

Exam Structural Design - solutions

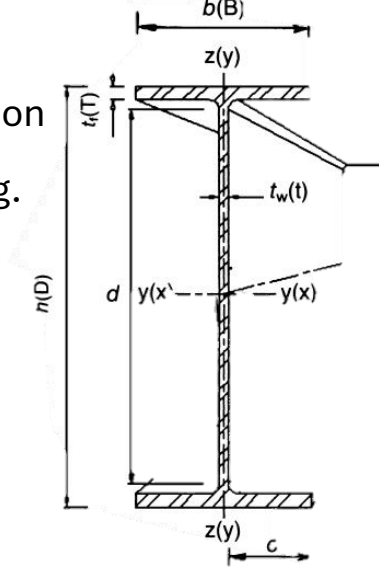
20/06/2025

Exercise 1 (30 points)

A simply supported beam of S275 steel has a $305 \times 165 \times 40 \text{ kg m}^{-1}$ UB section. The beam is fully laterally restrained and sits on 100 mm bearings at each end. The beam has a span length of 7 m, and it is subjected to a permanent load of 7 kN/m and a variable load of 5 kN/m all over its length. Verify the suitability of the beam for bending and deflection under the given loading.

The properties related to the selected sections are: Depth $d = 307.1 \text{ mm}$, Width $b = 165.1 \text{ mm}$, Flange thickness $t_f = 9.9 \text{ mm}$, Web thickness $t_w = 6.9 \text{ mm}$, Second moment of area: $I_x = 13,900 \times 10^3 \text{ mm}^4$, Plastic modulus $W_{pl,y} = 910 \times 10^3 \text{ mm}^3$, Yield strength $f_y = 275 \text{ N/mm}^2$.

Solution



A. Bending suitability (15)

Option 1 – Self weight included in the permanent load of 7kN/m

A1. Design load: (4)

$$w_d = \gamma_G g_k + \gamma_Q q_k = 1,35 * 7 \frac{\text{kN}}{\text{m}} + 1,5 * 5 \frac{\text{kN}}{\text{m}} = 16,95 \frac{\text{kN}}{\text{m}}$$

A2. Design Bending Moment: (4)

$$M_{ED} = \frac{w_d l^2}{8} = \frac{16,95 \frac{\text{kN}}{\text{m}} \cdot 7^2 \text{ m}^2}{8} = 103,82 \text{ kNm} = 1,04 * 10^8 \text{ Nmm}$$

A3. Design Resistance Moment: (4)

$$M_{pl,RD} = \frac{W_{pl,y} f_y}{\gamma_M} = \frac{910 * 10^3 \text{ mm}^3 * 275 \frac{\text{N}}{\text{mm}^2}}{1} = 2,5 * 10^8 \text{ Nmm}$$

A4. Bending verification (3)

$M_{ED} < M_{pl,RD} \rightarrow 1,04 * 10^8 \text{ Nmm} \text{ (or } 1,07 * 10^8 \text{ Nmm)} < 2,5 * 10^8 \text{ Nmm} \rightarrow$ the section is adequate for bending under the assigned load

Option 2 – Adding the Self weight to the permanent load of 7kN/m

A1. Design load: (4)

$$g_{sw} = 40 \frac{\text{kg}}{\text{m}} * 9,81 \frac{\text{m}}{\text{s}^2} = 0,4 \frac{\text{kN}}{\text{m}}$$

$$w_d = \gamma_G g_k + \gamma_Q q_k = 1,35 * (7 + 0,4) \frac{\text{kN}}{\text{m}} + 1,5 * 5 \frac{\text{kN}}{\text{m}} = 17,5 \frac{\text{kN}}{\text{m}}$$

A2. Design Bending Moment: (4)

$$M_{ED} = \frac{w_d l^2}{8} = \frac{17,5 \frac{\text{kN}}{\text{m}} \cdot 7^2 \text{ m}^2}{8} = 107,12 \text{ kNm} = 1,07 * 10^8 \text{ Nmm}$$

A3. Design Resistance Moment: (4)

$$M_{pl,RD} = \frac{W_{pl,y} f_y}{\gamma_M} = \frac{910 * 10^3 \text{ mm}^3 * 275 \frac{\text{N}}{\text{mm}^2}}{1} = 2,5 * 10^8 \text{ Nmm}$$

B. Deflection suitability (15)

B1. Serviceability load: (4)

$$w_s = g_k + q_k = 7 \frac{kN}{m} + 5 \frac{kN}{m} = 12 \frac{kN}{m}$$

B2. Maximum deflection (4)

$$\delta = \frac{5w_s l^4}{384EI_x} = \frac{5 * 12 * 10^3 \frac{N}{mm} (7000mm)^4}{384 * 2.1 * 10^5 \frac{N}{mm^2} * 1,39 * 10^7 mm^4} = 128.5mm$$

B3. limit deflection (4) –

If we assume that the beam has brittle finishes:

$$w_{lim} = \frac{l}{360} = \frac{7000}{360} = 19,4mm$$

If we assume no brittle finishes:

$$w_{lim} = \frac{l}{200} = \frac{7000}{200} = 35 \text{ mm}$$

B4. Verification (3)

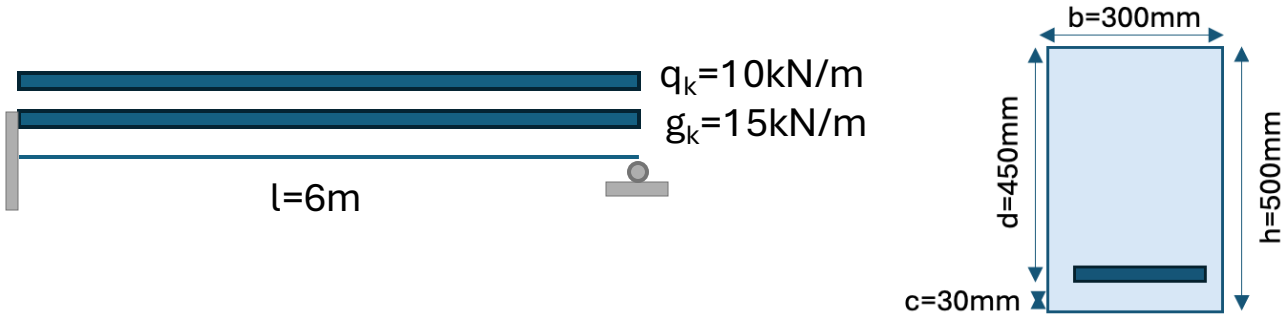
$$\delta_{max} < w_{lim} \rightarrow 128.5mm > 19,4mm \text{ (or } 35 \text{ mm) Not verified!}$$

In either case for assumption on the finishes of the beam, the beam is not verified at deflection.

Based on the load imposed, we should either use a stronger steel, or a thicker section

Exercise 2 (45 points)

A simply supported reinforced concrete beam has a span length of 6m, and it is designed with concrete class C25/30 and reinforcement steel B500. The beam is part of a residential building, and it is subjected to uniformly distributed loads g_k of 15 kN/m and q_k of 10 kN/m. The structure must comply with Eurocode 2. Calculate the needed reinforcement for bending and shear. The details of the section A-A of the beam are described in the figure below.



A. Bending suitability (25)

A1. Design load: (4)

$$w_d = \gamma_G g_k + \gamma_Q q_k = 1,35 * 15 \frac{\text{kN}}{\text{m}} + 1,5 * 10 \frac{\text{kN}}{\text{m}} = 35,25 \frac{\text{kN}}{\text{m}}$$

A5. Area of reinforcement

$$\text{Steel: } A_{s1} = \frac{M_{Ed}}{0,87 f_{yk} z} \quad (3)$$

$$\text{with } z = d \left[0,5 + \sqrt{0,25 - 3K_0/3,4} \right] \text{ and } K_0 = M_{Ed}/f_{ck} b d^2 \quad (1)$$

$$z \approx d \cdot \left(0,5 + \sqrt{0,25 - \frac{3 * 0,104}{3,4}} \right) \approx 403,9 \text{ mm} \quad (1)$$

$$K_0 = \frac{1576,16 \cdot 10^6 \text{ Nmm}}{25 \text{ N/mm}^2 \cdot 300 \text{ mm} \cdot 450^2 \text{ mm}^2} \approx 0,104$$

A2. Design Bending Moment: (3)

$$M_{Ed} = \frac{w_d l^2}{8} = \frac{35,25 \frac{\text{kN}}{\text{m}} \cdot 6^2 \text{ m}^2}{8} = 158,63 \text{ kNm} = 1,59 \cdot 10^8 \text{ Nmm}$$

A3. Design Resistance Moment: (5)

$$M_{RD} = 0,167 * f_{ck} * b * d^2 = 0,167 * 25 * 300 * 450^2 = 253,6 \text{ kNm}$$

A4. Moment verification: (4) $M_{Ed} < M_{RD}$

$$A_{s1} = \frac{Ed}{0,87 \cdot f_{yk} z} \approx \frac{158.625 \cdot 10^6 \text{ Nmm}}{0,87 \cdot 500 \text{ N/mm}^2 \cdot 404 \text{ mm}} \approx 902,94 \text{ mm}^2 \longrightarrow 3\phi 20 \text{ or } 5\phi 16 \quad (2)$$

Perform a spacing verification, and both option fits (2)

$$\text{Spacing} = \max(D_{50}, \phi, 20)$$

$$\text{Space needed} = 3 * 20 \text{ (bars)} + 5 * 20 \text{ (spacing)} = 160 \text{ mm} < 300 \text{ mm}$$

B. \Shear design suitability (20)

B1. Design shear (3-3)

$$V_{Ed} = \frac{w \cdot l}{2} \quad (3)$$

$$= \frac{35.25 \text{ kN/m} \cdot 6 \text{ m}}{2}$$

$$\approx 105.75 \text{ kN}$$

$$v_{Ed} = \frac{V_{Ed}}{b \cdot d} \quad (3)$$

$$= \frac{105.75 \cdot 10^3 \text{ N}}{300 \cdot 450 \text{ mm}^2}$$

$$\approx 0.784 \text{ N/mm}^2$$

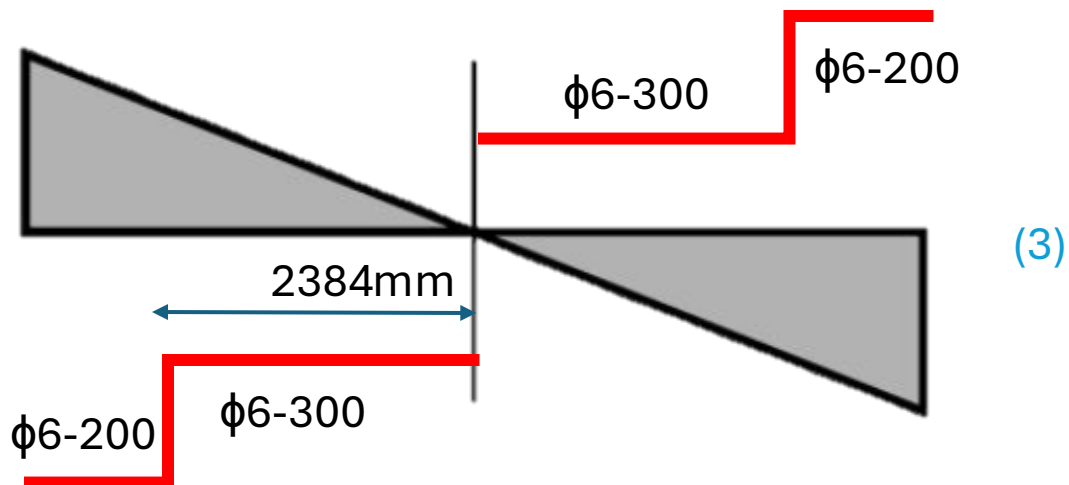
V min and v max from the tables

$$V_{\min} = 0.39 \text{ N/mm}^2 \quad (2)$$

$$V_{\max} = 2.79 \text{ N/mm}^2 \quad (2)$$

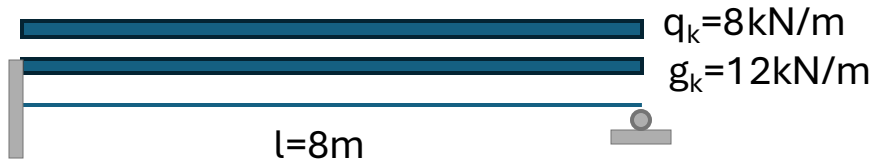
$V_{\min} < V_{Ed} < V_{\max} \rightarrow$ stirrups are needed (2) $\rightarrow$ from table: $\phi 6-200$ ($v = 0.92 \text{ N/mm}^2$) (2.5)

We can choose also $\phi 6-300$ ($v = 0.62 \text{ N/mm}^2$) starting from $x = 2384 \text{ mm}$ (2.5)



Exercise 3 (25 points)

Consider a simply supported reinforced concrete beam in a residential building. The span length of the beam is 8 m and it is subjected to a permanent load estimated to be 12kN/m, and a combined variable load of 8kN/m. The materials used for the beam are concrete class C30/37, and steel B500 for the reinforcement. Assume that the beam is cracked and has a cracked moment of inertia $I_{cr} = 2.86 \times 10^9 \text{ mm}^4$, and an effective Young modulus of $E_{eff} = 11000 \text{ N/mm}^2$. After calculating the serviceability load, by using the psi (ψ_2) from Table 1, calculate the deflection of the beam under quasi-permanent long-term loads of a simply supported reinforced concrete beam and verify if it complies with Eurocode 2 deflection limits.



Serviceability Load

$$\delta_{lim} = \frac{L}{500} = 16 \text{ mm} \quad 5$$

$$w_s = g + \psi_2 q = 12 \text{ kN/m} + 0.3 \cdot 8 \text{ kN/m} = 14.4 \text{ kN/m} \quad 5$$

5 (I give at least 2 if you know it is at SLS)

Immediate Deflection

$$\delta = \frac{5w_s l^4}{384EI} = \frac{(5)(14.4)(\frac{N}{mm})(8000^4)(mm^4)}{(384)(11000)(\frac{N}{mm^2})(2.86 \cdot 10^9)(mm^4)} = 24.4 \text{ mm} \quad 5$$

3 Verification $\rightarrow \delta_{lim} < \delta \rightarrow 16 \text{ mm} < 24.4 \text{ mm} \rightarrow \text{Beam is NOT safe}$