M8 - Formulas

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- Given a probability density function $f(x)$ and cumulative distribution function $F(x)$, then

$$F(x) = \int_{-\infty}^{x} f(z)dz$$

- Probability density function of a Poisson distribution is

$$f(k) = \frac{\mu^k}{k!}e^{-\mu} \quad k = 0, 1, .., n$$

- Probability density function of a negative exponential distribution with mean $1/q$ is

$$f(h) = qe^{-qh}$$

then cumulative function is:

$$F(h) = 1 - e^{-qh}$$

- Fraction of the time a gap is larger than x is

$$G_1(x) = \frac{\sum h_i | h_i > x}{\sum h_i}$$

- The fraction of time that a restgap is larger than x s, denoted as $G_2(x)$ is

$$G_2(x) = \frac{\sum (h_i - x) | h_i > x}{\sum h_i}$$

- Occupancy β is defined as

$$\beta = \frac{1}{T}\sum_{i=1}^{n}\beta_i$$

Per vehicle with length L_i and speed v_i it holds that $\beta_i = (L_i + L_{loop})/v_i$. Suppose $L_i \approx L$, $\forall i$, and $L_{tot} = L + L_{loop}$, then

$$\beta = \frac{1}{T}\sum_{i=1}^{n}\frac{L + L_{loop}}{v_i} = \frac{L_{tot}}{T}\sum_{i=1}^{n}\frac{1}{v_i} = L_{tot}\frac{q}{u_m} = L_{tot}k$$

- Greenshields assumes a linear relationship between speed and density, i.e.

$$u(k) = u_f\left(1 - \frac{k}{k_j}\right)$$

- Greenberg's model

$$u = u_o \ln(\frac{k_j}{k})$$

where

u	space mean speed
u_o	optimal speed
k_j	jam density
k	density

- Pipes safe distance model is

$$s_i = s(u_i) = s_0 + T_r u_i$$

- Forbes safe distance model is

$$h_i = T_r + \frac{L_i}{u_i(t)}$$

where L_i is vehicle length.

- GM-5 carfollowing model is

$$\ddot{x}_{n+1}(t+\Delta t) = a_{n+1}(t+\Delta t) = \frac{\alpha_{l,m}[u_{n+1}(t+\Delta t)]^m}{[x_n(t) - x_{n+1}(t)]^l}[u_n(t) - u_{n+1}(t)]$$

- shockwave speed is

$$\omega = \frac{\Delta q}{\Delta k}$$

- Little's law

$$N = \lambda W$$

where

N	average number of items in the queueing system
W	average waiting time in the system for an item
λ	average number of items arriving per unit of time

- average waiting time in a queue for a traffic light (no random arrivals)

$$W = \frac{(c-g)^2}{2c(1-\rho)}$$

where $\rho = \lambda/\mu$, g is greentime and c is cycletime

- average waiting time in a queue for a traffic light (Webster)

$$W = \frac{(c-g)^2}{2c(1-\rho)} + \frac{\rho c^2}{2g(\mu g - \lambda c)}$$