

Exam Soil Mechanics

Bachelor 2nd year, CiT/CTW

Module/course code: 202000065-1A: Safety and Risk in Delta's – Soil Mechanics

Date: 7 November 2024

Time: 13.45-15.45 (+25% for students who may use extra time)

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Type of test: Exercises, Open questions.

Allowed aids: Scientific calculator, formula sheet, English-Dutch dictionary.

Attachments: Formula sheet.

Exercise 1 [45 pt]

A layer of clay, 4 m thick, lies between two layers of sand, each 4 m thick. The top of the upper layer of sand is at ground level. The following conditions are given:

- Water table: 2 m below ground level.
- Artesian pressure: The lower layer of sand is under artesian pressure, with a piezometric surface located 4 m above ground level.
- Material properties: saturated unit weight of clay: 20 kN/m³, dry unit weight of sand: 14.5 kN/m³, porosity of sand: 45%, and unit weight of water: 10 kN/m³.

- ?/3.0 1.1 Calculate the saturated unit weight of sand.
- ?/5.0 1.2 Sketch the cross-sectional view of the soil layers as described above, showing the clay and sand layers, the water table, and the piezometric surface.
- ?/20.0 1.3 Calculate the following at the top, bottom, and middle of the clay layer:
- Total vertical stress
 - Pore pressure
 - Effective vertical stress
- ?/9.0 1.4 Create a sketch of the pressure profiles to illustrate the changes in total vertical stress, pore pressure, and effective vertical stress with depth.
- ?/8.0 1.5 Evaluate the safety against soil boiling (heaving) at the bottom of the clay layer. Consider what would happen to the safety if the upper sand layer is removed by excavation.

Solution:

1.1: Calculate the saturated unit weight of sand

Using the dry unit weight and porosity, we can calculate the saturated unit weight of sand with the formula:

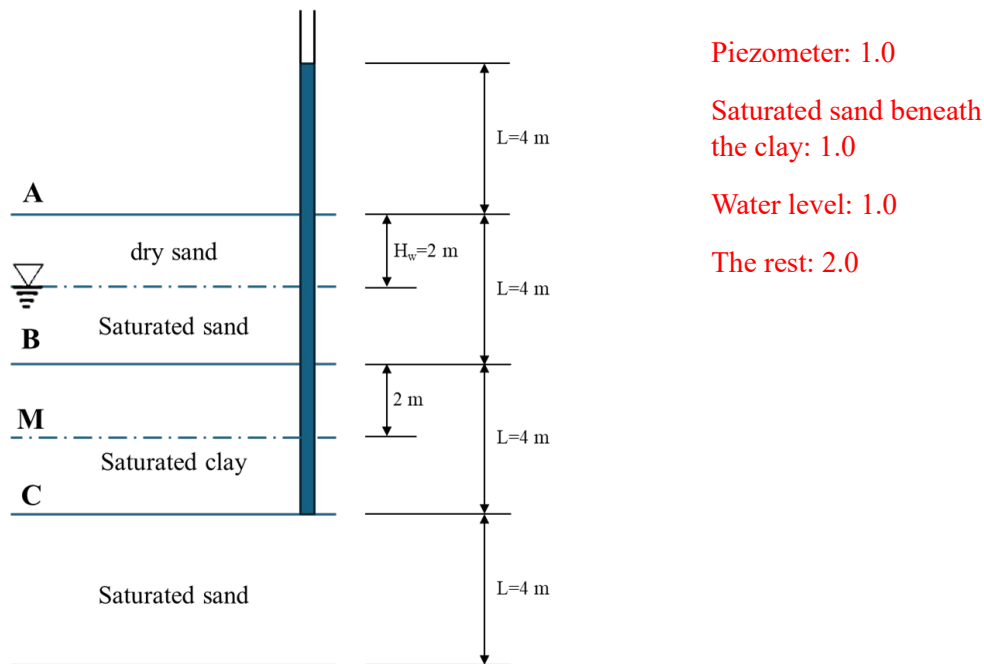
$$\gamma_{sat} = \gamma_d + n * \gamma_w \quad 1.0$$

Substitute the given values:

$$\gamma_{sat} = 14.5 + (0.45 * 10) = 14.5 + 4.5 = 19 \frac{kN}{m^3} \quad 1.0$$

So, the saturated unit weight of sand is 19 kN/m^3 .

1.2: Cross-sectional sketch



1.3: Calculate stresses at top, bottom, and middle of the clay layer

At the top of the clay layer (Point B)

1. Total Vertical Stress:

$$\sigma_B = \gamma_{sand(dry)} * HW + \gamma_{sat} * (L - HW) = (14.5 * 2) + (19 * 2) = 67 \frac{kN}{m^2}$$

2. Pore Pressure:

$$u_B = \gamma_w * (L - HW) = 10 * 2 = 20 \frac{kN}{m^2}$$

3. Effective Vertical Stress:

$$\sigma'_B = \sigma_B - u_B = 67 - 20 = 47 \frac{kN}{m^2}$$

At the bottom of the clay layer (Point C)

$$1. \text{ Total vertical stress: } \sigma_C = \sigma_B + \gamma_{clay} * L = 67 + (20 * 4) = 147 \frac{kN}{m^2}$$

$$2. \text{ Pore pressure: } u_C = h_c * \gamma_w = 12 * 10 = 120 \frac{kN}{m^2}$$

$$3. \text{ Effective vertical stress: } \sigma'_C = \sigma_C - u_C = 147 - 120 = 27 \frac{kN}{m^2}$$

At the middle of the clay layer (Point M)

1. Total vertical stress: $\sigma_M = \sigma_B + \gamma_{clay} * L_{BM} = 67 + (20 * 2) = 107 \frac{kN}{m^2}$ 1.0

2. Pore pressure:

Total head: $h = \frac{u}{\gamma_w} + z$ 1.0

Choosing the bottom of the clay layer (Point C) as datum, the head loss within the clay is

$$\Delta h = \left(\frac{u_C}{\gamma_w} + z_C \right) - \left(\frac{u_B}{\gamma_w} + z_B \right) = \frac{120}{10} - \left(\frac{20}{10} + 4 \right) = 6 \text{ m}$$
1.0

Therefore, the seepage flow is upward and hydraulic gradient $i = \frac{\Delta h}{L} = \frac{6}{4} = 1.5$ 1.0

The total head at Point M is,

$$h_M = h_B + i \times L_{BM} = 6 + 1.5 \times 2 = 9 \text{ m.}$$
1.0

or

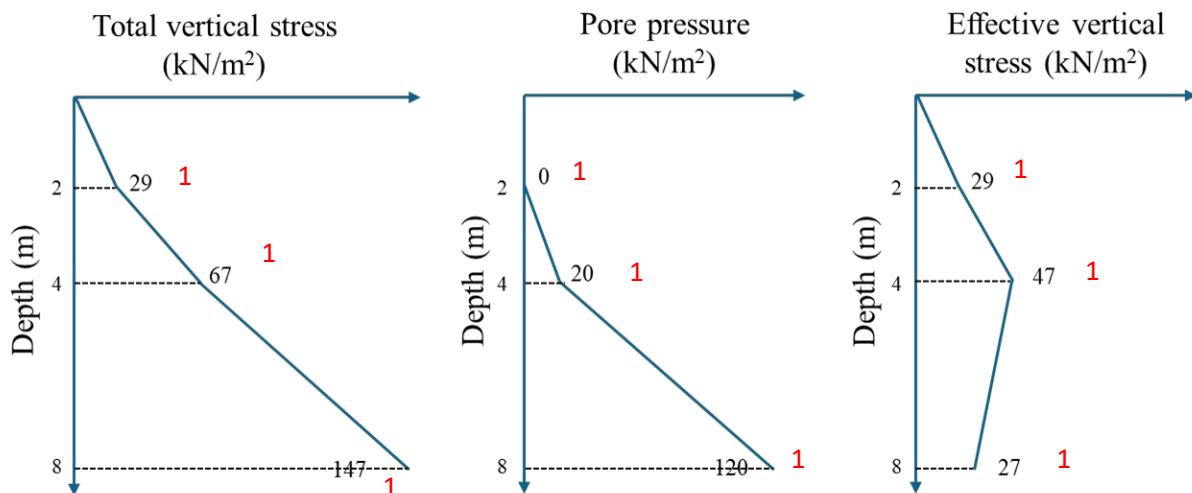
$$h_M = h_C - i \times L_{CM} = 12 - 1.5 \times 2 = 9 \text{ m.}$$

The pore water pressure at Point M:

$$u_M = \gamma_w(h_M - z_M) = 10 \times (9 - 2) = 70 \frac{kN}{m^2}$$
1.0

3. Effective vertical stress: $\sigma'_M = \sigma_M - u_M = 107 - 70 = 37 \frac{kN}{m^2}$ 1.0

1.4: Stress profiles



Each stress point needs to be correctly indicated on the stress and pressure profiles.

1.5: Safety against soil boiling

Effective vertical stress at Point C is $\sigma'_C = 27 \frac{kN}{m^2} > 0$. Therefore, heave will not happen.

1.0
1.0

At the bottom of the clay layer (Point C):

- Total vertical stress after excavation (removal of upper sand layer), taking into account that

the water level would not change:

$$\sigma_c = \underset{1.0}{\gamma_{clay}} * \underset{1.0}{L} + \underset{1.0}{\gamma_w} * (\underset{1.0}{L} - \underset{1.0}{HW}) = 20 * 4 + 10 * 2 = 100 \frac{kN}{m^2}$$

- Pore pressure remains at $u_c = 120 \text{ kN/m}^2$.

- Effective vertical stress after excavation:

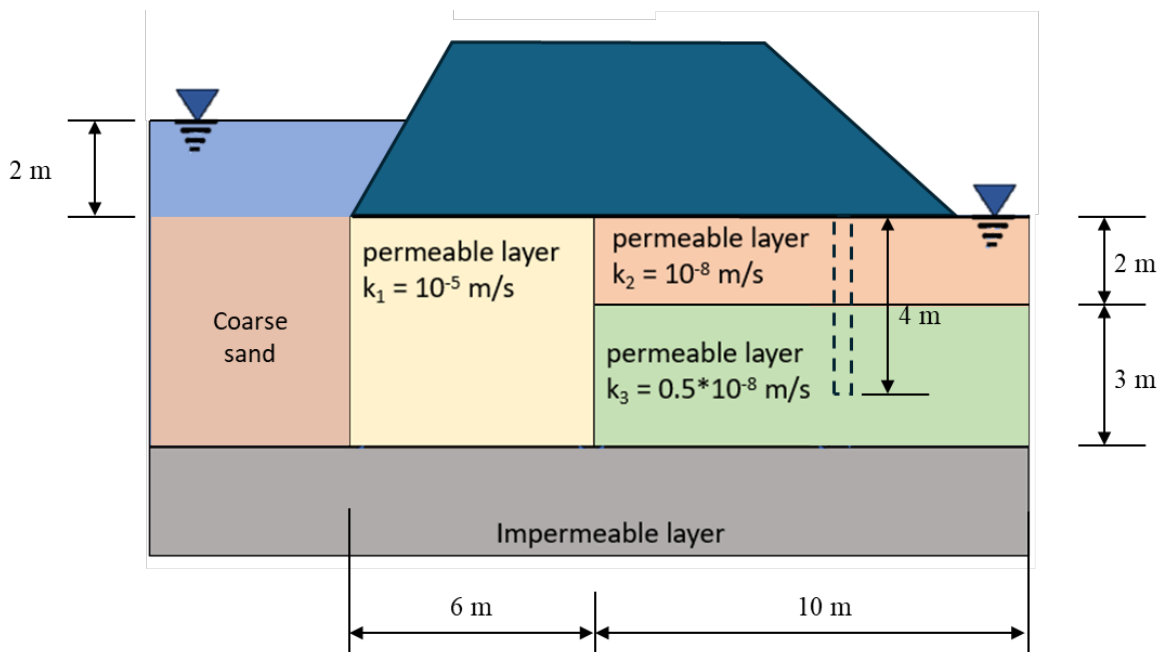
$$\sigma'_c = \sigma_c - u_c = 100 - 120 = -20 \frac{kN}{m^2} \quad 1.0$$

A **negative effective stress** means that the soil would lose stability and **upward heave** would happen under the artesian pressure. 1.0

Exercise 2 [25 pt]

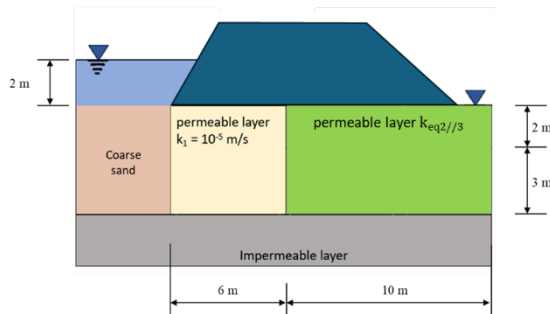
A dike with an impermeable clay core is constructed over three permeable soil layers. The upstream water level is 2 meters higher than the downstream water level. The impermeable clay core prevents water flow directly through the dike, but water can seep through the permeable soil layers below it. The areas on the left side of the clay core consist of highly permeable coarse sand. The geometry and hydraulic properties of the soil layers beneath the dike are provided in the figure below.

- 2.1 The coarse sand layer on the upstream side of the dike is highly permeable. Determine the hydraulic gradient of the seepage flow;
- 2.2 Determine the equivalent permeability of the three soil layers below the dike;
- 2.3 Determine the rate of the seepage flow;
- 2.4 If a 4-meter sheet pile wall is installed from the bottom of the dike to cut through permeable layers 2 and 3, determine the equivalent permeability of the soil adjacent to the sheet pile wall.



2.1 The hydraulic gradient is $i = \frac{\Delta h_{AB}}{L_{AB}} = 0.125$ 3.0

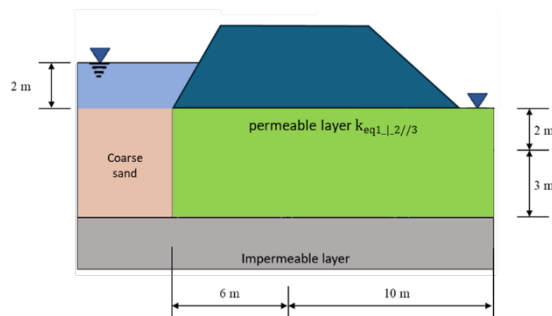
2.2 calculate the equivalent permeability across the 3 permeable layers
 a. we calculate the parallel equivalent permeability ($k_{eq2//3}$) between layer 2 and layer 3. 2.0
 b. we calculate the perpendicular equivalent permeability ($k_{eq1_|_2//3}$) between the layer 1 and the other two layers



$$k_{eq}^{\parallel} = \frac{\sum_i k_i H_i}{\sum_i H_i} \quad 2.0$$

$$k_{eq_23}^{\parallel} = \frac{10^{-8} \cdot 2 + 0.5 \cdot 10^{-8} \cdot 3}{2 + 3} = 0.7 \cdot 10^{-8} \text{ m/s} \quad 2.0$$

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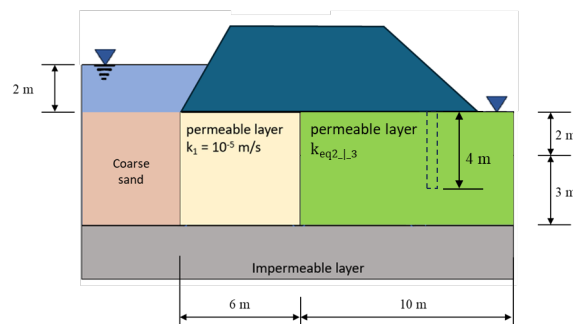


$$k_{eq}^{\perp} = \frac{\sum_i L_i}{\sum_i \frac{L_i}{k_i}} \quad 2.0$$

$$k_{eq_123}^{\perp} = \frac{6 + 10}{\frac{6}{10^{-5}} + \frac{10}{0.7 \cdot 10^{-8}}} = 1.12 \cdot 10^{-8} \text{ m/s} \quad 2.0$$

2.3 the flow rate is: $Q = k_{eq_123} \cdot A \cdot \frac{\Delta h_{AB}}{L_{AB}} = -1.12 \cdot 10^{-8} \frac{\text{m}}{\text{s}} \cdot (16 \text{ m} \cdot 5 \text{ m}) \cdot \frac{2 \text{ m}}{16 \text{ m}} = 1.12 \cdot 10^{-7} \frac{\text{m}^3}{\text{s}}$ 2.0

2.4 The vertical cutoff wall makes the ground water flow vertical next to it. Therefore, we calculate the perpendicular equivalent permeability ($k_{eq2_|_3}$) between layer 2 and layer 3 2.0



$$k_{eq}^{\perp} = \frac{\sum_i L_i}{\sum_i \frac{L_i}{k_i}} \quad 2.0$$

$$k_{eq}^{\perp} = \frac{2 + 2}{\frac{2}{10^{-8}} + \frac{2}{0.5 \cdot 10^{-8}}} = 6.67 \cdot 10^{-9} \text{ m/s} \quad 2.0$$

Exercise 3 [30 pt]

A series of laboratory tests has provided the following shear strength parameters (effective friction angle ϕ' and undrained cohesion c_u) for a particular type of soil:

$$\phi' = 28^\circ, \quad c_u = 35 \text{ kPa}$$

For the following triaxial tests conducted on the same soil sample, perform the requested calculations and sketches:

- 3.1 An unconsolidated undrained triaxial test (UU) where the radial stress at failure $\sigma_r^f = 170 \text{ kPa}$ is measured, determine the total axial stress at failure σ_a^f ,
- 3.2 A consolidated undrained triaxial test (CU) where the deviatoric stress at failure $\Delta\sigma_d = 215 \text{ kPa}$ and the excess pore pressure at failure $(\Delta u_D)_f = 200 \text{ kPa}$ are measured, determine the total axial and radial stresses at failure σ_a^f and σ_r^f and the internal frictional angle for the total stress envelope ϕ ,
- 3.3 Sketch the total and effective stress circles for the soil sample in both tests on a $\sigma - \tau$ plot. Add also the tangent line to each Mohr's circle to represent the failure envelope for each test. Indicate how the strength parameters ϕ , ϕ' , and c_u are obtained on the same plot.

Solution:

3.1: Solve for the UU test (unconsolidated undrained)

For an **unconsolidated undrained triaxial test**, the following relationship holds for the total axial stress at failure σ_a^f : **2.0**

$$\sigma_a^f - \sigma_r^f = 2 \times c_u \quad \text{2.0}$$

Substituting $c_u = 35 \text{ kPa}$ and $\sigma_r^f = 170 \text{ kPa}$:

$$\begin{aligned} \sigma_a^f - 170 &= 2 \times 35 \\ \sigma_a^f &= 170 + 70 = 240 \text{ kPa} \quad \text{2.0} \end{aligned}$$

3.2: Solve for the CU test (consolidated undrained)

2.0
In a **consolidated undrained test**, the analysis uses both total stresses and effective stresses, with excess pore pressure generated during the undrained loading phase. We will first calculate the total axial and radial stresses.

The deviatoric stress in a triaxial test is defined as:

$$\Delta\sigma_d = \sigma_a^f - \sigma_r^f \quad \text{2.0}$$

Therefore,

$$\sigma_a^f - \sigma_r^f = 215 \text{ kPa} \quad 2.0$$

We use the **effective friction angle** ϕ' to relate the effective principal stresses at failure. 2.0

$$\phi' = \sin^{-1} \left(\frac{\sigma_a - \sigma_r}{\sigma_a + \sigma_r - 2(\Delta u_D)_f} \right) \quad 2.0$$

Since $\phi' = 28^\circ$ and $\sin(28^\circ) \approx 0.4695$.

$$0.4695 = \frac{215}{\sigma_a + \sigma_r - 2 \times 200}$$

$$\sigma_a + \sigma_r = \frac{215}{0.4695} + 400 = 857.9 \quad 2.0$$

Therefore, we have:

$$\sigma_a^f + \sigma_r^f = 857.9 \text{ kPa},$$

$$\sigma_a^f - \sigma_r^f = 215 \text{ kPa}, \quad 2.0$$

Resulting in the total axial and radial stresses $\sigma_a^f = 536.45 \text{ kPa}$ and $\sigma_r^f = 321.45 \text{ kPa}$.

In the CU test, the **friction angle for the total stress envelope** ϕ can be calculated using the following relationship:

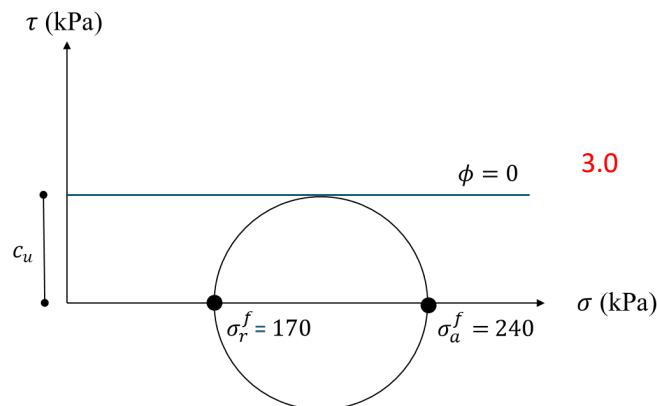
$$\phi = \sin^{-1} \left(\frac{\sigma_a - \sigma_r}{\sigma_a + \sigma_r} \right) \quad 2.0$$

$$\phi = \sin^{-1} \left(\frac{215}{867.9} \right)$$

Therefore, $\phi = \sin^{-1}(0.25) \approx 14.5^\circ$. 2.0

3.3 Total and effective stress circles

For the unconsolidated undrained triaxial test (UU)



For the consolidated undrained triaxial test (CU)

