

Course : Calculus 2

Module : 2B

Course code : 2020012::[19|20|21|22]

Date : Friday 3 July, 2026

Time : 13:45 – 15:45 (2 hours)

Reference : CAL2-2526-Q4-RESIT

Calculus 2

Resit

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- For exercises 1–6, you are only required to fill in the **final answer** on the answer form.
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Report all your answers on the answer form.

1. A function $f(x, y)$ has gradient $\nabla f(x, y) = \langle e^y, xe^y \rangle$. Find the directions $\mathbf{u}$ for which [3 pt]

$$D_{\mathbf{u}}f(-3, 0) = 1.$$

Note: There are two such direction vectors.

2. Let $f(x, y, z)$ be a differentiable function. For every integer number a , the partial derivatives of $f(x, y, z)$ evaluated at the point $(a, 2a, 0)$ compute to [3 pt]

$$f_x(a, 2a, 0) = 2\pi a, \quad f_y(a, 2a, 0) = 3a, \quad f_z(a, 2a, 0) = -4a.$$

The variables x , y and z are functions of s and t given by:

$$x(s, t) = \ln(s) + e^{2t}, \quad y(s, t) = t^2 + 2, \quad z(s, t) = \sin(\pi s) + \cos\left(\frac{\pi}{2}(t + 1)\right).$$

(a) Determine the point (x, y, z) at $(s, t) = (1, 0)$.

(b) Compute the partial derivative $\frac{\partial f}{\partial s}$ at $(s, t) = (1, 0)$.

3. For each of the following statements, determine whether it is **True** or **False**. [2 pt]

(a) $\mathbf{r}(t) = \langle 2\cos(t) - 2, 2\sin(t) + 3 \rangle$ for $0 \leq t \leq 2\pi$ is a parametrization of a circle centered at $(2, -3)$ with radius 2.

(b) Let L be the level curve of $f(x, y)$ containing the point (a, b) , and let $\mathbf{u}$ be a unit vector that is tangent to the level curve at (a, b) . Then the directional derivative of f at (a, b) in the direction of $\mathbf{u}$ is zero.

(c) Let $f(x, y)$ be a differentiable function. If $f_y(a, b) > 0$, then $f_{yy}(a, b) > 0$.

Continues on the following page.

4. The integral

$$I = \int_{-1}^1 \int_0^1 \int_{x^2}^1 (7z + 1) \, dy \, dz \, dx$$

is the triple integral of the function $f(x, y) = 7z + 1$ over some solid region E .

(a) Rewrite I as an equivalent iterated integral, using the order of integration $dx \, dz \, dy$. [2 pt]

(b) Knowing that $I = 6$ and $\iiint_E 7z \, dV = \frac{14}{3}$, compute the volume of E . [1 pt]

5. Consider the transformation of the plane given by: [3 pt]

$$\begin{cases} x = 4u + 4v \\ y = -3u + cv \end{cases}$$

where c is a constant. Find all the values of c such that the transformation turns a region G of area 1 of the (u, v) -plane into a region of area 16 of the Cartesian (x, y) -plane.

6. A straight thin rod L of mass $M = 2$ is positioned on the interval $[-1, 1]$ of the x -axis. Its [2 pt]
density is given by the function $\delta(x) = \frac{2x}{7} + 1$. Find the center of mass $\bar{x}$ of L .

Open questions

The full solutions to exercises 7—10 must be clearly written down on the answer form, including calculations and argumentations.

Points will not be awarded for reaching a correct result if this is not supported by a correct procedure and by a sound and clear argumentation.

7. The function [5 pt]

$$f(x, y) = xy - \frac{5}{2} \ln(x^2 + y^2 + 1)$$

has three critical points. One of them is $(0, 0)$.

(a) Determine the other critical points.

(b) Determine whether $(0, 0)$ is a saddle point for f , or f attains a local maximum at it, or f attains a local minimum at it. (You don't need to use the fact that $f(0, 0) = 0$).

8. Use the method of Lagrange multipliers to find the maximum and minimum values attained [5 pt]
by the function

$$f(x, y) = x^2 - \ln(2y + 3)$$

on the circle of equation $x^2 + y^2 = 1$. Clearly specify at which points these values are attained.

For the final computation, you may need to know that $\frac{3}{4} > \ln 2$.

9. The solid region D lies inside the cylinder $x^2 + (y - 1)^2 = 1$; it is bounded below by the plane [5 pt]
 $z = 0$, and above by the surface $z = 3 + xy$. Determine the volume of D .

10. The solid region D , described in spherical coordinates, lies above the cone $z = \frac{\sqrt{x^2 + y^2}}{\sqrt{3}}$ and [5 pt]
below the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

(a) Rewrite the equations for the given cone and sphere in spherical coordinates. Simplify [2 pt]
your equations as much as possible.

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Total: 36 pt

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5. Consider the transformation of the plane given by: [3 pt]

$$\begin{cases} x = 2u + 5v \\ y = -u + cv \end{cases}$$

where c is a constant. Find all the values of c such that the transformation turns a region G of area 1 of the (u, v) -plane into a region of area 9 of the Cartesian (x, y) -plane.

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Total: 36 pt

1	2	3	4	5	6	7	8	9	10	Σ

Do not fill in this table.

D

Only for sorting.

Answer form

Calculus 2 – Resit – Friday 3 July, 2026

Full name in BLOCK LETTERS _____

Student number _____

Programme ☐ BMT ☐ CE ☐ CSE ☐ IEM ☐ Other: _____

⚠ FILL IN YOUR DATA AS SOON AS YOU RECEIVE THE EXAM ⚠

Question 1. *The direction vectors are:*

Question 2(a). *The point (x, y, z) at $(s, t) = (1, 0)$ is:*

Question 2(b). *$\frac{\partial f}{\partial s}$ at $(s, t) = (1, 0)$:*

Question 3. *Write "TRUE" or "FALSE".*

Points are awarded only if all three answers are correct.

(a) is _____ (b) is _____ (c) is _____

Question 4(a). *Rewrite the integral using the order of integration $dx dz dy$:*

Question 4(b). *The volume of E is:*

Question 5. *The values of c are:*

Question 6. $\bar{x} =$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]