega_{BC} = \frac{q_B - q_C}{k_B - k_C} = \frac{5000 - 4000}{-62.25 - 200} = -7.3 \text{ km/h} \quad (5)$$

Then the demand reduces again and the inflow state returns to A. When lower demand hits the tail of the jam, the queue can solve from the tail. At the head of the queue, this change has no

influence yet, since drivers are still waiting to get out onto the smaller roadway segment. The boundary between C and A moves with a speed of

$$\omega_{CA} = \frac{q_C - q_A}{k_C - k_A} = \frac{4000 - 2500}{200 - 31.25} = 8.9 \text{ km/h} \quad (6)$$

Then, this wave arrives at the transition of the two lane road to a three lane road, and the congestion state C is dissolved. In the two lane part, the flow is now the same as the demand (the state in A, 2500 veh/h). The boundary between state D and A moves forward with a speed of

$$\omega_{DA} = \frac{q_D - q_A}{k_D - k_A} = \frac{4000 - 2500}{50 - 31.25} = 80 \text{ km/h} \quad (7)$$

So we get

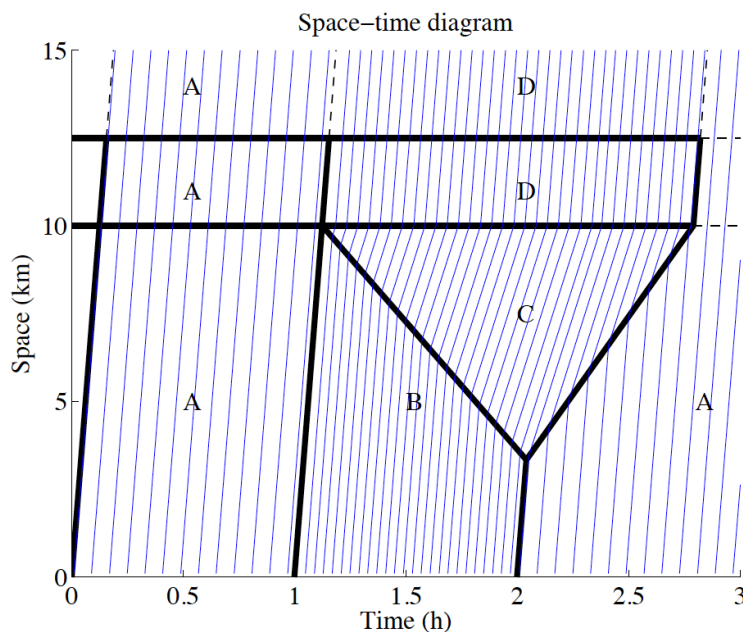
State	Flow	Density	Speed
A	2500	31.25	80
B	5000	62.5	80
C	4000	200	20
D	4000	50	80

- (b) [15 points] Draw a space-time diagram (time on x-axis) with all shockwaves

Solution: The speed of the shock waves is given in table 2, and the resulting traffic situation is shown in the space-time diagram below.

State1	State2	Speed ω
A	B	80
B	C	-7.3
C	A	8.9

and



4. [10 points] The so-called OVM, Optimal Velocity Model, is a car-following model specifying the acceleration a as follows:

$$a = a_0(v^* - v) \quad (8)$$

Where a_0 a reference acceleration (tuneable parameter, constant for a specific vehicle-driver combination), v is the speed of the vehicle, and v^* is determined as follows:

$$v^* = 16.8(\tanh(0.086(\Delta x - 25) + 0.913)) \quad (9)$$

where Δx is the distance headway.

In case you forgot or didn't know, the tanh function is depicted in figure 6

- (a) [4 points] Explain qualitatively the working of this car-following model; i.e. comment on these two equations.

Solution: Vehicles have a desired speed v^* , which is determined by the distance headway. The acceleration is proportional to the difference between their speed and the desired speed.

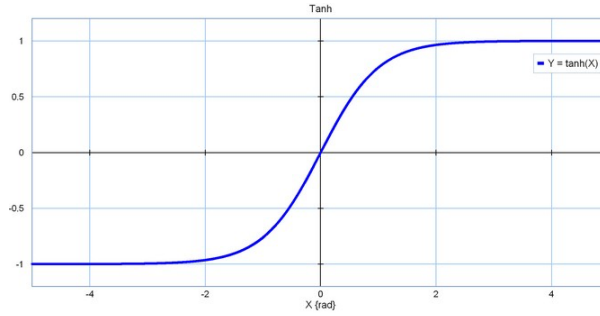


Figure 6: tanh

- (b) [4 points] What are the conditions for which a fundamental diagram holds?

Solution: Traffic must be stationary and homogeneous

- (c) [7 points] Derive the expression of a fundamental diagram (flow as function of density) for these conditions using the OVM model

Solution: If traffic is stationary, all vehicles drive the same speed and acceleration is 0, so $v = v^*$. Since $k = 1/\Delta x$ and correcting for the units (density is generally in veh/km, and distance headway in meters, we find $k = 1000/\Delta x$, leading to:

$$v = 16.8(\tanh(0.086((1/k) - 25) + 0.913)) \quad (10)$$

Since speed is not in m/s but rather in km/h we get

$$v = 3.6168(\tanh(0.086((1/k) - 25) + 0.913)) \quad (11)$$

and using $q = kv$

$$q = k \cdot 3.6168(\tanh(0.086((1/k) - 25) + 0.913)) \quad (12)$$