

Exam: Vector Calculus for BMT, CE, CSE, ME

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Provide suitable explanations of your answers!

The use of electronic devices is *not* allowed. A formula sheet is not handed out.

Question 1. Let $a > 0$ be a constant. Consider the curve C defined by the portion of the intersection of the cylinder $x^2 + y^2 = a^2$ and the plane $z = x$ that lies in the first octant.

- (a) Find a parametrization of C .
- (b) Calculate the mass of a wire shaped along C with mass density

$$\rho(x, y, z) = xy.$$

Question 2.

- (a) Let C be the straight line from $(0, 1, 1)$ to $(2, 1, 2)$. Calculate the value of

$$\int_C 2xe^{(z-1)^2} dx + 2y^2z dy + y^3 dz.$$

- (b) Is it possible to compute the integral in (a) using a suitable potential function?
(Do not forget to justify your answer)

Question 3. Evaluate a double integral to calculate the counterclockwise circulation of the vector field $\mathbf{F}(x, y) = y^2\mathbf{i}$ around the boundary of the triangle with vertices $(0, 0)$, $(2, 0)$ and $(0, 1)$.

Question 4. Let D be the solid cone $0 \leq z \leq 1 - \sqrt{x^2 + y^2}$. Its boundary S is composed of a conical surface A given by $z = 1 - \sqrt{x^2 + y^2}$ for $z \geq 0$ and the unit disk B in the xy -plane. Consider the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} + (z - 1)^2\mathbf{k}$.

- (a) Make a clear sketch of D .
- (b) Use a parametrization of A to calculate the flux of $\mathbf{F}$ across A in the direction leaving D .
- (c) Use a triple integral to calculate the flux of $\mathbf{F}$ across S in the direction leaving D .
- (d) Denote by b the result of part (b) and by c the result of part (c). Express the flux of $\mathbf{F}$ across B in the direction *entering* D in terms of b and c .

Points:

Q1.	(a) 1	Q2.	(a) 2.5	Q3.	2.5	Q4.	(a) 1
	(b) 3		(b) 1.5				(b) 4
							(c) 2
							(d) 0.5

Total: $18 + 2 = 20$ points