

Tag : Toetsen/21-22/I2MC1A.21-22[v4].Resit
Course : **Intro to Math + Calculus 1A**
Date : Friday November 12th, 2021
Time : 13:45 – 16:45

Motivate all your answers.

The use of electronic devices is not allowed.

The answer form

Use the answer form to write down your answers. Clearly fill out your name, student number and study programme. Any text outside a frame will be ignored.

Question types

Final answer

On the answer sheet, in the corresponding text frame you provide only one answer. Do not write down a calculation, explanation or motivation. If you do write down a calculation, explanation or motivation, it will not be taken into account for grading.

Open answer

You provide a full calculation or motivation in the text frame corresponding to the question. The calculation or motivation will be graded.

Extra writing space

If you need more space, you can write in the frame provided at the end of the answer form. Clearly refer to this space in the original answer.

1. [2 pt] *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Determine all subsets of $\{0, 4, \{0, 4\}\}$.

2. [2 pt] *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet.*

Describe each of the following statements using only the symbols

$A, B, \overline{A}, \overline{B}, \subseteq, \subset, \cup, \cap, =, \emptyset$.

(You can use one symbol more than once, and you don't have to use all symbols.)

(i) $\forall x : x \notin A$

(ii) $(\forall x : x \in A \rightarrow x \in B) \wedge (\exists x : x \in B \wedge x \notin A)$

3. [3 pt] Let $a \in \mathbb{Z}$. Prove the following statement:

$a^2 + a$ is even.

4. [4 pt] Use mathematical induction on n to prove that for all $n \in \mathbb{N}$,

$n^3 + 2n$ is divisible by 3.

5. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet.*

(a) [1.5 pt] Use Newton's binomial theorem to find the coefficient of x^2y^8 in $(x + y)^{10}$ and evaluate this coefficient.

(b) [1.5 pt] Use Newton's binomial theorem to find the coefficient of x^2y^8 in $(2x + 3y)^{10}$. You don't need to evaluate it. (For example, if the coefficient were $3! + 4!$, then $3! + 4!$ is a correct answer.)

6. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

(a) [2 pt] Given the vectors $\mathbf{u} = \langle -2, 1, 1 \rangle$ and $\mathbf{v} = \langle 2, -1, 2 \rangle$. Determine the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$.

(b) [2 pt] For which $\alpha \in \mathbb{R}$ is it true that

$$|\mathbf{v} \times \mathbf{u}| = |\mathbf{u}||\mathbf{v}|$$

if $\mathbf{u} = \langle -2, \alpha, -1 \rangle$ and $\mathbf{v} = \langle 5, -3, -1 \rangle$?

7. *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

(a) [2 pt] Calculate the distance from the point $P = (2, 2, -2)$ to the line through the origin and the point $Q = (-1, 0, 1)$.

(b) [2 pt] Find an equation for the plane through P , Q and the origin. Simplify the equation as much as possible.

8.
7. Define

$$f(x) = \begin{cases} \frac{1}{x} \sin(3x) & x < 0 \\ 3 & x = 0 \\ 2x + 3^x & x > 0 \end{cases}$$

(a) [2 pt] Does $\lim_{x \rightarrow 0} f(x)$ exist? Explain your reasoning.

(b) [1 pt] Is f continuous at $x = 0$? Explain your reasoning.

9

8. [2 pt] *This is a final answer question. Provide your final answer (and only your final answer) on the answer sheet. Do not use this sheet.*

Define the function

$$f(x) = 3\sqrt[3]{16 - x^2}.$$

Find all critical points of f within its domain.

10

9. [3 pt] Evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{-6 + 6 \cos(x^8)}{x^{16}}.$$

11

10. [3 pt] Let

$$f(x, y) = \frac{\sqrt{xy}\sqrt{x^2 + y^2}}{(x + y)^2 + (x - y)^2}.$$

Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist?

12

11. [3 pt] Find an equation for the tangent plane to the graph of the function

$$f(x, y) = y^3 e^{2xy}$$

at the point $(0, -1, f(0, -1))$. Simplify the equation as much as possible.