

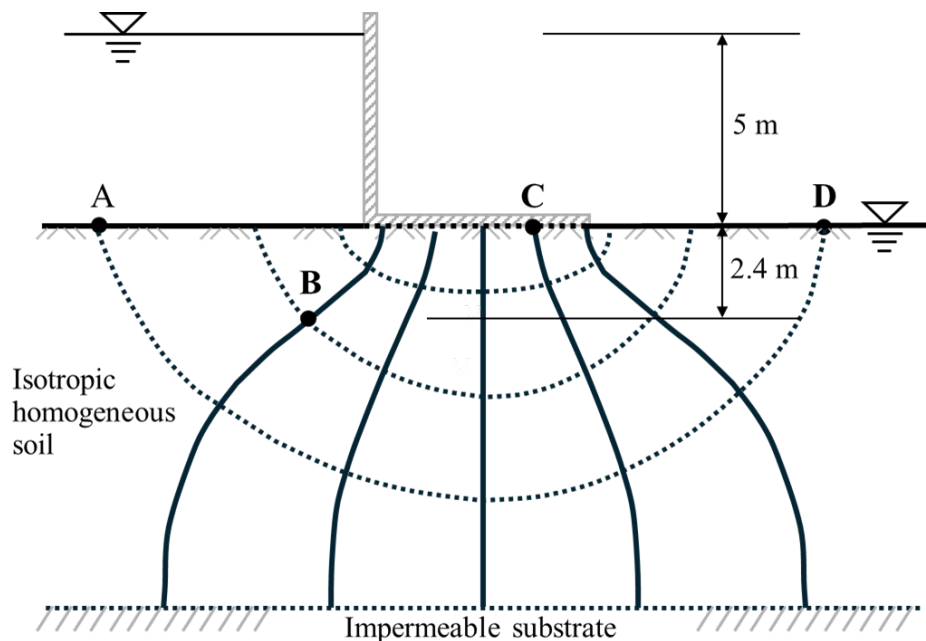
Solutions

Exercise 1 [35pt]

The figure shows the flow net in the foundation soil of a river weir. The soil is homogeneous and isotropic, and its permeability coefficient is $k = 10^{-5}$ m/sec, the unit weights: $\gamma_{dry} = 17 \frac{\text{kN}}{\text{m}^3}$, $\gamma_{sat} = 20,3 \frac{\text{kN}}{\text{m}^3}$, and $\gamma_w = 10 \frac{\text{kN}}{\text{m}^3}$

- ?/5 1. Indicate the equipotential and flow lines on your paper (draw them with different styles, e.g, solid for the equipotential and dotted for the flow lines).
- ?/10 2. Calculate the flow rate of the filtration motion in m^2/day per unit length.
- ?/10 3. Determine the pressure values at points A, B, C, and D.
- ?/10 4. Calculate the effective vertical stress at point B.

1.1



Solid lines and dotted lines indicate the equipotential and flow lines respectively.

3.0 for correct labeling of flow and equipotential lines.

2.0 for drawing them correctly.

1.2

Number of flow channels $N_f = 4$ 1.0

Number of equipotential drops $N_d = 6$ 1.0

The upstream water height $H = 5$ m 1.0

Head loss per equipotential drop $\Delta h = \frac{H}{N_d} = \frac{5}{6}$ 2.0

Flow rate of the infiltration $q = k \Delta h N_f = (10^{-5} \times 60 \times 60 \times 24 \text{ m/day}) \left(\frac{5}{6}\right) (4)$

2.0

1.0

$$\begin{aligned}
 &= (0.864)\left(\frac{5}{6}\right)(4) \\
 &= (0.864)\left(\frac{5}{6}\right)(4) \\
 &= \underline{2.88} \text{ m}^3/\text{day per unit length} \\
 &\quad \underline{2.0}
 \end{aligned}$$

1.3

The head loss per equipotential drop is $\Delta h = \frac{H}{N_d} = \frac{5}{6}$.

At a given point P , the total head $h_P = H - n_{d,P}\Delta h$. 1.0

Therefore, $h_A = H - (0)\Delta h = 5 \text{ m}$ 1.0

$$h_B = H - (1)\Delta h = 5 - 1 \times \frac{5}{6} = 4.17 \text{ m} \quad 1.0$$

$$h_C = H - (4)\Delta h = 5 - 4 \times \frac{5}{6} = 1.67 \text{ m} \quad 1.0$$

$$h_D = 0 \text{ m} \quad 1.0$$

Choosing the soil surface line AD as the datum, the pore pressure at a given point P can be calculated as

$$u_P = \gamma_W(h_P - z_P) \quad 1.0$$

The elevation head and pore pressure at Point A, B, C, and D:

$$z_A = 0 \text{ m}, u_A = \gamma_W(5 - 0) = 10 \times 5 = 50 \text{ kPa} \quad 1.0$$

$$z_B = -H_{BC} = -2.4 \text{ m}, u_B = \gamma_W(4.17 + 2.4) = 10 \times 6.57 = 65.7 \text{ kPa} \quad 1.0$$

$$z_C = 0 \text{ m}, u_B = \gamma_W(1.67 - 0) = 10 \times 1.67 = 16.7 \text{ kPa} \quad 1.0$$

$$z_D = 0 \text{ m}, u_B = \gamma_W(0 - 0) = 0 \text{ kPa} \quad 1.0$$

1.4

The total vertical stress at Point B:

$$\sigma_{v,B} = \gamma_{sat}H_{BC} + \gamma_W H = 20.3 \times \underline{2.4} + 10 \times \underline{5} = 98.72 \text{ kPa}$$

2.0
1.0
1.0
2.0

The effective vertical stress:

$$\sigma'_{v,B} = \sigma_{v,B} - u_B = 98.72 - 65.7 = 33.02 \text{ kPa}$$

2.0
2.0

Exercise 2 [25pt]

The table below shows the normal force N and shear forces measured at the peak T_f (failure) and the ultimate state T_{ult} from direct shear tests. The tests are performed on specimens of undisturbed clays, with a circular section of diameter $d = 60$ mm.

- ?/5 1. Sketch the graphs showing the shear stress-strain response of the soil specimens.
- ?/10 2. Determine the shear strength parameters that can be derived from the available data (round cohesion to the nearest integer). Are the clay specimens normally consolidated, overconsolidated or underconsolidated?
- ?/10 3. If a standard consolidated drained triaxial test (CD) were performed on the same soil, where the deviatoric stress $\Delta\sigma_d$ is 180 kPa, use the ultimate shear strength from the direct shear tests to determine the effective axial and radial stresses, σ'_a and σ'_r , and the angle θ that the failure plane makes with the major principal plane.

Test	N (N)	T_f (N) at 5% strain	T_{ult} (N) at 15% strain
1	138.57	83.74	45.08
2	277.15	139.02	89.21

2.1

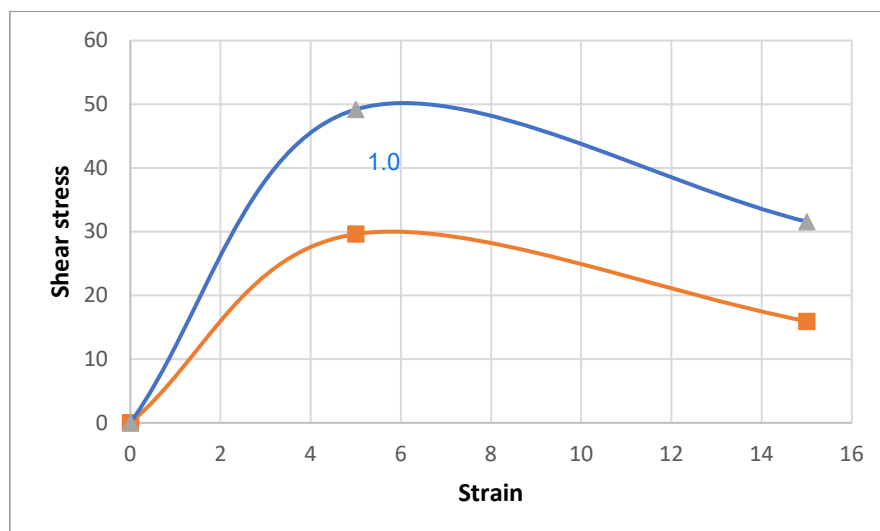
Making use of the area of the circular cross-section of the soil specimen

$$A = \frac{\pi d^2}{4} = 3.14 \times \frac{0.06^2}{4} = 0.002826 \text{ m}^2 \quad 1.0$$

The normal and shear stresses are computed as follows.

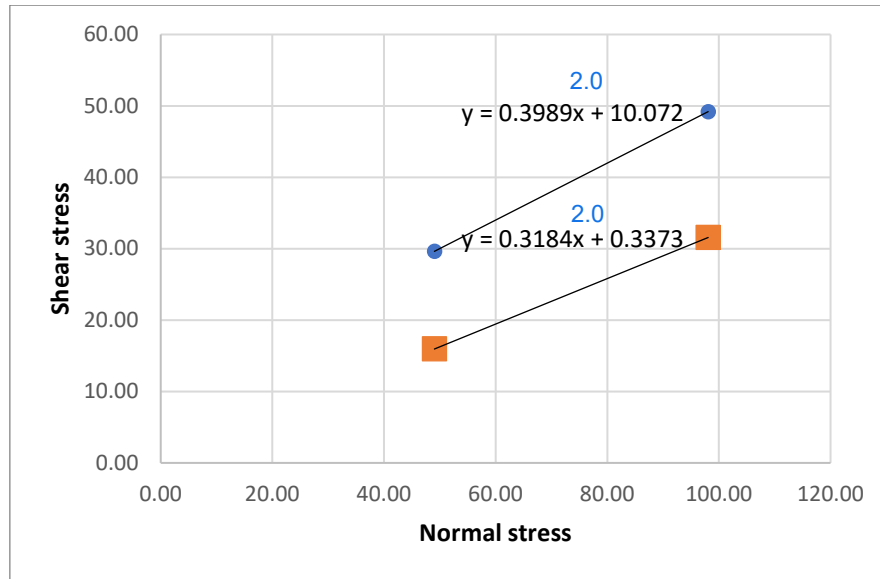
Test	σ (kPa)	τ_f (kPa) at 5% strain	τ_{ult} (kPa) at 15% strain
1	49.03 0.5	29.63 0.5	15.95 0.5
2	98.07 0.5	49.19 0.5	31.57 0.5

A sketch of the stress-strain response can be found below. Because of the peak shear stresses (larger than the residual) is measured, the specimens are overconsolidated. 1.0



2.2

Plotting the normal versus shear stresses in the same figure,



The shear strength parameters for the soil **at failure** are

$$c_f = 10 \text{ kPa} \quad 1.0$$

$$\phi'_f = \tan^{-1} 0.4 = 21.8 \text{ degree} \quad 1.0$$

The ultimate shear strength parameters of the soil

$$c = 0 \quad 1.0$$

$$\phi' = \tan^{-1} 0.32 = 17.7 \text{ degree} \quad 1.0$$

2.3

For consolidated drained (CD) triaxial tests, the major and minor principal stresses are the axial and radial effective stresses, friction angle can be expressed as

$$\phi' = \sin^{-1} \left(\frac{\sigma'_a - \sigma'_r}{\sigma'_a + \sigma'_r} \right) \quad 2.0$$

$$\sigma'_a + \sigma'_r = \frac{\sigma'_a - \sigma'_r}{\sin \phi'} \quad 2.0$$

The deviatoric stress $q = \sigma'_a - \sigma'_r = 180 \text{ kPa}$ 1.0

$$\sigma'_a + \sigma'_r = \frac{180}{\sin(17.7)} = \frac{180}{0.3} = 600 \text{ kPa} \quad 1.0$$

Therefore, $\sigma'_a = 390 \text{ kPa}$ 1.0 and $\sigma'_r = 210 \text{ kPa}$ 1.0

The failure plane will be inclined at an angle of $45 + \frac{\phi'}{2} = 53.85$ degrees to the major principal plane. 1.0 1.0

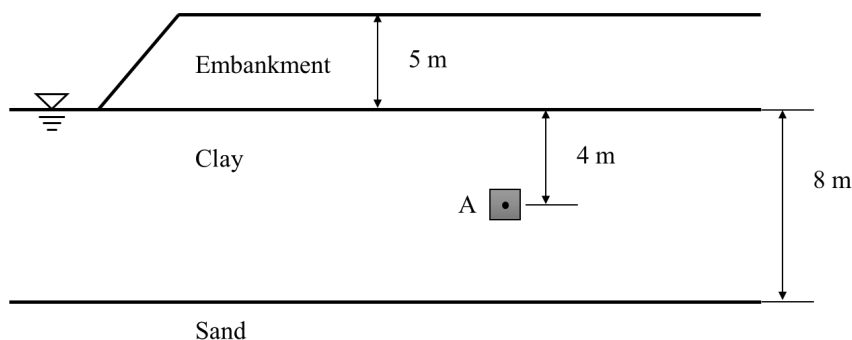
Exercise 3 [40pt]

A long embankment has to be constructed on an 8 m thick clay deposit, as shown in the Figure below. The average unit weight of the clay is $\gamma_{sat} = 20 \text{ kN/m}^3$. Preliminary consolidation tests on a clay sample taken 4 m below the ground surface, provided the following properties of the soil:

- the soil is overconsolidated with $OCR = 2$;
- the initial void index is $e_0 = 0.685$;
- the compression and recompression coefficients are $C_c = 1.3$ and $C_e = 0.05$, respectively.

The new embankment, 5 m in height is to be constructed on the clay layer. The unit weight of the embankment fill material is $\gamma_e = 18 \text{ kN/m}^3$.

- ?/26 1. Determine the settlement of the clay layer due to the embankment. Use the soil properties provided above and assume one-dimensional consolidation conditions.
- ?/14 2. Several years after the construction, it is decided to build a new structure on top of this embankment. Before constructing the new structure, a 2 m thick layer of soil has to be excavated from the surface. Calculate the expected rebound of the ground (in cm) after the excavation.



3.1

The clay sample is taken 4 m below the ground surface. At a depth of 4m, before the embankment is constructed, the total vertical stress at Point A is

$$\sigma_0^A = \gamma_{sat} \times 4 = 20 \text{ kN/m}^3 \times 4 \text{ m} = 80 \text{ kPa},$$

and the effective vertical stress is

$$\sigma_0'^A = \sigma_0^A - u_A = 80 - \gamma_w \times 4 = 40 \text{ kPa}.$$

The construction of the embankment causes additional surcharge load on top of the clay layer,

$$\Delta\sigma = \gamma_e \times 5 = 18 \text{ kN/m}^3 \times 5 \text{ m} = 90 \text{ kPa}.$$

The final total vertical stress becomes

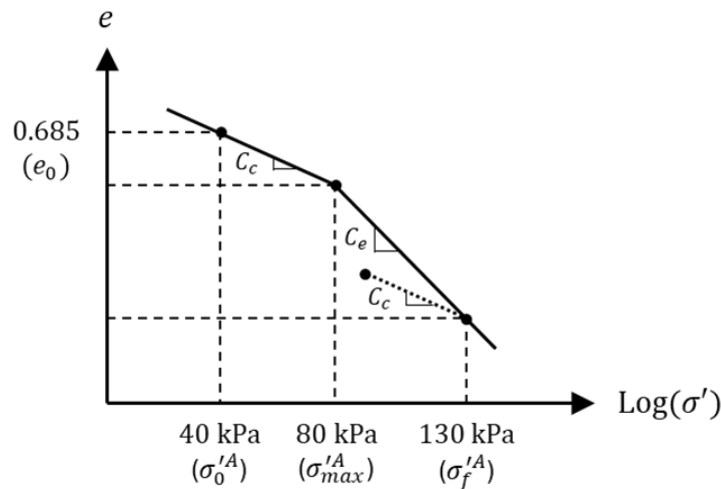
$$\sigma_f'^A = \sigma_0'^A + \Delta\sigma = 40 + 90 = 130 \text{ kPa}.$$

The clay is overconsolidated with $OCR = 2$, which means $\sigma_{max}'^A = 2\sigma_0'^A = 80 \text{ kPa}$.

1.0

1.0

Because $OCR > 1$ and $\sigma_f'^A > \sigma_{max}'$, the change of void index has to be calculated from first the decompression-recompression line and then the virgin compression line, as shown in the figure below.



2.0

2.0

$$\begin{aligned}\Delta e &= -C_e \log\left(\frac{\sigma_{max}'^A}{\sigma_0'^A}\right) - C_c \log\left(\frac{\sigma_f'^A}{\sigma_{max}'^A}\right) \\ &= -0.05 \times \log\left(\frac{80}{40}\right) - 1.3 \times \log\left(\frac{130}{80}\right) \\ &= -0.29\end{aligned}$$

2.0

The total strain within the clay layer

$$\varepsilon_a = -\frac{\Delta e}{1 + e_0} = -\frac{0.29}{1 + 0.685} = 0.17$$

2.0

2.0

The settlement of the clay layer due to the embankment

$$S = \varepsilon_a H = 0.17 \times 8 = 1.36 \text{ m}$$

2.0

2.0

3.2

Several years after the construction, the consolidation process has completed. The clay layer reaches a new height

$$H = 8 - 1.36 = 6.64 \text{ m}$$

1.0

and a new value for the void index

$$e_0 = 0.685 - 0.29 = 0.395$$

1.0

The vertical stress that gets removed due to the 2-meter excavation of the embankment is,

$$\Delta\sigma = -18 \text{ kN/m}^3 \times 2 \text{ m} = -36 \text{ kPa.}$$

2.0

Therefore, the new vertical stress on the clay layer becomes

$$\widehat{\sigma_f'^A} = \sigma_f'^A + \Delta\sigma = 130 - 36 = 94 \text{ kPa,}$$

2.0

1.0

The change of void index has to be calculated from the decompression-recompression line using the recompression coefficient $C_e = 0.05$, as shown in the figure above.

1.0

1.0

$$\Delta e = -C_e \log\left(\frac{\widehat{\sigma'_f{}^A}}{\sigma'_f{}^A}\right) = -0.05 \times \log\left(\frac{94}{130}\right) = 0.007$$

2.0 1.0

Using the new height and the new void index for the clay layer, the total strain is

$$\varepsilon_a = -\frac{\Delta e}{1 + e_0} = -\frac{0.007}{1 + 0.395} = -0.005$$

1.0

The settlement of the clay layer due to the embankment

$$S = \varepsilon_a H = -0.005 \times 6.64 = -0.0332 \times 100 = -3.32 \text{ cm.}$$

1.0

Therefore, the expected rebound of the ground after the excavation is 3.32 cm.