

Grading Scheme : Calculus 1A (24/10/2025)

- ③ Version A : Horizontal asymptote : $y = \frac{7}{3}$
Vertical asymptotes: $x = 1$ and $x = \frac{1}{3}$
- Version B : Horizontal asymptote : $y = \frac{7}{2}$
Vertical asymptotes : $x = 1$ and $x = \frac{1}{2}$
- Version C : Horizontal asymptote : $y = \frac{7}{5}$
Vertical asymptotes : $x = -1$ and $x = \frac{1}{5}$
- Version D : Horizontal asymptote : $y = \frac{7}{2}$
Vertical asymptotes : $x = 2$ and $x = \frac{1}{2}$

- ④ Version A : (a) $-6x + 3y + z = -2$
(b) $\frac{\sqrt{46}}{2}$
(c) $\frac{2}{\sqrt{50}}$ or $\frac{\sqrt{2}}{5}$
- Version B : (a) $2x + 3y - 3z = 2$
(b) $\frac{\sqrt{22}}{2}$
(c) $-\frac{2}{\sqrt{26}}$ or $-\frac{\sqrt{26}}{13}$

Version C : (a) $x - 2y + z = -1$

(b) $\frac{\sqrt{6}}{2}$

(c) $\frac{3}{\sqrt{15}}$ or $\frac{\sqrt{15}}{5}$

Version D : (a) $3x - 2y + 2z = 3$

(b) $\frac{\sqrt{17}}{2}$

(c) $\frac{-3}{\sqrt{26}}$ or $-\frac{3\sqrt{26}}{26}$

⑤

Version A :

$A \rightarrow 1$

$B \rightarrow 3$

$C \rightarrow 4$

$D \rightarrow 2$

Version B :

$A \rightarrow 3$

$B \rightarrow 1$

$C \rightarrow 2$

$D \rightarrow 4$

Version C :

$A \rightarrow 4$

$B \rightarrow 2$

$C \rightarrow 1$

$D \rightarrow 3$

Version D :

$A \rightarrow 2$

$B \rightarrow 4$

$C \rightarrow 3$

$D \rightarrow 1$

$$(9) \quad f(x) = \frac{1}{x^2 + x + 1}$$

We start by finding the domain of f : Since $x^2 + x + 1 > 0$ for all $x \in \mathbb{R}$, we see that $\text{Dom } f = \mathbb{R}$.

Asymptotic behaviour:

$\lim_{x \rightarrow \pm\infty} f(x) = 0$, so $y = 0$ is a horizontal asymptote

$x^2 + x + 1 > 0$ for all $x \in \mathbb{R}$, so f has no vert. asymptote

f is differentiable for all $x \in \mathbb{R}$:

$$f'(x) = \frac{-(2x+1)}{(x^2+x+1)^2}$$

$$f'(x) = 0 \Rightarrow x = -\frac{1}{2} \text{ (critical point)}$$

$$f''(x) = \frac{-2(x^2+x+1)^2 + (2x+1) \cdot 2(x^2+x+1) \cdot (2x+1)}{(x^2+x+1)^4}$$

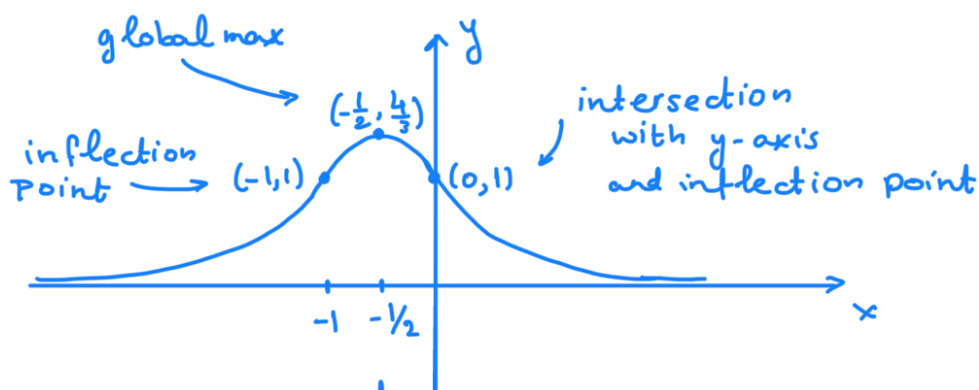
$$f''(x) = 0 \Rightarrow (x^2+x+1) (-2(x^2+x+1) + 2(2x+1)^2) = 0$$

$$x^2+x+1 \neq 0$$

$$\Rightarrow -2x^2 - 2x - 2 + 8x^2 + 8x + 2 = 0$$

$$\Rightarrow 6x^2 + 6x = 0 \Rightarrow x^2 + x = 0$$

$$\Rightarrow x(x+1) = 0 \Rightarrow x = 0 \text{ or } x = -1$$



f'	> 0		< 0
f''	> 0	< 0	> 0

f is increasing for $x \in (-\infty, -\frac{1}{2})$

f is decreasing for $x \in (-\frac{1}{2}, +\infty)$

f is concave up for $x \in (-\infty, -1) \cup (0, +\infty)$

f is concave down for $x \in (-1, 0)$

$$\begin{aligned} \textcircled{10} \quad \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+3} - 2} &= \lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+3} - 2} \cdot \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2} \\ &= \lim_{x \rightarrow 1} \frac{(x^2 - 1)(\sqrt{x+3} + 2)}{x + 3 - 4} = \lim_{x \rightarrow 1} \frac{(x+1)(x-1)(\sqrt{x+3} + 2)}{x - 1} \\ &= \lim_{x \rightarrow 1} (x+1)(\sqrt{x+3} + 2) = 2 \cdot (2+2) = 8 \end{aligned}$$

Alternative approach:

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{\sqrt{x+3} - 2} \xrightarrow{\text{indeterminate form } \frac{0}{0}, \text{ use L'Hôpital}} \lim_{x \rightarrow 1} \frac{2x}{\frac{1}{2\sqrt{x+3}}} = \lim_{x \rightarrow 1} 2x \cdot 2\sqrt{x+3} = 8$$

$$\textcircled{11} \quad f(x) = \sin(x) \cdot e^{-\cos^2(x)}$$

$$(a) \quad f'(x) = \cos(x) \cdot e^{-\cos^2(x)} + \sin(x) e^{-\cos^2(x)} \cdot 2(-\cos(x))(-\sin(x))$$

$$= \cos(x) e^{-\cos^2(x)} (1 + 2\sin^2(x))$$

Since f is differentiable everywhere, its critical points are found by setting $f'(x) = 0$:

$$f'(x) = 0 \Rightarrow \cos(x) e^{-\cos^2(x)} (1 + 2\sin^2(x)) = 0$$

$\Rightarrow \cos(x) = 0 \Rightarrow x = \frac{\pi}{2}$ is the only critical point in the interval $[\frac{\pi}{4}, \pi]$

since $e^{-\cos^2(x)} > 0$
and $1 + 2\sin^2(x) > 0$

(b) Since f is a continuous function on a closed and bounded interval, we have by the Extreme Value Theorem that it attains an absolute max and an absolute min value.

<u>Candidate points</u>	<u>$f(x)$ values</u>
$x = \frac{\pi}{4}$	$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} e^{-(\frac{\sqrt{2}}{2})^2} = \frac{\sqrt{2}}{2} e^{-\frac{1}{2}} = \frac{\sqrt{2}}{2\sqrt{e}} < 1$
$x = \frac{\pi}{2}$	$f(\frac{\pi}{2}) = 1 \cdot e^0 = 1 \rightarrow \text{absolute max}$
$x = \pi$	$f(\pi) = 0 \rightarrow \text{absolute min}$

⑫ $f(x) = -x^3 + 2x$

$y = \frac{5}{4}x - \frac{1}{4}$ tangent line of f at $(a, f(a))$

$$\text{So } f'(a) = \frac{5}{4} \Rightarrow -3a^2 + 2 = \frac{5}{4} \Rightarrow -3a^2 = -\frac{3}{4} \\ \Rightarrow a^2 = \frac{1}{4} \Rightarrow a = \pm \frac{1}{2}$$

We have to check whether the given tangent line goes through the point $(\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, \frac{7}{8})$ or through the point $(-\frac{1}{2}, f(-\frac{1}{2})) = (-\frac{1}{2}, -\frac{7}{8})$

It can easily be checked that $x = \frac{1}{2}$ and $y = \frac{7}{8}$ do not satisfy the equation $y = \frac{5}{4}x - \frac{1}{4}$.

On the other hand, $x = -\frac{1}{2}$ and $y = -\frac{7}{8}$ satisfy the given tangent line equation.

Hence $a = -\frac{1}{2}$.

(One could of course also compute the tangent lines of f at the points $(\frac{1}{2}, \frac{7}{8})$ and $(-\frac{1}{2}, -\frac{7}{8})$ and reach the same conclusion.)

(13) $f(x) = \ln(x^2 - 4x + 5)$

Taylor polynomial of order 2 of f at $x = 2$:

$$P_2(x) = f(2) + f'(2)(x-2) + \frac{1}{2}f''(2)(x-2)^2.$$

$$f(2) = \ln(4 - 8 + 5) = \ln(1) = 0$$

$$f'(x) = \frac{1}{x^2 - 4x + 5} \cdot (2x - 4)$$

$$f'(2) = 0$$

$$f''(x) = \frac{2(x^2-4x+5) - (2x-4)(2x-4)}{(x^2-4x+5)^2}$$

$$f''(2) = 2$$

$$\text{Hence, } P_2(x) = (x-2)^2$$