

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

A

Only for sorting.

Answer form

Calculus 2 – Exam – Friday 13 June, 2025

Full name in BLOCK LETTERS

CORRECTION MODEL

Student number

Programme

☐

BMT

☐

CE

☐

CSE

☐

IEM

☐

Other: _____



FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM



Question 1. The directional derivative is:

4

Question 2(a). $f_{xy}(0,0) =$

3

Question 2(b). Mark your answer with an X.

☐

local maximum

☐

local minimum

☒

saddle point

Question 3. Mark the correct option with an X.

↓ Use this space for corrections. ↓

☐ A

☒ B

☐ C

☐ D

☐ E

Question 4(a). Rewrite the integral using the order of integration $dydx$:

$$\int_2^6 \int_0^{4-(x-4)^2} (y-4) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_0^4 \int_{4-\sqrt{4-y}}^{4+\sqrt{4-y}} (y-4) dx dy$$

Question 5. The area of R is:

$$2\pi + \frac{2}{19}$$

Question 6. Mark all and only the correct options with an X.

↓ Use this space for corrections. ↓

☒ A

☒ B

☐ C

☐ D

☐ E

☐ F

☐ G

☐ H

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

B

Only for sorting.

Answer form

Calculus 2 – Exam – Friday 13 June, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. The directional derivative is:

8

Question 2(a). $f_{xy}(0,0) =$

-3

Question 2(b). Mark your answer with an X.

☐ local maximum ☐ local minimum ☒ saddle point

Question 3. Mark the correct option with an X. ↓ Use this space for corrections. ↓

☒ A ☐ B ☐ C ☐ D ☐ E

Question 4(a). Rewrite the integral using the order of integration $dydx$:

$$\int_3^7 \int_0^{4-(x-5)^2} (y-4) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_0^4 \int_{5-\sqrt{4-y}}^{5+\sqrt{4-y}} (y-4) dx dy$$

Question 5. The area of R is:

$2\pi + \frac{2}{17}$

Question 6. Mark all and only the correct options with an X. ↓ Use this space for corrections. ↓

☐ A ☐ B ☒ C ☒ D ☐ E ☐ F ☐ G ☐ H

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

C

Only for sorting.

Answer form

Calculus 2 – Exam – Friday 13 June, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. The directional derivative is:

-8

Question 2(a). $f_{xy}(0,0) =$

2

Question 2(b). Mark your answer with an X.

☐ local maximum ☐ local minimum ☒ saddle point

Question 3. Mark the correct option with an X. $\downarrow$ Use this space for corrections. $\downarrow$

☐ A ☐ B ☐ C ☒ D ☐ E

Question 4(a). Rewrite the integral using the order of integration $dydx$:

$\int_4^{10} \int_0^{9-(x-7)^2} (y-9) dy dx$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$\int_0^9 \int_{7-\sqrt{9-y}}^{7+\sqrt{9-y}} (y-9) dx dy$

Question 5. The area of R is:

$2\pi + \frac{2}{15}$

Question 6. Mark all and only the correct options with an X. $\downarrow$ Use this space for corrections. $\downarrow$

☐ A ☐ B ☐ C ☐ D ☒ E ☒ F ☐ G ☐ H

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

D

Only for sorting.

Answer form

Calculus 2 - Exam - Friday 13 June, 2025

Full name in BLOCK LETTERS _____

Student number _____

Programme

☐ BMT

☐ CE

☐ CSE

☐ IEM

☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. The directional derivative is:

-4

Question 2(a). $f_{xy}(0,0) =$

-2

Question 2(b). Mark your answer with an X.

☐ local maximum

☐ local minimum

☒ saddle point

Question 3. Mark the correct option with an X.

↓ Use this space for corrections. ↓

☐ A

☐ B

☒ C

☐ D

☐ E

Question 4(a). Rewrite the integral using the order of integration $dydx$:

$$\int_3^9 \int_0^{9-(x-6)^2} (y-9) dy dx$$

Question 4(b). Rewrite the integral using the order of integration $dx dy$:

$$\int_0^9 \int_{6-\sqrt{9-y}}^{6+\sqrt{9-y}} (y-9) dx dy$$

Question 5. The area of R is:

$$2\pi + \frac{2}{13}$$

Question 6. Mark all and only the correct options with an X.

↓ Use this space for corrections. ↓

☐ A

☐ B

☐ C

☐ D

☐ E

☐ F

☒ G

☒ H

Question 7. Write a full calculation/argumentation clearly in the box.

• Interior of D

$$\nabla f(x,y) = \left\langle \frac{2x(x^2+y^2+1) - (x^2-y^2-2) \cdot 2x}{(x^2+y^2+1)^2}, \frac{-2y(x^2+y^2+1) - (x^2-y^2-2) \cdot 2y}{(x^2+y^2+1)^2} \right\rangle$$

$$= \left\langle \frac{2x(2y^2+3)}{(x^2+y^2+1)^2}, -\frac{2y(2x^2-1)}{(x^2+y^2+1)^2} \right\rangle.$$

$$\nabla f(x,y) = \langle 0, 0 \rangle \text{ iff } \begin{cases} 2x(2y^2+3) = 0 \\ 2y(2x^2-1) = 0 \end{cases} \text{ iff } \begin{cases} x = 0 \\ -2y = 0 \end{cases}$$

iff $(x,y) = (0,0)$, the only critical point in the interior of D.

• Boundary of D

* Method 1: parametrize the boundary, e.g. as $\underline{r}(t) = \langle 2\cos t, 2\sin t \rangle$ for $t \in [0, 2\pi]$. Then the restricted function is

$$g(t) = f(\underline{r}(t)) = \frac{4\cos^2 t - 4\sin^2 t - 2}{5} = \frac{4}{5}\cos(2t) - \frac{2}{5}.$$

$g'(t) = -\frac{16}{5}\sin(t)\cos(t)$, so $g'(t) = 0$ iff $t = 0, t = \frac{\pi}{2}, t = \pi, t = \frac{3\pi}{2}$ in the domain, corresponding to the points $(2,0)$, $(0,2)$, $(-2,0)$ and $(0,-2)$.

OR * Method 2: On the boundary,

$$g(x) = f(x,y) \Big|_{x^2+y^2=4} = \frac{x^2-4+x^2-2}{4+1} = \frac{2}{5}x^2 - \frac{6}{5} \text{ for } x \in [-2, 2].$$

$g'(x) = \frac{4}{5}x$, so $g'(x) = 0$ iff $x = 0$, so when $0+y^2=4$, so at the points $(0,2)$ and $(0,-2)$.

Need to check the endpoints of the domain, i.e. when $x = \pm 2$.

In that case $(\pm 2)^2 + y^2 = 4$, so the points are $(2,0)$ and $(-2,0)$.

• Evaluation

$f(0,0) = -2$, $f(\pm 2, 0) = 2/5$, $f(0, \pm 2) = -6/5$, so the maximum is $2/5$ attained at $(2,0)$ and $(-2,0)$, and the minimum is -2 attained at $(0,0)$.

Question 8. Write a full calculation/argumentation clearly in the box.

$$f(x,y) = y^2 - (x-2)^2. \quad \text{Constraint: } g(x,y) = (x-2)^2 + 2(y-3)^2 - 8.$$

• System of equations: $\nabla f = \lambda \nabla g$ and $g=0$:

$$\begin{cases} -2(x-2) = 2\lambda(x-2) \\ 2y = 4\lambda(y-3) \\ (x-2)^2 + 2(y-3)^2 = 8 \end{cases} \quad \text{iff} \quad \begin{cases} (x-2)(\lambda+1) = 0 \quad \text{branch} \\ y = 2\lambda(y-3) \\ (x-2)^2 + 2(y-3)^2 = 8 \end{cases}$$

$$\text{Case 1: } \begin{cases} x-2=0 \\ \dots \\ 2(y-3)^2 = 8 \end{cases} \quad \text{iff} \quad \begin{cases} x=2 \\ \dots \\ y-3 = \pm 2 \end{cases} \quad \text{yielding } (2,1) \text{ and } (2,5) \text{ as critical points.}$$

$$\text{Case 2: } \begin{cases} \lambda = -1 \\ y = -2y + 6 \\ (x-2)^2 + 2(y-3)^2 = 8 \end{cases} \quad \text{iff} \quad \begin{cases} \lambda = -1 \\ y = 2 \\ (x-2)^2 + 2 = 8 \end{cases} \quad \text{iff}$$

$$\begin{cases} \lambda = -1 \\ y = 2 \\ x-2 = \pm\sqrt{6} \end{cases} \quad \text{yielding } (2-\sqrt{6}, 2) \text{ and } (2+\sqrt{6}, 2) \text{ as critical points}$$

• Evaluation

$$f(2,1) = 1, \quad f(2,5) = 25, \quad f(2 \pm \sqrt{6}, 2) = -2,$$

so the maximum is 25 attained at (2,5),
and the minimum is -2 attained at $(2-\sqrt{6}, 2)$ and $(2+\sqrt{6}, 2)$

Question 9. Write a full calculation/argumentation clearly in the box.

• Region: $y=2x$ iff $y/x=2$ i.e. $u=2$
 $y=3x$ iff $y/x=3$ i.e. $u=3$
 $y=-x+4$ iff $x+y=4$ i.e. $v=4$
 $y=-x+6$ iff $x+y=6$ i.e. $v=6$

So the transformed region on the uv -plane is $[2,3] \times [4,6]$.

• Jacobian: $J(u,v) = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} =$
 $= -\frac{v}{(u+1)^2} \left(1 - \frac{1}{u+1}\right) - \frac{1}{u+1} \cdot \frac{v}{(u+1)^2} =$
 $= -\frac{v}{(u+1)^2} + \frac{v}{(u+1)^3} - \frac{v}{(u+1)^3} = -\frac{v}{(u+1)^2}.$

Note that $J(u,v) < 0$ for $u \in [2,3]$ and $v \in [4,6]$, so
 $|J(u,v)| = v/(u+1)^2.$

• Computation: $\iint_R \left(\frac{y}{x} + 1\right)^2 dA = \int_2^3 \int_4^6 (u+1)^2 \cdot |J(u,v)| dv du =$

$$= \int_2^3 \int_4^6 (u+1)^2 \cdot \frac{v}{(u+1)^2} dv du = \int_2^3 \int_4^6 v dv du =$$

$$= \int_2^3 (18-8) du = 10(3-2) = 10.$$

Question 10. Write a full calculation/argumentation clearly in the box.

(a)

$M_{xy} = 0$, because B is symmetric wrt the coordinate plane xy , and $\delta(x, y, -z) = \delta(x, y, z)$.

[Alternatively: $\int_{-1}^1 \int_{-1}^1 \int_{-3}^3 (x^2 z + y^2 z - 2xz - 4yz + 6z) dz dy dx = 0$]

$$M_{yz} = \int_{-1}^1 \int_{-1}^1 \int_{-3}^3 x \delta(x, y, z) dz dy dx = \int_{-1}^1 \int_{-1}^1 \int_{-3}^3 x^3 + xy^2 - 2x^2 - 4xy + 6x dz dy dx$$

$$= (\text{using symmetries}) = \int_{-1}^1 \int_{-1}^1 \int_{-3}^3 -2x^2 dz dy dx =$$

$$= -2 \cdot 6 \cdot 2 \cdot \left[\frac{1}{3} x^3 \right]_{-1}^1 = -2 \cdot 6 \cdot 2 \cdot \left(\frac{1}{3} + \frac{1}{3} \right) = -16.$$

$$(b) \quad \text{avg}(\delta) = \frac{\iiint_B \delta(x, y, z) dV}{\text{vol}(B)} = \frac{M}{2 \cdot 2 \cdot 6} = \frac{160}{24} = 20/3.$$

(c) No: the center of mass of C is the origin, while for B we have $M_{yz} \neq 0$ so $\bar{x} \neq 0$.