

Exam Structure Mechanics2

Bachelor Civil Engineering 1st year

Module code/name: 202000060-2B M4 B-CE Designing Construction

Course code/name: 202000061 Structure Mechanics 2

Date: 23 May 2025

Time: 8.45-10.45 (+25% for students who may use extra time)

Instructors: Vanessa Magnanimo

Type of test: Exercises

INSTRUCTIONS & PRACTICALITIES

- All Rules of Order for Testing as established by the examination board apply to this test. In case of inconsistencies with other rules set by the examiner, the Rules of Order for Testing prevail.
- Please read each question carefully before answering the question.
- All paperwork must be handed in at the end of the test, including scrap paper.
- For answering questions, use the working sheets handed to you. If one sheet is not enough, use the sheet with the text "Continuation of ..." in the top right corner.
- The elaboration of the question must be clear. Show what you do, so use a lot of drawings, sketches, or schemas. Make sure to write/draw your answers in a readable form.

ALLOWED RESOURCES

Allowed material to be used during the exam:

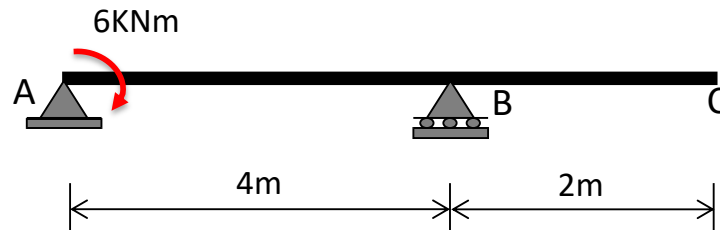
- Scientific calculator (not a communication device)
- Formula sheet
- English-Dutch dictionary
- Pen or/and pencil, coloured pens
- Ruler

Attachments: Formula sheet, Forget-me-Not

Exercise 1 (25 pt)

The beam in the figure is supported by pin supports in *A* and *B*. An external moment is applied in *A*. Dimensions are reported in the figure. EI is constant.

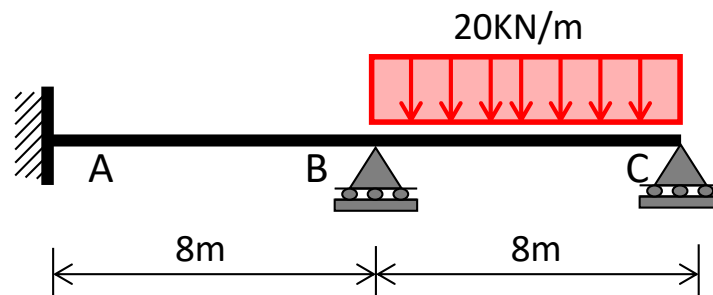
- Derive the equation of shear force, bending moment, rotation, vertical displacement. In your calculation state explicitly the boundary conditions applied.
- Plot the diagrams of shear force, bending moment and displacement.
- Calculate the rotation in *A* and the vertical displacement in *C*, in terms of EI .



Exercise 2 (35 pt)

The statically indeterminate beam in the figure is supported by a fixed support in *A* and rollers in *B* and *C*. Take $E=200\text{ GPa}$ and $I=100\times 10^6\text{ mm}^4$.

- Determine the reaction forces/moments in *A*, *B* and *C*, using the method of forces.
- Sketch the diagrams of shear force and bending moment.



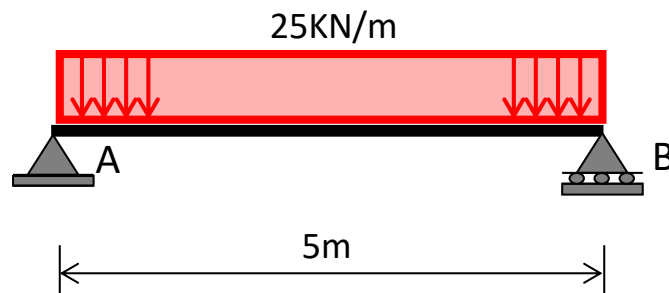
Exercise 3 (40 pt)

The beam is subjected to the 25 kN/m distributed load. The cross section of the beam is rectangular with dimensions $b \times h = 250\text{ mm} \times 400\text{ mm}$.

- Identify the most critical point(s) of the beam.
- Sketch the distribution of normal and tangential stress in the cross-sections where the shear load is maximum (positive).
- Determine the principal stresses and principal stress directions in the middle of the cross-section. Check the results using Mohr's circle graphical procedure.

Now consider the case where a horizontal compressional load $P = 100\text{ kN}$ is applied to the beam.

- Repeat the calculations as before:
 - Sketch the distribution of normal and tangential stress in the cross-sections where the shear load is maximum (positive).
 - Determine the principal stresses and principal stress directions in the middle of the cross-section. Check the results using Mohr's circle graphical procedure.
- Compare the values of principal stresses and the principal stress directions. Comment on the effect of the compressional load on those.



Formula Sheet Mechanics

Axial load

$$\sigma = \frac{P}{A} \quad \varepsilon = \frac{\Delta l}{l} \quad \sigma = E\varepsilon \Rightarrow \Delta l = \frac{Pl}{EA}$$

Area moment of Inertia

$$I_x = \int_A y^2 dA \quad I_y = \int_A x^2 dA$$

$$I_x = I_{x_{zw.p.}} + Ad_y^2 \quad I_y = I_{y_{zw.p.}} + Ad_x^2$$

Rectangular sections:

$$I_{x_{zw.p.}} = \frac{1}{12}bh^3 \quad I_{x_{basis}} = \frac{1}{3}bh^3$$

Bending

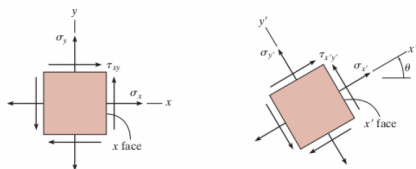
$$\sigma = -\frac{My}{I} \quad \sigma_{max} = \pm \frac{Mh/2}{I}$$

Shear

$$\tau_{avg} = \frac{V}{A} \quad \tau = \frac{VQ}{It} \quad Q = \int_A y dA$$

$$\text{Rectangular sections} \quad \tau_{max} = 1.5 \frac{V}{A}$$

Stress transformations



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2(\theta + 90) + \tau_{xy} \sin 2(\theta + 90)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \sigma_{avg} \pm R \quad \tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\tau_{max} = \pm R \quad \tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

Relations between q, V, M

$$V(x) = \int q(x) dx, \quad M(x) = \int V(x) dx$$

Elastic curve

$$\theta(x) = \frac{1}{EI} \int M(x) dx$$

$$v(x) = \int \theta(x) dx = \frac{1}{EI} \iint M(x) dx$$

Cross method

Stiffness factors:

$$\text{pin-end } K = 3EI/l$$

$$\text{fixed-end } K = 4EI/l$$

Distribution factor:

$$\mu_i = \frac{K_i}{\sum K}, M_i = \frac{K_i}{\sum K} M$$

Carry-over factors:

$$\text{pin-end } M_{end} = 0$$

$$\text{fixed-end } M_{end} = 1/2 M_j$$

Concrete Design

Safety factor loads (CC2):

$$\text{Permanent: } \gamma_G = 1,35 \quad \text{Variable: } \gamma_Q = 1,5$$

$$\text{Serviceability: } \gamma_G = \gamma_Q = 1$$

Safety factor strength:

$$\text{Concrete: } \gamma_c = 1,5$$

$$\text{Steel: } \gamma_s = 1,15$$

Bending:

$$\text{Moment of resistance: } M_{Rd} = 0.167 f_{ck} b d^2$$

single reinforcement (or tables):

$$\text{Steel: } A_{s1} = \frac{M_{Ed}}{0.87 f_{yk} z}$$

$$\text{with } z = d[0.5 + \sqrt{0.25 - 3K_0/3.4}] \text{ and } K_0 = M_{Ed}/f_{ck} b d^2$$

double reinforcement [condition $d_2/x \leq 0.38$]

$$\text{Steel: } A_{s2} = \frac{M_{Ed} - M_{Rd}}{0.87 f_{yk} (d - d_2)}, \quad A_{s1} = \frac{M_{Rd}}{0.87 f_{yk} z} + A_{s2}$$

$$\text{with } z = d[0.5 + \sqrt{0.25 - 3K'_0/3.4}], \quad K'_0 = 0.167, \quad x = (d - z)/0.4$$

Spacing > max(ϕ , dgrain+5, 20)

Shear:

Shear resistance:

$$v_{Rd,c} = 0.035 k^{3/2} \sqrt{f_{ck}}, \text{ with } k = 1 + \sqrt{200/d}$$

$$v_{Rd,max} = \frac{z}{d} v_1 f_{cd} / (\cot \theta + \tan \theta), \text{ with } v_1 = 0.6 \left[1 - \frac{f_{ck}}{250} \right]$$

$$\text{Steel: } v_{Rd,s} = 0.9 A_{sw} f_{yd} / b s \tan \theta \text{ (or tables)}$$

Anchoring:

$$l_{bd} = \alpha_1 l_{b,rqd} > l_{b,min}, \quad l_{b,rqd} = \frac{\phi \sigma_{sd}}{4 f_{bd}}$$

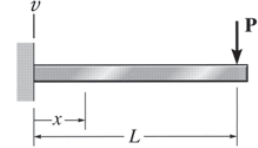
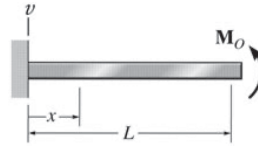
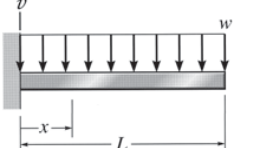
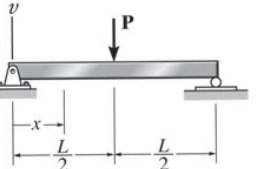
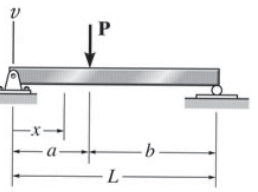
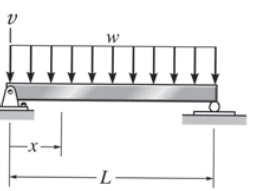
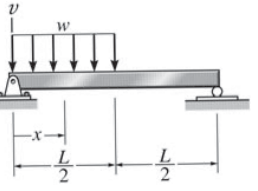
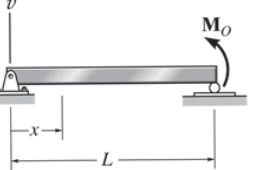
$$\text{(or tables)} \quad l_{b,min} = \max(0.3 l_{b,rqd}; 10\phi; 100)$$

Cover: $c = c_{min} + \Delta c_{dev}$

Bending:

$$\text{Ultimate: } v_{max} < l/250 \quad \text{Serviceability: } v_{max} < l/500$$

Beam Deflections and Slopes

Loading	$v + \uparrow$	$\theta + \curvearrowright$	Equation $+ \uparrow + \curvearrowright$
	$v_{\max} = -\frac{PL^3}{3EI}$ at $x = L$	$\theta_{\max} = -\frac{PL^2}{2EI}$ at $x = L$	$v = \frac{P}{6EI}(x^3 - 3Lx^2)$
	$v_{\max} = \frac{M_O L^2}{2EI}$ at $x = L$	$\theta_{\max} = \frac{M_O L}{EI}$ at $x = L$	$v = \frac{M_O}{2EI}x^2$
	$v_{\max} = -\frac{wL^4}{8EI}$ at $x = L$	$\theta_{\max} = -\frac{wL^3}{6EI}$ at $x = L$	$v = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
	$v_{\max} = -\frac{PL^3}{48EI}$ at $x = L/2$	$\theta_{\max} = \pm \frac{PL^2}{16EI}$ at $x = 0$ or $x = L$	$v = \frac{P}{48EI}(4x^3 - 3L^2x),$ $0 \leq x \leq L/2$
		$\theta_L = -\frac{Pab(L+b)}{6LEI}$ $\theta = \frac{Pab(L+a)}{6LEI}$	$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $0 \leq x \leq a$
	$v_{\max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta_{\max} = \pm \frac{wL^3}{24EI}$	$v = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
		$\theta_L = -\frac{3wL^3}{128EI}$ $\theta_R = \frac{7wL^3}{384EI}$	$v = -\frac{wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \leq x \leq L/2$ $v = -\frac{wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \leq x \leq L$
	$v_{\max} = -\frac{M_O L^2}{9\sqrt{3}EI}$	$\theta_L = -\frac{M_O L}{6EI}$ $\theta_R = \frac{M_O L}{3EI}$	$v = -\frac{M_O x}{6EIL}(L^2 - x^2)$

Fixed End Moments

$(FEM)_{AB} = \frac{PL}{8}$ $(FEM)_{BA} = \frac{PL}{8}$	$(FEM)'_{AB} = \frac{3PL}{16}$
$(FEM)_{AB} = \frac{Pb^2a}{L^2}$ $(FEM)_{BA} = \frac{Pa^2b}{L^2}$	$(FEM)'_{AB} = \left(\frac{P}{L^2}\right)(b^2a + \frac{a^2b}{2})$
$(FEM)_{AB} = \frac{2PL}{9}$ $(FEM)_{BA} = \frac{2PL}{9}$	$(FEM)'_{AB} = \frac{PL}{3}$
$(FEM)_{AB} = \frac{5PL}{16}$ $(FEM)_{BA} = \frac{5PL}{16}$	$(FEM)'_{AB} = \frac{45PL}{96}$
$(FEM)_{AB} = \frac{wL^2}{12}$ $(FEM)_{BA} = \frac{wL^2}{12}$	$(FEM)'_{AB} = \frac{wL^2}{8}$
$(FEM)_{AB} = \frac{11wL^2}{192}$ $(FEM)_{BA} = \frac{5wL^2}{192}$	$(FEM)'_{AB} = \frac{9wL^2}{128}$
$(FEM)_{AB} = \frac{wL^2}{20}$ $(FEM)_{BA} = \frac{wL^2}{30}$	$(FEM)'_{AB} = \frac{wL^2}{15}$
$(FEM)_{AB} = \frac{5wL^2}{96}$ $(FEM)_{BA} = \frac{5wL^2}{96}$	$(FEM)'_{AB} = \frac{5wL^2}{64}$
$(FEM)_{AB} = \frac{6EI\Delta}{L^2}$ $(FEM)_{BA} = \frac{6EI\Delta}{L^2}$	$(FEM)'_{AB} = \frac{3EI\Delta}{L^2}$